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DINESH VERMA-LAPLACE TRANSFORM OF SOME MOMENTOUS FUNCTIONS

DINESH VERMA and AFTAB ALAM

Department of Mathematics
NIILM University, Kaithal
Haryana, India

Department of mathematics
Swami Vivekanand Subharti University
Meerut, U.P. India
E-mail: drdinesh.maths@ gmail.com

Abstract

The paper inquires the Dinesh Verma-Laplace transform of some momentous functions which can be used for solving various differential and integral equations. Both transform is a powerful mathematical tool for the engineering to solve engineering problem. The purpose of this paper is to prove the applicability of obtaining Dinesh Verma-Laplace transform of some momentous functions.

I. Introduction

Dinesh Verma transform (DVT) and Laplace Transform approaches play a significant role in solving various problems in science and engineering separately[1], [2], [3], [4], [5]. Thedifferential and integral equations are generally solved by adopting Laplace transform method or Dinesh verma Transform method or Fourier Transform [6], [7], [8], [9], [10], [11]. The Dinesh Verma Transform (DVT) and Laplace Transform is applicable in so many fields and effectively solving linear differential equations, Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Dinesh Verma Transform (DVT) and Laplace transform without finding their general solutions [12], [13] [14], [15], [16], [17], [18]. In this paper, we present a new approach called Dinesh Verma

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transform (DVT)-Laplace transform for obtaining Dinesh Verma transform (DVT)-Laplace transform of some momentous functions.

II. Basic Definitions

The Laplace Transform with parameter p of $u(x)$

$$L\{u(x)\} = \int_0^\infty e^{-px} u(x) dx$$

for Parameter $p > 0$.

The Dinesh Verma transforms (DVT) with parameter q of $v(x)$

$$D\{v(y)\} = q^5 \int_0^\infty e^{-qy} v(y) dy.$$

The usual Laplace-Dinesh Verma Transform (DVT) is defined as

$LD\{f(x, y)\} = \bar{f}(p, q).$

$$= q^5 \int_0^\infty \int_0^\infty f(x, y) R(x, y)\, dx dy$$

Where,

$$R(x, y) = e^{-(px+qy)}.$$

III. Methodology

Dinesh Verma-Laplace Transform of some Momentous Functions

[A]

$$DL\{1\} = q^5 \int_0^\infty \int_0^\infty 1 \cdot e^{-(px+qy)} dx dy$$

$$DL\{1\} = q^5 \int_0^\infty \int_0^\infty e^{-(px+qy)} dx dy$$

$$DL\{1\} = \left[q^5 \int_0^\infty e^{-qy} dy\right]\left[\int_0^\infty e^{-qx} dy\right]$$

$$q^5 \left[\frac{e^{-qy}}{-q}\right]_0^\infty \left[\frac{e^{-px}}{-p}\right]_0^\infty$$

$$DL\{1\} = \frac{q^4}{p}$$

[B]

$$DL\{xy\} = q^5 \int_0^\infty \int_0^\infty xye^{-(px+qy)} dx dy$$

$$DL\{xy\} = \left[q^5 \int_0^\infty ye^{-qy} dy\right]\left[\int_0^\infty xe^{-px} dx\right]$$

$$= q^5 \left[\frac{e^{-qy}}{-q^2}\right]_0^\infty \left[\frac{e^{-px}}{-p^2}\right]_0^\infty$$

$$DL\{xy\} = \frac{q^3}{p^2}.$$

[C]

$$DL\{e^{ax+by}\} = q^5 \int_0^\infty e^{ax+by} \cdot e^{-(px+qy)} dx dy$$

$$= \left[q^5 \int_0^\infty e^{by} e^{-qy} dy\right]\left[\int_0^\infty e^{ax} e^{-px} dx\right]$$

$$= \left[q^5 \int_0^\infty e^{-y(q-b)} dy\right]\left[\int_0^\infty e^{-(p-a)x} dx\right]$$

$$= q^5 \left[\frac{e^{-y(q-b)}}{-(q-b)}\right]_0^\infty \left[\frac{e^{-(p-a)x}}{-(p-a)}\right]_0^\infty$$

$$= \left[\frac{q^5}{q-b}\right]\left[\frac{1}{p-a}\right]$$

$$DL\{e^{ax+by}\} = \frac{q^5}{(q-b)(p-a)}.$$

[D]

$$DL\{\sin ax \sin by\} = q^5 \int_0^\infty \{\sin ax \sin by\} e^{-(px+qy)} dx dy$$

$$= \left[q^5 \int_0^\infty e^{-qy} \sin by dy\right]\left[\int_0^\infty e^{-px} \sin ax dx\right]$$

$$= q^5 \left[\left\{e^{-qy} \frac{-q \sin by - b \cos by}{q^2 + b^2}\right\}\right]_0^\infty$$

$$= \left[e^{-px} \frac{(-p \sin ax - a \cos ax)}{p^2 + a^2}\right]_0^\infty$$

$$= \left[q^5 \left\{\frac{b}{q^2 + b^2}\right\}\right]\left[\left\{\frac{a}{p^2 + a^2}\right\}\right]$$

$$DL\{\sin ax \sin by\} = \frac{abq^5}{\{q^2 + b^2\}(p^2 + a^2)}$$

[E]

$$DL\{\cos ax \cos by\} = q^5 \int_0^\infty \{\cos ax \cos by\} e^{-(px+qy)} dx dy$$

$$= \left[q^5 \int_0^\infty e^{-qy} \cos y dy\right]\left[\int_0^\infty e^{-px} \sin ax dx\right]$$

$$= q^5 \left[\left\{e^{-qy} \frac{-q \cos by - b \sin by}{q^2 + b^2}\right\}\right]_0^\infty$$

$$= \left[e^{-px} \frac{(-p \cos ax - a \sin ax)}{p^2 + a^2}\right]_0^\infty$$

$$= \left[q^5 \left\{ \frac{q}{q^2 + b^2} \right\} \right] \left[\left\{ \frac{p}{p^2 + a^2} \right\} \right]$$

$$DL\{\cos ax \cos by\} = \frac{pq^6}{(q^2 + b^2)(p^2 + a^2)}.$$

[F]

$$DL\{\sinh ax \sinh by\} = q^5 \int_0^\infty \{\sinh ax \sinh by\} e^{-(px+qy)}\, dxdy$$

$$= \left[q^5 \int_0^\infty e^{-qy} \sinh by\, dy \right] \left[\int_0^\infty e^{-px} \sinh ax\, dx \right]$$

$$= \left[q^5 \int_0^\infty e^{-qy} \left(\frac{e^{by} - e^{-by}}{2} \right) dy \right] \left[\int_0^\infty e^{-px} \left(\frac{e^{ax} - e^{-ax}}{2} \right) dx \right]$$

$$= \left[q^5 \int_0^\infty \frac{1}{2} \{ e^{-y(q-b)} - e^{-y(q+b)} \} dy \right]$$

$$* \left[\int_0^\infty \frac{1}{2} \{ e^{-x(p-a)} - e^{-y(p+a)} \} dy \right]$$

$$= \frac{q^5}{2} \left[\left\{ \frac{e^{-y(q-b)}}{-(q-b)} + \frac{e^{-y(q+b)}}{(q+b)} \right\} \right]_0^\infty$$

$$* \frac{1}{2} \left[\left\{ \frac{e^{-x(p-a)}}{-(p-a)} + \frac{e^{-x(p+a)}}{(p+a)} \right\} \right]_0^\infty.$$

On solving, we get,

$$DL\{\sinh ax \sinh by\} = \frac{abq^5}{(q^2 - b^2)(p^2 - a^2)}.$$

[G]

$$DL\{\cosh ax \cosh by\} = q^5 \int_0^\infty \{\cosh ax \cosh by\} e^{-(px+qy)}\, dxdy$$

$$= \left[q^5 \int_0^\infty e^{-qy} \cosh by dy \right] * \left[\int_0^\infty e^{-px} \cosh ax dx \right]$$

$$= \left[q^5 \int_0^\infty e^{-qy} \left(\frac{e^{by} + e^{-by}}{2} \right) dy \right] * \left[\int_0^\infty e^{-px} \left(\frac{e^{ax} + e^{-ax}}{2} \right) dx \right]$$

$$= \left[\frac{q^5}{2} \left\{ \frac{e^{-y(q-b)}}{-(q-b)} + \frac{e^{-y(q+b)}}{(q+b)} \right\} \right]_0^\infty * \left[\frac{1}{2} \left\{ \frac{e^{-x(p-a)}}{-(p-a)} - \frac{e^{-x(p+a)}}{(p+a)} \right\} \right]_0^\infty$$

$$= \left[\frac{q^5}{2} \left\{ \frac{1}{(q-b)} + \frac{1}{(q+b)} \right\} \right]_0^\infty * \left[\frac{1}{2} \left\{ \frac{1}{-(p-a)} + \frac{1}{(p+a)} \right\} \right].$$

On solving, we get,

$$DL\{\cosh ax \cosh by\} = \frac{bq^6}{(q^2 - b^2)(p^2 - a^2)}.$$

[H]

$$DL\{x^n y^n\} = q^5 \int_\infty^\infty \int_0^\infty x^n y^n e^{-(px+qy)} dx dy$$

$$DL\{xy\} = \left[q^5 \int_0^\infty y^n e^{-qy} dy \right] \left[\int_0^\infty x^n e^{-px} dx \right]$$

$$= \left[q^5 \left\{ \frac{n}{q} \int_0^\infty y^{n-1} e^{-qy} dy \right\} \right] * \left[\frac{n}{p} \int_0^\infty x^{n-1} e^{-px} dx \right]$$

$$= \left[q^5 \left\{ \frac{n}{q} \cdot \frac{n-1}{q} \int_0^\infty y^{n-2} e^{-qy} dy \right\} \right]$$

$$* \left[\frac{n}{p} \cdot \frac{n-1}{p} \int_0^\infty x^{n-2} e^{-px} dx \right].$$

Expand up to n terms

$$= \left[q^5 \left\{ [n(n-1)(n-2) \dots 2.1] \frac{1}{q^n} \int_0^\infty e^{-qy} dy \right\} \right]$$

$$* \left[[n(n-1)(n-2)\dots 2.1] \frac{1}{p^n} \int_0^\infty e^{-px} dx \right]$$

$$= \left[n! \frac{1}{q^n} \frac{1}{q} \right] \left[n! \frac{1}{p^n} \frac{1}{p} \right]$$

$$= \left[n! \frac{1}{q^{n+1}} \right] \left[n! \frac{1}{p^{n+1}} \right]$$

$$DL\{x^n y^n\} = \frac{(n!)^2 q^{n+2}}{q^{n+1} \cdot p^{n+1}}.$$

IV. Conclusion

In this paper, we present a new approach called Dinesh Verma-Laplace transform for obtaining Dinesh Verma-Laplace transform of some significant functions. It may be finished that the technique is a ccomplished for obtaining Dinesh Verma-Laplace transform of some significant functions.

S.No.	$DL\{f(x, y)\}$	$\bar{f}(q, p)$
1.	$DL\{1\}$	$\frac{q^4}{p}$
2.	$DL\{xy\}$	$\frac{q^3}{p^2}$
3.	$DL\{x^n y^n\}$	$\frac{(n!)^2 q^{n+2}}{q^{n+1} \cdot p^{n+1}}$
4.	$DL\{e^{ax+by}\}$	$\frac{q^5}{(q-b)(p-a)}$
5.	$DL\{\sin ax \sin by\}$	$\frac{abq^5}{(q^2+b^2)(p^2+a^2)}$
6.	$DL\{\cos ax \cos by\}$	$\frac{pq^6}{(q^2+b^2)(p^2+a^2)}$

7.	$DL\{\sinh ax \sinh by\}$	$\dfrac{abq^5}{(q^2 - b^2)(p^2 - a^2)}$
8.	$DL\{\cos hax \cosh by\}$	$\dfrac{pq^6}{(q^2 - b^2)(p^2 - a^2)}$

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