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TOPOLOGICAL ASPECTS ON *prw*-COMPACT AND *prw*-LINDELOF SPACES VIA GROUP ACTION

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Abstract

The main aim of this paper is to introduce and investigate group acting on *prw*-compact and *prw*-Lindelof spaces in topological spaces. Also some of the properties have been investigated. We also obtain some new results of group acting on *prw*-compact and *prw*-Lindelof spaces.

1. Introduction

Compactness is one of the most important, useful and fundamental concepts in topology. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. In 1981, Dorsett [4] introduced and studied the concept of semi-compactness. In 1991, K. Balachandran, P. Sundaram and J. Maki, [2] introduced a class of compact spaces called GO-compact spaces. The notion of μ-compactness in generalized topological space was introduced by Jyothi Thomas and Sunil Jacob John [5]. In [3] Á. Császár has also introduced the concept of γ-compact in generalized topological space. The concept of topological groups is introduced by Mikhail Tkaccnko [7]. In this paper we defined and studied on group action on *prw*-compact and *prw*-Lindelof spaces in topological spaces. Throughout this paper, group (G, τ) (or simply G)

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always means a topological group on which no separation axioms are assumed unless explicitly stated. For a subset M of a group G, $cl(M)$, $pcl(M)$ and $\text{int}(M)$ denote the closure of M, pre-closure of M, the interior of M respectively.

2. Preliminaries

We require the following definitions.

Definition 2.1. A topological group is a set G with two structures: (i) G is a group, and (ii) G is a topological space, such that the two structures are compatible, i.e., the multiplication map $\mu : G \times G \to G$ and the inversion map $v : G \to G$ are both continuous.

Definition 2.2. A group G acting on a subset M is called regular open (briefly r-open) [6] set if $M = \text{int}(cl(M))$ and regular closed (briefly r-closed) [6] set if $M = cl(\text{int}(M))$.

Definition 2.3. A group G acting on a subset M is called pre-open [9] set if $M \subseteq \text{int}(cl(M))$ and pre-closed [9] set if $cl(\text{int}(M)) \subseteq M$.

Definition 2.4. A topological group (G, τ) acting on a subset M is said to be pre-regular weakly closed (briefly prw-closed) [8] set if $pcl(M) \subseteq U$ whenever $M \subseteq U$ and U is rs-open in G.

Definition 2.5. A group acting on a subset M of G is said to be b-open [1] if $M \subseteq Int(cl(M)) \cup Cl(Int(M))$. The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of (G, τ) is denoted by $bO(G, \tau)$ [respectively $bCl(G, \tau)$].

Definition 2.6. If M be a subset of G, then

(i) b-interior [1] of M is the union of all b-open sets contained in M.

(ii) b-closure [1] of M is the intersection of all b-closed sets containing M. The b-interior [respectively b-closure] of M is denoted by $b - Int(M)$ [respectively $b - Cl(M)$].

3. Group Action on *prw*-Compact and *prw*-Lindelof Spaces

Definition 3.1. A collection $\{M_i : i \in \wedge\}$ of *prw*-open sets in topological group (G, τ) is called *prw*-open cover of a subset *N* of *G* if $N \subseteq \bigcup\{M_i : i \in \wedge\}$ holds.

Definition 3.2. A topological group (G, τ) is called *prw*-compact if every *prw*-open cover of *G* has a finite subcover.

Theorem 3.3. *A topological group* (G, τ) *acting on every prw-closed subset of a prw-compact space is prw-compact relative to G.*

Proof. Let *M* be *prw*-closed subset of a *prw*-compact space (G, τ). Then $G - M$ is *prw*-open in (G, τ). Let $V = \bigcup\{A_i : i \in \wedge\}$ be a cover of *M* by *prw*-open sets. Therefore, $M \cup (G - M)$ is a *prw*-open cover of *G*. Since *G* is *prw*-compact, there exists a finite subset $\wedge_\circ$ of $\wedge$ such that $M \cup (G - M)$ $= \bigcup\{A_i : i \in \wedge_0\}$. Therefore $M \subseteq \bigcup\{A_i : i \in \wedge_\circ\}$. Hence *M* is *prw*-compact relative to *G*.

Definition 3.4. A group *G* is said to be *prw*-Lindelof space if every cover of *G* by *prw*-open sets contains a countable subcover.

Theorem 3.5. *A group acting on a function* $g : G \to G'$ *is prw-open and* G' *is prw-Lindelof space, then G is Lindelof space.*

Proof. Let $\{U_i\}$ be an open cover for *G*. Then $\{g(U_i)\}$ is a cover of G' by *prw*-open sets. Since G' is *prw*-Lindelof, $\{g(U_i)\}$ contains a countable subcover, namely $\{g(U_{ij})\}$. Then $\{(U_{ij})\}$ is a countable subcover for *G*. Thus *G* is Lindelof space.

Theorem 3.6. *A group G acting on a function* $g : (G, \tau) \to (G', \tau')$ *be prw-irresolute onto and G be prw-Lindelof, then* G' *is prw-Lindelof space.*

Proof. Let $g : G \to G'$ be *prw*-irresolute onto and *G* be *prw*-Lindelof. Let $\{U_i\}$ be an open cover for G'. Then $\{g^{-1}(U_i)\}$ is a cover of *G* by *prw*-open sets. Since *G* is *prw*-Lindelof, $\{g^{-1}(U_i)\}$ contains a countable subcover, namely $\{g^{-1}(U_{ij})\}$. Then $\{(U_{ij})\}$ is a countable subcover for G'. Thus G' is *prw*-Lindelof space.

4. Conclusion

In this paper, group action on *prw*-compact and *prw*-Lindelof spaces in topological spaces are introduced and some new results are analyzed.

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