



DOMINATION AND EDGE DOMINATION IN SINGLE VALUED NEUTROSOPHIC GRAPH

J. MALARVIZHI and G. DIVYA

Govt. Arts College
Ariyalur, Tamilnadu, India

Research Scholar
PG and Research Department of Mathematics
K.N. Govt. Arts College (Autonomous) for Women
Thanjavur-7, Tamilnadu, India

Abstract

In this paper, the concepts of cardinality, complete bipartite, dominating set, domination number, independence number and total domination number of Single Valued Neutrosophic Graph are introduced and some properties are investigated. Also the concept of edge domination in Single Valued Neutrosophic Graph has also been discussed.

1. Introduction

Fuzzy set theory and intuitionistic fuzzy set theory are useful models in dealing with uncertainty and incomplete information. But they may not be sufficient for indeterminate and inconsistent information that happens in the real world. In order to overcome this, neutrosophic set theory was proposed by Smarandache as a generalization for fuzzy sets and intuitionistic fuzzy sets. Somasundaran [6] discussed about domination in fuzzy graphs. The concept of domination in Intuitionistic fuzzy graphs [5] was investigated by Parvathi and Thamizhendhi and the concept of domination in Bipolar fuzzy graphs was investigated by Akram and Karunambigai [7].

In this paper, the concept of domination and independence on Single

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Valued Neutrosophic graph has been discussed. Further definitions and some theorems are given.

2. Preliminaries

Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set X , then $B = (T_B, I_B, F_B)$ is called a single valued neutrosophic relation on $A = (T_A, I_A, F_A)$ if

$$T_B(x, y) \leq \min \{T_A(x), T_A(y)\},$$

$$I_B(x, y) \leq \min \{I_A(x), I_A(y)\},$$

$$F_B(x, y) \leq \max \{F_A(x), F_A(y)\}.$$

A single valued neutrosophic relation B is called symmetric if $T_B(x, y) = T_B(y, x)$, $I_B(x, y) = I_B(y, x)$ and $F_B(x, y) = F_B(y, x)$ for all $x, y \in X$.

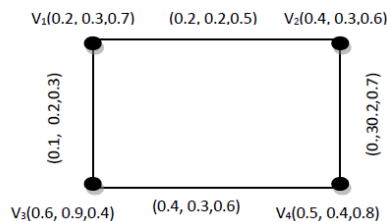
A Single-Valued Neutrosophic graph (SVN graph) is a pair $G = (A, B)$ of the crisp graph $G^* = (V, E)$ (i.e., with underlying set V), where $A : V \rightarrow [0, 1]$ is single-valued neutrosophic set in V and $B : V \times V \rightarrow [0, 1]$ is single-valued neutrosophic relation on V such that

$$T_B(xy) \leq \min \{T_A(x), T_A(y)\},$$

$$I_B(xy) \leq \min \{I_A(x), I_A(y)\},$$

$$F_B(xy) \leq \max \{F_A(x), F_A(y)\},$$

for all $x, y \in V$. A is called single-valued neutrosophic vertex set of G and B is called single-valued neutrosophic edge set of G , respectively.



Let $G = (A, B)$ be a SVNG. Then the cardinality of G is defined to be

$$|G| = \left| \sum_{x \in A} \frac{1 + T_A(x) + I_A(x) - F_A(x)}{3} + \sum_{x, y \in A} \frac{1 + T_B(x, y) + I_B(x, y) - F_B(x, y)}{3} \right|$$

Let $G = (A, B)$ be a SVNG. Then the vertex cardinality of G is defined to be

$$|A| = \left| \sum_{x \in A} \frac{1 + T_A(x) + I_A(x) - F_A(x)}{3} \right|.$$

Let $G = (A, B)$ be a SVNG. Then the edge cardinality of G is defined to be

$$|B| = \left| \sum_{x, y \in A} \frac{1 + T_B(x, y) + I_B(x, y) - F_B(x, y)}{3} \right|.$$

Given a single-valued neutrosophic graph $G = (A, B)$ of a crisp graph $G^* = (V, E)$, the order of G is defined as $\text{Order}(G) = \frac{1 + O_T(G) + O_I(G) - O_F(G)}{3}$ where $O_T(G) = \sum_{v \in V} T_A(v)$, $O_I(G) = \sum_{v \in V} I_A(v)$, $O_F(G) = \sum_{v \in V} F_A(v)$.

Given a single-valued neutrosophic graph $G = (A, B)$ of a crisp graph $G^* = (V, E)$, the size of G is defined as $\text{Size}(G) = \frac{1 + S_T(G) + S_I(G) - S_F(G)}{3}$ where $S_T(G) = \sum_{u \neq v} T_B(u, v)$, $S_I(G) = \sum_{u \neq v} I_B(u, v)$, $S_F(G) = \sum_{u \neq v} F_B(u, v)$.

The vertex degree in a SVNG, $G = (A, B)$ is defined to be sum of the weights of the strong edges incident at x . It is denoted by $d_G(x)$.

$\delta(G) = \min \{d_G(x) | x \in V\}$ is minimum degree of G .

$\Delta(G) = \max \{d_G(x) | x \in V\}$ is maximum degree of G .

Two vertices x and y are said to be neighbors in $SVNG$ if any one of the following condition hold

- i. $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) > 0$
- ii. $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) > 0$
- iii. $T_B(x, y) = 0, I_B(x, y) = 0, F_B(x, y) > 0$
- iv. $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) = 0$
- v. $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) > 0$
- vi. $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) = 0$
- vii. $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) = 0$ for $x, y \in A$.

A path of a SVN graph G is a sequence of vertices and edges $v_1 e_1 v_2 e_2 v_3 e_3, \dots, v_n$ such that either if the conditions is satisfied

- i. $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) > 0$ for some x and y in A
- ii. $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) > 0$ for some x and y in A
- iii. $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) > 0$ for some x and y in A
- iv. $T_B(x, y) > 0, I_B(x, y) > 0, F_B(x, y) = 0$ for some x and y in A
- v. $T_B(x, y) = 0, I_B(x, y) > 0, F_B(x, y) > 0$ for some x and y in A
- vi. $T_B(x, y) > 0, I_B(x, y) = 0, F_B(x, y) = 0$ for some x and y in A
- vii. $T_B(x, y) = 0, I_B(x, y) = 0, F_B(x, y) = 0$ for some x and y in A

The length of the path $v_1 e_1 v_2 e_2 v_3 e_3, \dots, v_n$ is $n - 1$ for $n > 0$.

Let P be path in SVN graph $G = (A, B)$ The T -strength for the path P is $\min (T_B(x, y))$ for all x and y and is denoted St_T . The I -strength for the path P is $\min (I_B(x, y))$ for all x and y and is denoted St_I . The F -strength for the path P is $\max (F_B(x, y))$ for all x and y and is denoted St_F . If the same edge posses all the T -strength, I -strength and F -strength values then it is said to

be the strength for the path. The path strength $(St_T, St_I, St_F) = (\min (T_B(x, y)), (\min (I_B(x, y)), \max (F_B(x, y))$.

Let $G = (A, B)$ be a SVN graph. If x, y belongs to A , the T - Strength of connectedness between x and y is $T_B^\infty(x, y) = \sup \{T_B^m(x, y) : m = 1, 2, 3, \dots, n\}$, I -Strength of connectedness between x and y is $I_B^\infty(x, y) = \sup \{I_B^m(x, y) : m = 1, 2, 3, \dots, n\}$, and F -Strength of connectedness between x and y is $F_B^\infty(x, y) = \inf \{F_B^m(x, y) : m = 1, 2, 3, \dots, n\}$, where $T_B^m(x, y) = \sup \{T_B(x, y_1) \wedge T_B(y_1, y_2) \wedge T_B(y_2, y_3) \dots$

$$T_B(y_{m-1}, y) : x, y_1, y_2, \dots, y_{m-1}y \in V\}$$

$$I_B^m(x, y) = \sup \{I_B(x, y_1) \wedge I_B(y_1, y_2) \wedge I_B(y_2, y_3) \dots$$

$$I_B(y_{m-1}, y) : x, y_1, y_2, \dots, y_{m-1}y \in V\}$$

$$F_B^m(x, y) = \inf \{F_B(x, y_1) \vee F_B(y_1, y_2) \vee F_B(y_2, y_3) \dots$$

$$F_B(y_{m-1}, y) : x, y_1, y_2, \dots, y_{m-1}y \in V\}.$$

Two vertices are connected if they are joined by path.

A single-valued neutrosophic graph $G = (A, B)$ is called complete if the following conditions are satisfied:

$$T_B(x, y) = \min \{T_A(x), T_A(y)\},$$

$$I_B(x, y) = \min \{I_A(x), I_A(y)\},$$

$$F_B(x, y) = \max \{F_A(x), F_A(y)\},$$

for all $x, y \in A$.

The complement of a single-valued neutrosophic graph $G = (A, B)$ of a crisp graph $G^* = (V, E)$ is a single-valued neutrosophic graph $\bar{G} = (\bar{A}, \bar{B})$, where

$$1. \quad \bar{A} = A = (T_A, I_A, F_A) \quad \text{where} \quad T_A(v) = T_A(v), I_A(v) = I_A(v),$$

$$F_A(v) = F_A(v), \text{ for all } v \in V$$

2. $\bar{B} = (T_{\bar{B}}, I_{\bar{B}}, F_{\bar{B}})$ where $T_{\bar{B}}(u, v) = \min \{T_A(u), T_A(v)\}$ if $T_B(u, v) = 0$,

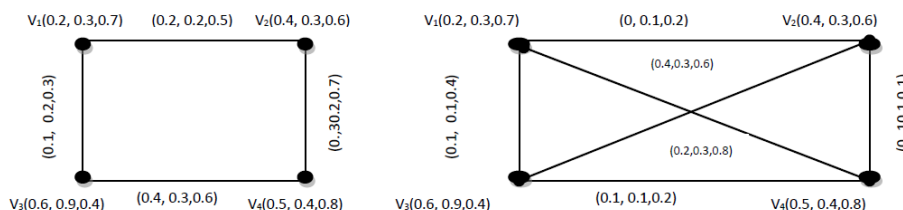
$= \min \{T_A(u), T_A(v)\} - T_B(u, v)$ if $T_B(u, v) > 0$

$I_{\bar{B}}(u, v) = \min \{I_A(u), I_A(v)\}$ if $I_B(u, v) = 0$,

$= \min \{I_A(u), I_A(v)\} - I_B(u, v)$ if $I_B(u, v) > 0$,

$F_{\bar{B}}(u, v) = \max \{F_A(u), F_A(v)\}$ if $F_B(u, v) = 0$,

$= \max \{F_A(u), F_A(v)\} - F_B(u, v)$ if $F_B(u, v) > 0$.



SVN graph G Complement of SVN graph $G : \bar{G}$.

A SVN graph $G = (A, B)$ is said to be bipartite if the vertex set A can be partitioned into two nonempty sets A_1 and A_2 such that

i. $T_B(x, y) = 0$ $I_B(x, y) = 0$, $F_B(x, y) = 0$ if $(x, y) \in A_1$ or $(x, y) \in A_2$

ii. $T_B(x, y) > 0$ $I_B(x, y) > 0$, $F_B(x, y) > 0$ if $x \in A_1$ and $y \in A_2$ (OR)

$T_B(x, y) = 0$ $I_B(x, y) > 0$, $F_B(x, y) > 0$ if $x \in A_1$ and $y \in A_2$ (OR)

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$T_B(x, y) = 0$ $I_B(x, y) = 0$, $F_B(x, y) > 0$ if $x \in A_1$ and $y \in A_2$ (OR)

$T_B(x, y) = 0$ $I_B(x, y) > 0$, $F_B(x, y) = 0$ if $x \in A_1$ and $y \in A_2$ (OR)

$T_B(x, y) > 0$ $I_B(x, y) = 0$, $F_B(x, y) = 0$ if $x \in A_1$ and $y \in A_2$ (OR)

A bipartite SVN graph $G = (A, B)$ is said to be complete if

$$T_B(x, y) = \min \{T_A(x), T_A(y)\}$$

$$I_B(x, y) = \min \{I_A(x), I_A(y)\}$$

$$F_B(x, y) = \max \{F_A(x), F_A(y)\}$$

for all $x \in A_1$ and $y \in A_2$.

3. Domination in A Single Valued Neutrosophic Graph

Definition 3.1. An edge (x, y) is said to be strong edge in *SVN* graph $G = (A, B)$ if $T_B(x, y) \geq T_B^\infty(x, y)$, $I_B(x, y) \geq I_B^\infty(x, y)$ and $F_B(x, y) \geq F_B^\infty(x, y)$.

Definition 3.2. Let x be a vertex in a *SVNG*, $G = (A, B)$, then $N(x) = \{y : y \in A \text{ and } (x, y) \text{ is strong in } G\}$ is the neighborhood of x in G .

Definition 3.3. A vertex $x \in A$ of a *SVNG* graph is an isolated vertex if $T_B(x, y) = 0$, $I_B(x, y) = 0$ and $F_B(x, y) = 0$ for every $y \in V$. That is $N(x) = \{ \}$. An isolated vertex does not dominate any of the other vertex in it.

Definition 3.4. Let $G = (A, B)$ be a *SVN* graph of the crisp graph G^* . Let $x, y \in A$, x dominates y in G if we find a strong edge between them.

Note:

1. For any $x, y \in A$, if x dominates y then y dominates x , domination is symmetric relation on V .
2. For any $y \in A$, $N(y)$ is the set of all vertices in A that are dominated by y .
3. If $T_B(x, y) < T_B^\infty(x, y)$, $I_B(x, y) < I_B^\infty(x, y)$ and $F_B(x, y) < F_B^\infty(x, y)$ for all x, y in A , then the only dominating set is A .

Definition 3.5. A subset C of A is called a dominating set in $G = (A, B)$ if for every $y \in A - C$, there exists $x \in C$ such that x dominates y .

Definition 3.6. A dominating set C of a *SVNG* is said to be minimal dominating set if no proper subset of C is a dominating set.

Definition 3.7. Minimum cardinality among all minimal dominating set is called lower domination number of *SVN* graph G , and is denoted by $d_N(G)$. Maximum cardinality among all minimal dominating set is called upper domination number of *SVN* graph G , and is denoted by $D_N(G)$.

Definition 3.8. Two vertices in a *SVN* graph, $G = (A, B)$ are said to be independent if there is no strong edge between them.

Definition 3.9. A subset C of A is said to be independent set of G if $T_B(x, y) < T_B^\infty(x, y)$, $I_B(x, y) < I_B^\infty(x, y)$ and $F_B(x, y) < F_B^\infty(x, y)$ for all x, y in C .

Definition 3.10. An independent set S of G in a *SVN* graph G is said to be maximal independent, if for every vertex $y \in A - C$ the set $C \cup \{y\}$ is not independent.

Definition 3.11. The minimum cardinality among all maximal independent set is called lower independence number of *SVN* graph G , and it is denoted by $i_N(G)$. The maximum cardinality among all maximal independent set is called upper independence number of *SVN* graph G , and it is denoted by $I_N(G)$.

Definition 3.12. Let $G = (A, B)$ a *SVNG* without isolated vertices. A set C is a total dominating set if for every vertex $y \in A$, there exists a vertex $x \in C$, $x \neq y$, such that x dominates y .

Definition 3.13. The minimum cardinality of a total dominating set is called lower total domination number of *SVN* graph G , and it is denoted by $t_N(G)$. The maximum cardinality of a total dominating set is called upper total domination number of *SVN* graph G , and it is denoted by $T_N(G)$.

Theorem 3.1. A dominating set Q of a *SVN* graph, $G = (A, B)$ is a minimal dominating set if and only if for each $q \in Q$ any one of the following hold.

- (i) q is not strong neighbour of any vertex in Q .
- (ii) There is a vertex $x \in A - Q$ such that $N(x) \cap Q = q$.

Proof. Assume that D is a minimal dominating set of G . Then for every vertex $q \in Q$, $Q - q$ is not a dominating set and hence there is $x \in A - (Q - \{q\})$ which is not dominated by any of the vertex in $Q - \{q\}$. If $x = q$, we get, x not strong neighbor of any vertex in Q . If $x \neq q$, x is not dominated by $Q - \{x\}$, but is dominated by Q , then the vertex x is strong neighbour only to q in Q . That is, $N(x) \cap Q = \{q\}$.

Conversely, assume Q , a dominating set and for each vertex $q \in Q$, one of the conditions hold. Suppose Q is not a minimal dominating set, then there exists a vertex $q \in Q$, $Q - \{q\}$ is a dominating set. Hence q is a strong neighbour for at least one vertex in $Q - \{q\}$, the first condition does not hold. If $Q - \{q\}$ is a dominating set then every vertex in $A - Q$ is a strong neighbour to at least one vertex in $Q - \{q\}$, the condition two does not hold which contradicts our assumption. So Q is a minimal dominating set.

Theorem 3.2. Let $G = (A, B)$ be a SVNG without isolated vertices and Q is a minimal dominating set. Then $A - Q$ is a dominating set of G .

Proof. Let Q be a minimal dominating set. Let y be any vertex of Q . Since G is without isolated vertices, there is a vertex $x \in N(y)$. y must be dominated by at least one vertex in $Q - \{y\}$, that is $Q - \{y\}$ is a dominating set. By theorem 3.1, it follows that $x \in A - Q$. Thus every vertex in Q is dominated by at least one vertex in $A - Q$, and $A - Q$ is a dominating set.

Theorem 3.3. For any SVN graph $G = (A, B)$, $d_N(G) + d_N(\overline{G}) \leq$ twice the order of G where $d_N(\overline{G})$ is the lower domination number of \overline{G} and equality holds if and only if $0 < T_B(x, y) < T_B^\infty(x, y)$, $0 < I_B(x, y) < I_B^\infty(x, y)$ and $0 < F_B(x, y) < F_B^\infty(x, y)$ for all $x, y \in A$.

Proof. The result is obvious when $d_N(G) + d_N(\overline{G}) < 2O(G)$. $d_N(G) = O(G)$ if and only if $T_B(x, y) < T_B^\infty(x, y)$, $I_B(x, y) < I_B^\infty(x, y)$ and

$F_B(x, y) < F_B^\infty(x, y)$ for all $x, y \in A$. $d_N(\overline{G}) = O(G)$ if and only if
 $T_{\overline{B}}(x, y) - T_B(x, y) < T_B^\infty(x, y)$, $I_{\overline{B}}(x, y) - I_B(x, y) < I_B^\infty(x, y)$ and
 $F_{\overline{B}}(x, y) - F_B(x, y) < F_B^\infty(x, y)$ for all $x, y \in A$ which gives
 $T_B(x, y) > 0$, $I_B(x, y) > 0$ and $F_B(x, y) > 0$. Hence
 $d_N(G) + d_N(\overline{G}) \leq 2O(G) = \text{twice order of } G$.

Corollary 3.4. *For any SVN graph $G = (A, B)$ without isolated vertices, $d(G) \leq$ Half the order of G .*

Corollary 3.5. *Let G be an SVN graph G such that both G and \overline{G} have no isolated vertices. Then $d_N(G) + d_N(\overline{G}) \leq O(G)$, where $d_N(G)$ is the lower domination number of G . Further equality holds if and only if $d_N(G) = d_N(\overline{G}) = O(G)$.*

Theorem 3.6. $d_N((G)) \leq O(G) - \Delta$.

Proof. Let x be a vertex in an SVN graph, $G = (A, B)$. Assume that $|N(x)| = \Delta$. Then $A - N(x)$ is a dominating set of G , so that $d_N(G) \leq |A - N(x)| = O(G) - \Delta$.

Theorem 3.7. *An independent set of SVN graph, $G = (A, B)$ is a maximal independent set iff it is both independent and dominating.*

Proof. Let Q be a maximal independent set in a SVNG, and hence for every vertex $x \in A - Q$, the set $Q \cup \{x\}$ is not independent. For every vertex $x \in A - Q$, there is a vertex $a \in Q$ such that a is a strong neighbour of x . Thus Q is dominating set. Hence Q is dominating and independent set.

Conversely, assume Q is independent and dominating set. Suppose Q not maximal independent, then there is a vertex $y \in A - Q$, the set $Q \cup \{y\}$ is independent. If $Q \cup \{y\}$ is independent then no vertex in Q is strong neighbour of y . Hence Q cannot be a dominating set, which is a contradiction. Hence Q is a maximal independent set.

Theorem 3.8. *Every maximal independnet set in a SVN graph, $G = (A, B)$ is a minimal dominating set.*

Proof. Let C be a maximal independent set in a *SVNG*. By the above theorem, C is a dominating set. Suppose C is not a minimal dominating set, then there exists at least one vertex $x \in C$ for which $C - \{x\}$ is a dominating set. But if $C - \{x\}$ dominates $A - \{C - (x)\}$, then at least one vertex in $C - \{x\}$ must be strong neighbour to x . This contradicts the fact that C is an independent set of G . Therefore, C must be a minimal dominating set.

Theorem 3.9. *To a SVN graph $G = (A, B)$, $t_N(G) = O(G)$ iff every vertex in G has a unique neighbour.*

Proof. If every vertex of G has a unique neighbor then, the vertex set A is the only dominating set of G , then $t_N(G) = O(G)$.

Conversely, assume that $t_N(G) = O(G)$. If there exists a vertex x with neighbours y and z then $A - \{x\}$ is a total dominating set of G , so that $t_N(G) < O(G)$ which leads to contradiction. Therefore every vertex of G has unique neighbour.

4. Edge Domination in Single Valued Neutrosophic Graph

Definition 4.1. An edge $e = (u, v)$ of a *SVN* graph is said to be an effective edge if $T_B(u, v) = \min \{T_A(u), T_A(v)\}$, $I_B(u, v) = \min \{I_A(u), I_A(v)\}$, and $F_B(u, v) = \max \{F_A(u), F_A(v)\}$.

Definition 4.2. Let $G = (A, B)$ be *SVN* graph of the crisp graph $G^* = (V, E)$. A subset of E is said to be an edge dominating set in G if for every edge not in E , there is an effective edge and is adjacent to some edge in E .

Definition 4.3. A minimal edge dominating set of a *SVN* graph is the edge dominating set if none of its proper subset.

Definition 4.4. The minimum cardinality of edge dominating set is the edge domination number of *SVN* graph.

Definition 4.5. Isolated edge of a *SVN* graph is the edge (a, b) which has no effective edge incident on the vertex of the (a, b) .

Note: An isolated vertex cannot dominate any other vertex of the graph.

Remark. An edge dominating set Y is minimal iff for edge $f \in Y$, any of the following conditions hold

(a) $N(f) \cap Y = \varnothing$

(b) there exists an edge $e \in E - Y$ such that $N(e) \cap Y = \{f\}$ and f is an effective edge.

Theorem 4.1. *G be a SVN graph with no isolated edges then for every minimal edge dominating set Y , $E - Y$ is an edge dominating set.*

Proof. Let y be any edge in Y . Since G has no isolated edges, there is an edge $e \in N(y)$. Then $e \in E - Y$. Thus every element of Y is dominated by some element of $E - Y$.

5. Conclusion

In this paper, we have developed the concept of domination and independence on Single Valued Neutrosophic graph. Definitions and some theorems have been given. Have just defined the edge dominating set and further investigation has to done in the edge domination field.

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