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PERIODIC SUCTION ON CONVECTIVE HEAT TRANSFER PAST A SEMI-INFINITE VERTICAL POROUS WALL: COMPUTATION AL STUDY

ANIL KUMAR

Department of Mathematics and Statistics
Swami Vivekanand Subharti University
Meerut, UP, India
E-mail: dranilkumar73@rediffmail.com

Abstract

The unstable two-dimensional laminar flow of a viscous incompressible electrically directed fluid via a semi-infinite vertical permeable moving plate with periodic suction has been explored in this paper in the presence of a transverse magnetic field. The mathematical expressions for dimensionless velocity profile, temperature, and skin friction at the wall have been acquired and numerically worked out for various estimations of the parameters engaged with the solution. A uniform magnetic field acts opposite to the permeable plate and there is a suction on the plate changing intermittently with time. The induced magnetic field is ignored. For various estimations of the parameters involved with the solution, mathematical equations for dimensionless velocity profile, temperature, and skin friction at the wall have been acquired and numerically worked out. A uniform magnetic field changes the direction of the permeable plate, generating a suction that varies over time. The induced magnetic field is not properly considered. The temperature increases significantly in the boundary layer. The influence of suction becomes more noticeable as the estimation of the combined Prandtl number increases. Surface heat transfer is reduced by extending the size of suction velocity for various Prandtl numbers. As a generalization of the problems, the study could be extended to a suction speed $f(t)$.

1. Introduction

Due to the immense importance and continuous interest of scientists and mathematicians in many engineering, technological, and medical disciplines, convective flows across porous plate have become quite popular among

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research workers in this area. During the last two decades, magneto-hydrodynamic (MHD) flows have received a lot of attention. The important applications of MHD have been reported, relating to the MHD generators, MHD flow- meter, MHD pump, nuclear reactors and MHD marine propulsion. A solution to the problem of transient heat transfer from a sphere at creeping flow $(\mathrm{Re} = 0)$ appears in Carslow and Jaeger [9]. Acrivos and Taylor [2] investigated a singular perturbation scheme for a Stokesion velocity profile around the sphere, to derive their well-known solution for the steady-state heat transfer from a sphere at small but finite Peclet numbers. A very useful proposed application which involves MHD is the lithium cooling blanket in a nuclear fusion reactor was made by Branover [8]. With the high-temperature plasma contained in the reactor using a tropical magnetic field, liquid lithium flows in channels (blankets) between the plasma and magnetic windings to absorb the thermal energy released by the fusion reaction. Singh et al. [17] investigated a mathematical analysis of wall shear stress and heat transfer of the fluid flow affected by the periodic suction velocity opposite to the flow side. Acrivos [3] obtained a solution for $\mathrm{Pe} > 5$ with the Reynolds number of the flow being very small. Abramzon and Elata [1] conducted a numerical study for the transient heat transfer from a rigid sphere at a wide range of Peclet numbers. Al- Khawaja et al. [4] studied a laminar, steady and fully developed flow with the constant vertical magnetic field applied to electrically conducting flow in an electrically insulated magnetic-fluid-mechanic (MFM) pipe. Al-Nimr et al. [6], [7] have researched numerically, the convection in the passageway locale of either a container of an annulus, when a period - savvy step change of divider temperature is forced, for Darcy and non- Darcy models. An exploratory investigation of warmth move from a chamber inserted in a bed of circular particles, with cross-progression of air that has been made by Nasr et al. [16]. Shenoy [15] has reviewed studies of flow in non-Newtonian fluids in porous media, with attention, concentrated on Power-law fluids. Unsteady forced convection produced by small-amplitude variations in the wall temperature and free stream velocity along a flat plate has been studied by Hossain et al. [13] Michaelides and Feng [14] observed that the numerical solution agrees very well with the asymptotic steady-state solutions while studying the analogies between the transient momentum and energy equations of the particles. The models for stagnant thermal conductivity have been developed by Cheng and Hsu [10]. Feng and Michaelides [11] considered transient transfer from a molecule with subjective shape movement.

Al-Khawaja et al. [5] made the numerical study of magnetic - fluid-mechanic (MFM) pipe flow with constant heat flux at the wall with combined free and forced convection heat transfer. Feng et al. [12] studied numerically the transient heat transfer from a sphere at high Reynolds and Peclet numbers. Kim (2000) has considered the case of a semi-infinite moving porous plate in a porous medium in the presence of pressure gradient and constant velocity in the flow direction when the magnetic field is imposed transverse to the plate. Mann et al. [18] investigated the effects of radiation on unsteady MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium with an oscillatory heat flux in the presence of a uniform transverse magnetic field. Vyas et al. [19] investigated abdominal entropy generation of radiative micropolar liquid flow in permeable medium and they are utilized the FDM technique. Barnoonet et al. [21] examined of investigation of various nanofluid flows in the space between two concentric flat channels within the sight of the magnetic field. Bulinda et al. [22] made an MHD free convection flow of incompressible liquids over a layered vibrating surface with corridor ebbs and flows and heat and mass exchanges. For the study of the Darcian porous MHD flow, it is necessary to consider in detail the distribution of velocity and temperature distribution across the boundary layer with the moving wall. The present study investigates the effect of periodic suction on the flow through the whole analysis applies to general suction velocity as a function of time. 2. Mathematical model: In the present study, the two-dimensional unsteady flow of laminar, incompressible fluid past a semi-infinite vertical porous moving plate having periodic suction velocity with transverse Magnetic hydrodynamic field is studied. The transverse applied magnetic field and Reynolds number are quite small so that the induced magnetic field is negligible. It is assumed that there is no applied voltage and this implies the absence of an electric field. The viscous and Darcy's resistance terms are taken into account with constant permeability of the porous medium. The effect of Lorentz force is taken into consideration with an order of magnitude analysis of the full Navier-Stokes equations. Agrawal and Anil Kumar [22] studied an effect of viscous dissipation on MHD unsteady flow though vertical porous medium with constant suction and their applications. Kumar et al. [23] investigated finite difference technique for reliable MHD steady flow though channels permeable boundaries and their applications.

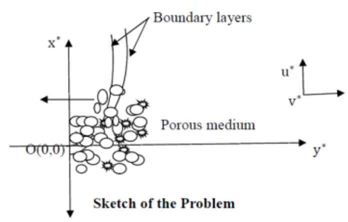

Figure 1. Geometrical description of the mathematical model.

The continuity, momentum, and energy equations governing the fluid motion are:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + v\frac{\partial^2 u^*}{\partial {y^*}^2} + g\beta(T - T_\infty) - v\frac{u^*}{K^*} - \frac{\sigma}{\rho}B_0^2 u^* \tag{2}$$

$$\frac{\partial T}{\partial t^*} v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial {y^*}^2} \tag{3}$$

where x^* and y^* are the dimensional separations along and opposite to the plate, respectively. ρ is the density, K^* is the permeability of the porous medium, ν is kinematic viscosity, α is the fluid thermal diffusivity.

The boundary conditions are:

$$u^* = u_p^*,\ T = T_w + \varepsilon(T_w - T_\infty)e^{i\omega^* t^*}\ \ at\ y^* = 0 \tag{4}$$

$$u^* \to U_\infty^* = U_0(1 + \varepsilon e^{i\omega^* i^*}), T \to T_\infty\ \ as\ y^* \to \infty \tag{5}$$

As is evident from (1), the suction velocity is the function of time only, we assume

$$v^* = V_0(1 + A\varepsilon e^{i\omega^* t^*}), \tag{6}$$

where A is a real positive constant, ε is small less than unity, V_0 is the suction velocity. Outside the boundary layer, from equation (2) we have

$$-\frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{dU_\infty^*}{dt^*} + \frac{\upsilon}{K^*}U_\infty^* + \frac{\sigma}{\rho}B_0^2 U_\infty^* \tag{7}$$

Let us introduce the following non-dimensional parameters

$$u = \frac{u^*}{U_0}, v = \frac{v^*}{V_0}, y = \frac{V_0 y^*}{\upsilon}, U_\infty \frac{U_\infty^*}{U_0}, U_p = \frac{u_p^*}{U_0}, t = \frac{t^* V_0}{\upsilon}, \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

$$\omega = \frac{\omega^* \upsilon}{V_0^2}, K = \frac{K^* V_0^2}{\upsilon^2}, \mathrm{Pr} = \frac{\upsilon \rho C_p}{k} = \frac{\upsilon}{\alpha}, M = \frac{\sigma B_0^2 \upsilon}{\rho V_0^2}, G = \frac{\upsilon \beta g (T_w - T_\infty)}{U_0 V_0^2}$$

The governing equations (2) and (3) using equations (6)-(8), in the non-dimensional form are as follows:

$$\frac{\partial u}{\partial t}(1 + \varepsilon A e^{i\omega t})\frac{\partial u}{\partial t} = \frac{dU_\infty}{dy} + \frac{\partial^2 u}{\partial y^2} + G\theta + S(U_\infty - u) \tag{9}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} \tag{10}$$

Where $S = \left(M + \frac{1}{K}\right)$

The corresponding boundary conditions are:

$$u = U_p, \; \theta = 1 + \varepsilon e^{i\omega t} \quad \text{at } y = 0, \tag{11}$$

$$u \to U_\infty, \; \theta \to 0 \quad \text{as } y = \infty. \tag{12}$$

3. Numerical solution of the Problem

We solve the equations (9) and (10), by using perturbation techniques. We assume that the velocity profile and temperature profile is given by

$$u = f_0(y) + \varepsilon e^{i\omega t} f_1(y) + O(\varepsilon^2) + \ldots \tag{13}$$

$$\theta = g_0(y) + \varepsilon e^{i\omega t} g_1(y) + O(\varepsilon^2) + \ldots \tag{14}$$

Substituting these equations (13) and (14) into the equations (9) and (10) and equating the harmonic and non-harmonic oscillating terms, neglecting the higher coefficient of $O(\varepsilon^2)$, we have

$$\frac{\partial^2 f_0}{\partial y^2} + \frac{\partial f_0}{\partial y} - Sf_0 = -S - Gg_0 \tag{15}$$

$$\frac{\partial^2 f_1}{\partial y^2} + \frac{\partial f_1}{\partial y} - (S + \omega) f_1 = -(S + \omega) - A\frac{\partial f_0}{\partial y} - Gg_1 \tag{16}$$

$$\frac{\partial^2 g_0}{\partial y^2} + \Pr\frac{\partial g_0}{\partial y} = 0 \tag{17}$$

$$\frac{\partial^2 g_1}{\partial y^2} + \Pr\frac{\partial g_1}{\partial y} - \omega \Pr g_1 = -A\Pr\frac{\partial g_0}{\partial y} \tag{18}$$

The appropriate boundary condition is given by

$$f_0 = U_p,\ f_1 = 0,\ g_0 = 1,\ g_1 = 1, \quad \text{at } y = 0 \tag{19}$$

$$f_0 = 1,\ f_1 = 1,\ g_0 \to 0,\ g_1 \to 0 \quad \text{as } y \to \infty \tag{20}$$

From solving equations (15)-(18) and satisfying boundary conditions (19) and (20) are given by

$$f_0(y) = 1 + (U_p + Q - 1)e^{-h_1 y} - Qe^{-\Pr y}, \tag{21}$$

$$f_1(y) = 1 + Ce^{-h_1 y}(1 + C + D + E)e^{-h_2 y} + De^{-h_3 y} + Ee^{-\Pr y}, \tag{22}$$

$$g_0(y) = e^{-\Pr y} \tag{23}$$

$$g_1(y) = e^{-h_3 y} + \frac{A}{\omega}\Pr(e^{-h_3} y - e^{-\Pr y}), \tag{24}$$

where $h_1 = \frac{1}{2}(1 + \sqrt{1 + 4S}),\ h_2 = \frac{1}{2}[1 + \sqrt{1 + 4(S + \omega)}]$,

$$h_3 = \frac{\Pr}{2}\left(1 + \sqrt{1 + \frac{4\omega}{\Pr}}\right),\ C = \frac{Ah_1}{\omega}(U_p + Q - 1),$$

$$D = -G\left(1 + \frac{A\,\mathrm{Pr}}{\omega}\right)\frac{1}{h_3^2 - h_3 - (S - \omega)},\ E = \frac{AG\,\mathrm{Pr}}{\omega}\frac{1}{\mathrm{Pr}^2 - \mathrm{Pr} - S}$$

$$Q = \frac{G}{\mathrm{Pr}^2 - \mathrm{Pr} - S}$$

Substituting the above values in (13) and (14) we get the velocity profile and temperature profile are given by

$$u(y, t) = 1 + (U_p Q - 1)e^{-h_1 y} - Qe^{-\mathrm{Pr}\,y} + \varepsilon(\cos\omega t + i\sin\omega t)$$

$$\{1 + Ce^{-h_1 y} - (1 + C + D + E)e^{-h_2 y} + De^{-h3y} + Ee^{-\mathrm{Pr}\,y}\} \tag{25}$$

$$\theta(y, t) = e^{-\mathrm{Pr}\,y}\varepsilon(\cos\omega t + i\sin\omega t)\left\{e^{-h_3 y} + \frac{A}{\omega}\mathrm{Pr}\left(e^{-h_3 y} - e^{-\mathrm{Pr}\,y}\right)\right\} \tag{26}$$

The real parts represent the velocity u and θ in equation (25) and (26).

The skin friction (τ_w) at the wall of $y = 0$ is given by

$$\tau_w = \frac{\tau_w^*}{\rho U_0 V_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= -h_1(U_p + Q - 1) + Q\,\mathrm{Pr} + \varepsilon(\cos\omega t + i\sin\omega t)$$

$$\{h_2(1 + C + D + E) - Ch_1 - Dh_3 - E\,\mathrm{Pr}\}$$

The heat transfer coefficient at the wall in terms of Nusselt number is given by

$$Nu = x\frac{(\partial T/\partial y^*)_w}{T_w - T_\infty}$$

$$NuR_x^{-1} = \left.\frac{\partial\theta}{\partial y}\right|_{y=0}$$

$$= -\mathrm{Pr} + \varepsilon(\cos\omega t + i\sin\omega t)\left[\frac{A}{\omega}\mathrm{Pr}^2 - h_3\left(1 + \frac{A}{\omega}\mathrm{Pr}\right)\right],$$

where $\mathrm{Re}_x = \dfrac{V_0 x}{v}$ is the Reynolds number.

$$\therefore |\, Nu\, R_{ex}^{-1} \,| = [\{-\Pr + M_1 \in \cos \omega t\}^2 + \{M_1 \in \sin \omega t\}^2]^{1/2}$$

$$= [P^2 r^2 - 2\Pr \in M_1 \cos \omega t + M_1^2 \in^2]^{1/2}$$

where $M_1 = \frac{A}{\omega}\Pr^2 - h_3\left(1 + \frac{A}{\omega}\Pr\right).$

4. Results and Discussion

The numerical computations for the velocity profile and temperature profile for different values of the material parameters have been an investigation. Figure 2 shows the velocity profiles for different values of the index number (ω). It is clear that an increase in the values of ω leads to a decrease in the velocity distribution along with the boundary layer. Figure 3 explains the variation of temperature function with spanwise coordinate ω. for different values of ω. For various values of the magnetic field parameter (M), the velocity profiles are depicted in figure 4. The increase of the magnetic field decreases the velocity profile.

Figure 5 shows the velocity distribution for several values of the permeability parameter (k) The velocity profiles for several values of Grashof number (natural convection) are shown in Figure 6, Figure 7 depicts the temperature profiles with spanwise co-ordinates (y) for different values of the Prandtl number. Figure 8 displays the velocity profiles for several values of the Prandtl number (Pr). In the beginning, the velocity increases with an increase in y but settles down later to constant free stream velocity.

Figure 9 depicts the variation of the velocity profiles along with the boundary layer for different values of moving velocity of the plate in the direction of fluid flow. Figure (10) depicts the surface skin friction decreases by increasing the plate moving velocity (Up).

Figure 11 displays the skin friction on the porous plate against the suction velocity parameter. It is clear that for small values of the dimensionless index number (ω), the increment of the surface skin friction is almost negligible.

Figure 12. depicts the variation of surface heat transfer against the

suction velocity parameter (A) for different values of the Prandtl number.

5. Conclusion

Numerical results for free convective laminar flow of an incompressible electrically driving liquid on a planning fluid past a semi-infinite vertical permeable moving plate with intermittent suction inside the nearness of transverse magnetic field have been investigated. The impact of the magnetic field and suction velocity shifting occasionally with time about a non zero reliable mean on the stream and heat transfer of an incompressible fluid along a semi-perpetual vertical porous moving plate have been examined. Doubtlessly the velocity profiles diminish with an extension in the Prandtl number and Hartmann parameter. The twists show that the speed increases compellingly near the different Grashof number augmentations and a short time later reducing to secure a consistent free speed profile. It is demonstrated that development in the parameters reasonable causes a decline in skin friction.

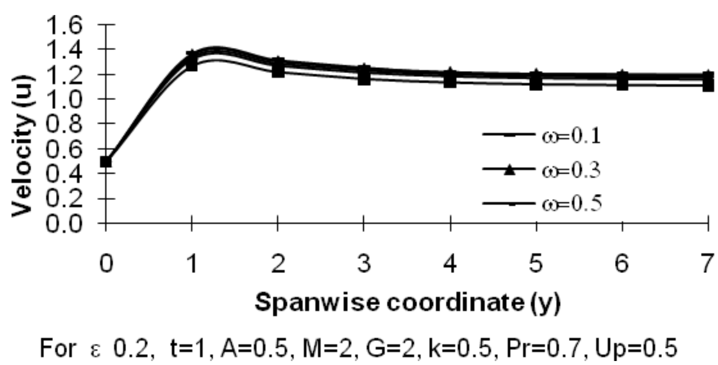

Figure 2. Velocity profiles against spanwise coordinate different values of ω.

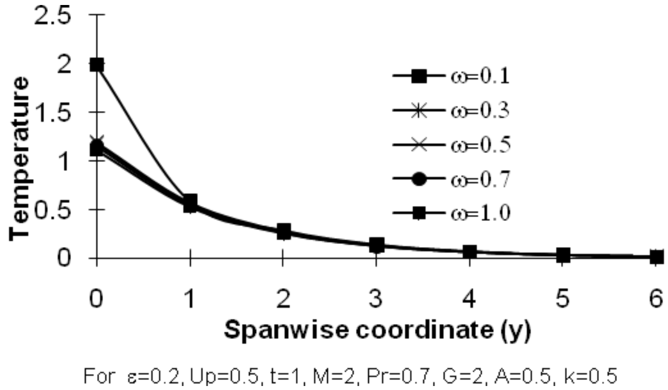

Figure 3. Temperature profiles for different values of ω.

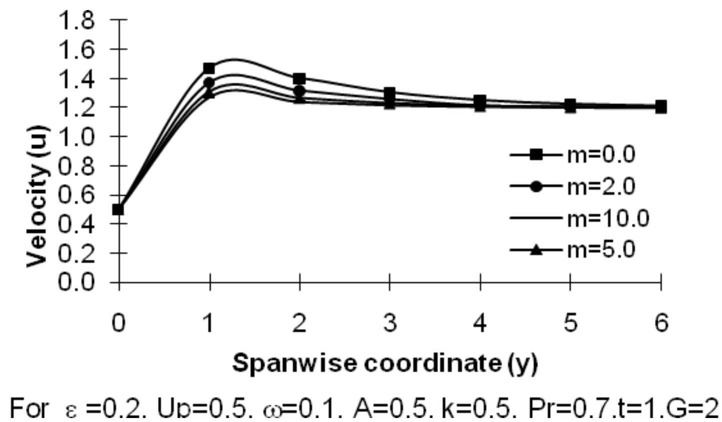

Figure 4. Velocity profiles for different values of *M*.

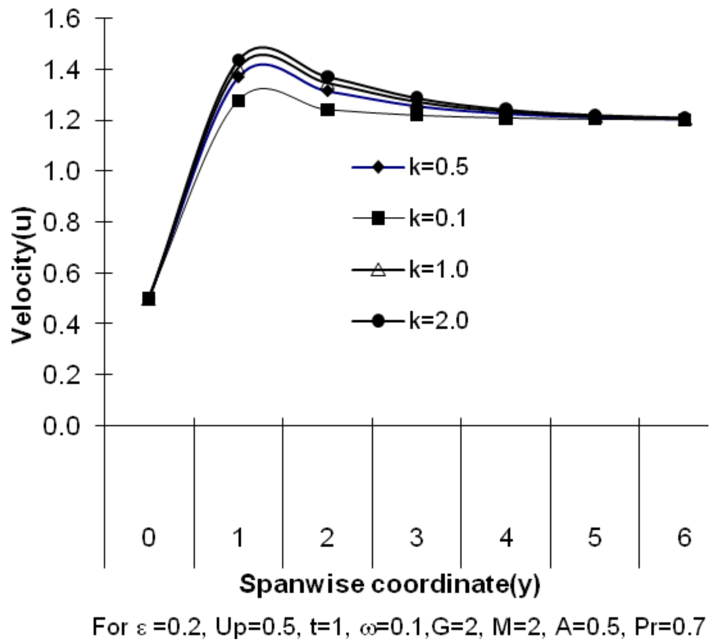

Figure 5. Velocity profiles for different values of K.

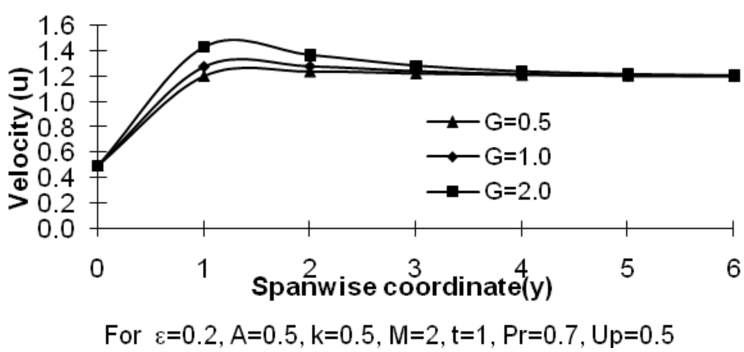

Figure 6. velocity profiles for different values of G.

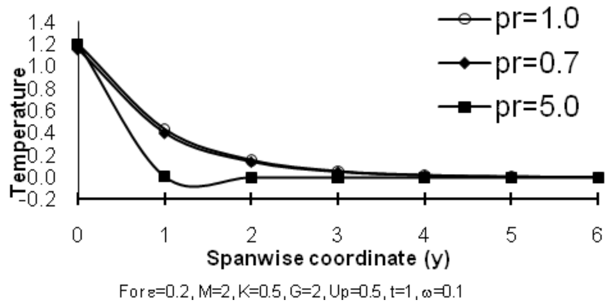

Figure 7. Temperature profiles for different values of Pr.

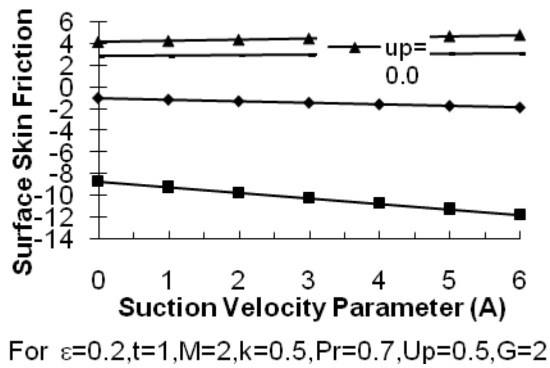

Figure 8. Variation of the surface skin friction with the suction velocity Parameter (A) for different values of Up.

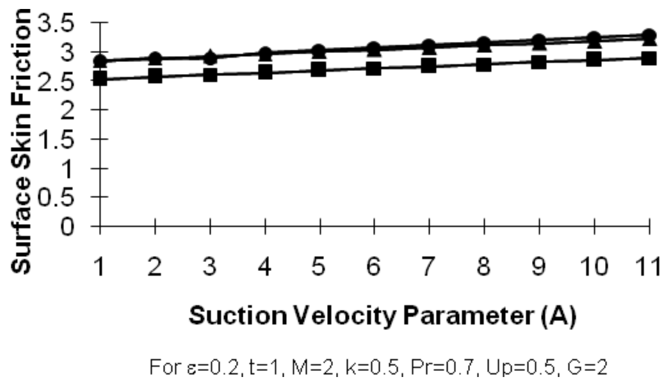

Figure 9. Variation of the surface skin friction with the suction velocity parameter for different values of ω.

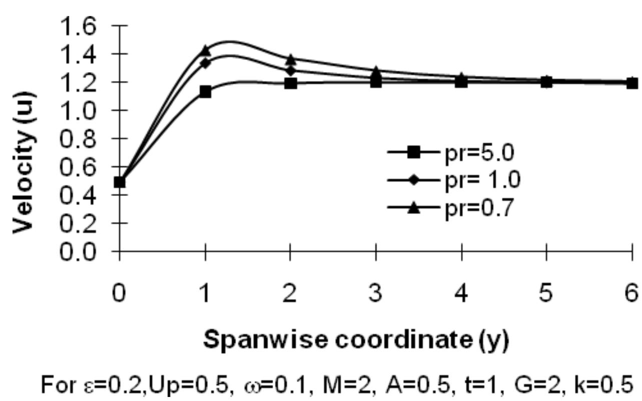

Figure 10. velocity profiles for different values of Pr.

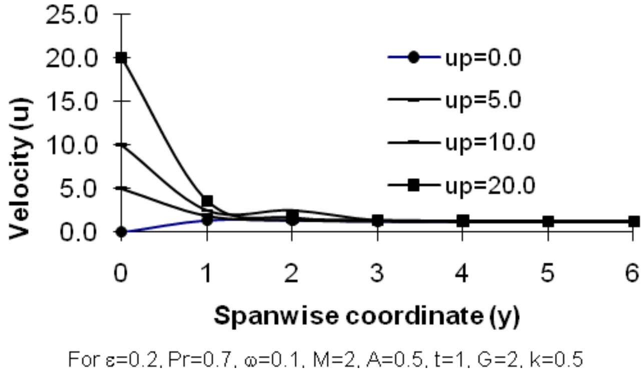

Figure 11. Velocity profiles for different values of Up.

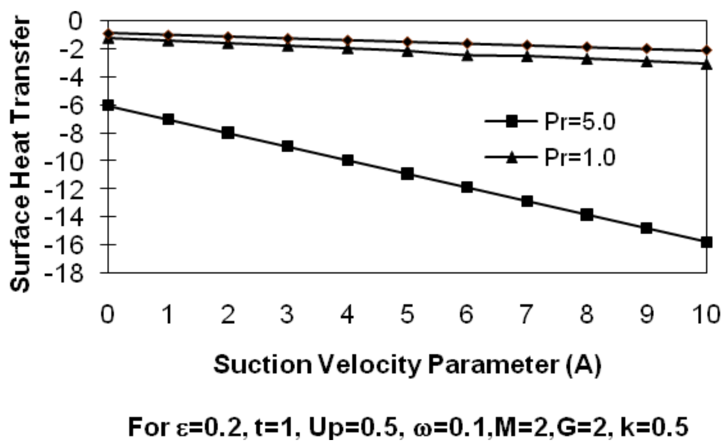

Figure 12. Variation of the surface the heat transfer with suction velocity Parameter (A) for different values of Pr.

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