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AN APPROACH FOR SOLVING INTUITIONISTIC FUZZY ASSIGNMENT PROBLEMS USING SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS OF REACTION-DIFFUSION TYPE

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Abstract

The goal of a general assignment problem is to identify an optimal machine to job assignment without allocating an agent more than once and ensuring that all jobs are finished. The Intuitionistic Fuzzy Assignment Problem (IFAP) is investigated here. To solve IFAP a new method is proposed. Here, the weights obtained using the exact solution of singularly perturbed problem is applied in solving IFAP. An illustration is provided to demonstrate the method and compare it to the existing method.

1. Introduction

Bellman and Zadeh (1970) and Zadeh (1965) established the fuzzy set theory. "Sakthi and Kajla (2010) presented a method for solving fuzzy assignment problems". Later, "Atanassov (1986) suggested Intuitionistic Fuzzy Set (IFS)". "Singh and Yadav (2014) introduced an approach to solve Intuitionistic fuzzy transportation problems". "Sagaya and Henry (2015)

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proposed methods to solve IFAP". "Paramasivam et al. (2010) developed a method for a linear Singular Perturbation Problem (SPP), utilized to develop numerical approaches for the problem". "Robinson et al. (2019) discussed MAGDM problems using SPPs". "Asha and Joseph (2021) introduced a method to solve Fuzzy Assignment Problems". This paper proposes a "method to solve IFAP where the weights are derived from SPPs, normalized and utilized in IFAPs".

2. Preliminaries

Here, "basic definitions are reviewed (Atanassov, (1986, 1989), Nehi (2010)").

Definition 1. An IFS $\tilde{F}^I$ in U is $\tilde{F}^I = \{(u, \mu_{\tilde{F}^I}(u), \nu_{\tilde{F}^I}(u))/u \in U\}$ where $\mu_{\tilde{F}^I}, \nu_{\tilde{F}^I} : U \to [0, 1]$ are functions such that $0 \leq \mu_{\tilde{F}^I}(u) + \nu_{\tilde{F}^I}(u) \leq 1 \; \forall \, u \in U$. The hesitation degree for the element $u \in U$ being in $\tilde{F}^I$ is given by $h(u) = 1 - \mu_{\tilde{F}^I}(u) - \nu_{\tilde{F}^I}(u) \leq 1 \; u \in U$.

Definition 2. An Intuitionistic Fuzzy subset $\tilde{F}^I = \{(u, \mu_{\tilde{F}^I}(u), \nu_{\tilde{F}^I}(u))/u \in U\}$, of the real line R is called an Intuitionistic Fuzzy Number if

(i) "$\exists u \in R$ such that $\mu_{\tilde{F}^I}(u) = 1$ and $\nu_{\tilde{F}^I}(u) = 0$"

(ii) "$\mu_{\tilde{F}^I}$ and $\nu_{\tilde{F}^I}$ are piecewise continuous mappings from R to $[0, 1]$ and $0 \leq \mu_{\tilde{F}^I}(u) + \nu_{\tilde{F}^I}(u) \leq 1 \; u \in U$ holds".

Definition 3. A Trapezoidal Intuitionistic Fuzzy Number (TIFN) $\tilde{F}^I$ with parameters $u_1' \leq u_1 \leq u_2' \leq u_2 \leq u_3 \leq u_3' \leq u_4 \leq u_4'$ is defined as an IFS $\tilde{F}^I = (u_1', u_1, u_2', u_2, u_3, u_3', u_4, u_4')$. In this case,

$$\mu_{\widetilde{F}^I}(x) = \begin{cases} 0 & \text{if } x \le u_1 \\ \frac{x - u_1}{u_2 - u_1} & \text{if } u_1 < x < u_2 \\ 1 & \text{if } u_2 \le x \le u_3 \\ \frac{x - u_4}{u_3 - u_4} & \text{if } u_3 < x < u_4 \\ 0 & \text{if } u_4 \le x \end{cases} \text{ and}$$

$$\nu_{\widetilde{F}^I}(x) = \begin{cases} 1 & \text{if } x \le u_1' \\ \frac{x - u_1'}{u_2' - u_1'} & \text{if } u_1' < x < u_2' \\ 0 & \text{if } u_2' \le x \le u_3' \\ \frac{x - u_4'}{u_3' - u_4'} & \text{if } u_3' < x < u_4' \\ 1 & \text{if } u_4' \le x \end{cases}$$

If in $\widetilde{F}^I$, $u_2' = u_3'(u_2 = u_3)$ then it gives a Triangular Intuitionistic Fuzzy Number (TrIFN) with parameters $u_1' \le u_1 \le u_2'(u_2 = u_3 = u_3') \le u_4 \le u_4'$ and denoted by $\widetilde{F}^I = (u_1', u_1, u_2', u_4, u_4')$.

3. Arithmetic Operations of Intuitionistic Fuzzy Numbers

Let $\widetilde{F}_1^I = (u_1', u_1, u_2', u_2, u_3, u_3', u_4, u_4')$ and

$\widetilde{F}_2^I = (w_1', w_1, w_2', w_2, w_3, w_3', w_4, w_4')$ be TIFNs and p be a real number.

Then

(i) Intuitionistic Fuzzy numbers addition $(\oplus)$:

$\widetilde{F}_1^I \oplus \widetilde{F}_2^I =$

$(u_1' + w_1', u_1 + w_4, u_2' + w_2', u_2 + w_2, u_3 + w_3 + w_3, u_3' + w_3', u_4 + w_4, u_4' + w_4')$

(ii) Intuitionistic Fuzzy numbers subtraction $(\ominus)$:

$\widetilde{F}_1^I \ominus \widetilde{F}_2^I =$

$(u_1' - w_4', u_1 - w_4, u_2' - w_3', u_2 - w_3, u_3 - w_2, u_3' - w_2' - u_4 - w_1, u_4' - w_4')$

(iii) Intuitionistic Fuzzy numbers multiplication $(\otimes)$:

$$\widetilde{F}_1^I \otimes \widetilde{F}_2^I =$$

$$= \begin{cases} (u_1'w_1', u_1w_1, u_2'w_2', u_2w_2, u_3w_3, u_3'w_3', u_4w_4, u_4'w_4') \text{ if } \widetilde{F}_1^I > 0 \text{ and } \widetilde{F}_2^I > 0 \\ (u_1'w_4', u_1w_4, u_2'w_3', u_2w_3, u_3w_2, u_3'w_2', u_4w_1, u_4'w_1') \text{ if } \widetilde{F}_1^I < 0 \text{ and } \widetilde{F}_2^I > 0 \\ (u_4'w_4', u_4w_4, u_3'w_3', u_3w_3, u_2w_2, u_2'w_2', u_1w_1, u_1'w_1') \text{ if } \widetilde{F}_1^I < 0 \text{ and } \widetilde{F}_2^I < 0 \end{cases}$$

(iv) Intuitionistic Fuzzy numbers scalar multiplication.

$$p\widetilde{F}_1^I = \begin{cases} (pu_1',\ pu_1,\ pu_2',\ pu_2,\ pu_3,\ pu_3',\ pu_4,\ pu_4')\, if\ p > 0 \\ (pu_4',\ pu_4,\ pu_3',\ pu_3,\ pu_2,\ pu_2',\ pu_1,\ pu_1')\, if\ p < 0 \end{cases}$$

4. Intuitionistic Fuzzy Assignment Problem

The IFAP can be expressed in the form of $n \times n$ Intuitionistic fuzzy cost table as follows.

Table 1.

Jobs

Persons	1	2	…	j	…	n
1	$\widetilde{c}_{11}$	$\widetilde{c}_{12}$	…	$\widetilde{c}_{1j}$	…	$\widetilde{c}_{1n}$
2	$\widetilde{c}_{21}$	$\widetilde{c}_{22}$	…	$\widetilde{c}_{2j}$	…	$\widetilde{c}_{1n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	$\widetilde{c}_{i1}$	$\widetilde{c}_{i2}$	…	$\widetilde{c}_{ij}$	…	$\widetilde{c}_{in}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$\widetilde{c}_{n1}$	$\widetilde{c}_{n2}$		$\widetilde{c}_{nj}$		$\widetilde{c}_{nn}$

where $\widetilde{c}_{kl}$ are the Intuitionistic fuzzy costs.

5. Weights Determination for Resolving IFAPs using SPP

The weighting vector for solving the IFAP is represented in the form of the following SPP by the manufacturer.

$-\varepsilon u''(x) + u(x) = 1 + x,\ x \in (0, 1)$ with $u(0) = 2,\ u(1) = 1.$

The exact solution is $u(x) = \left(\dfrac{1 + e^{\frac{1}{\sqrt{\varepsilon}}}}{e^{\frac{-1}{\sqrt{\varepsilon}}} - e^{\frac{1}{\sqrt{\varepsilon}}}} \right) e^{\frac{x}{\sqrt{\varepsilon}}} + \left(\dfrac{1 + e^{\frac{1}{\sqrt{\varepsilon}}}}{e^{\frac{-1}{\sqrt{\varepsilon}}} - e^{\frac{1}{\sqrt{\varepsilon}}}} \right) e^{\frac{-x}{\sqrt{\varepsilon}}} + 1 + x$

The weight vectors can be achieved for $\varepsilon = 0.01$ by normalizing the exact solution and is as follows.

Table 2. Exact solution for $-\varepsilon u''(x) + u(x) = 1 + x.$

x	$u(x)$	Normalization
0.2	8.4861	0.40866
0.4	6.1682	0.29704
0.6	4.0537	0.19521
0.8	2.0578	0.09910

As a result, the manufacturer's weighting vector is constructed as

$$(0.40866,\ 0.29704,\ 0.19521,\ 0.09910)^T$$

6. Proposed Methodology to Solve IFAP using SPP

We now present a new strategy for determining an Intuitionistic fuzzy optimal assignment of IFAP.

Step 1. Check whether the IF cost matrix obtained is balanced or not.

(a) If it is balanced proceed to step 3.

(b) If it is not balanced proceed to step 2.

Step 2. Convert the IF cost matrix to the IF square matrix by adding dummy rows or dummy columns with Intuitionistic fuzzy zero entries.

Step 3. The Formulation of the IFAP is as follows

$$Minimize\ \tilde{z} = \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{c}_{kl} x_{kl} k = 1 \text{ to } n; l = 1 \text{ to } n$$

S. to

$$\sum_{l=1}^{n} x_{kl} = 1,\ k = 1 \text{ to } n$$

$$\sum_{k=1}^{n} x_{kl} = 1,\ l = 1 \text{ to } n$$

$x_{kl} \in \{0, 1\}$, where

$$x_{kl} = \begin{cases} 1, & \text{if the } k\text{th individual is allocated the } l\text{th job} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

is the decision variable of allocating the job l to individual k.

$\tilde{c}_{kl}$ is the allocating cost of the l^{th} job to the k^{th} individual.

Step 4. Transform the Intuitionistic fuzzy linear programming to the multi objective linear programming problem (MOLPP) with Intuitionistic fuzzy coefficients as shown below:

$Min_{x \in X}\{f_1(x), f_2(x), \ldots, f_k(x)\}$ S. to

$$\sum_{l=1}^{n} x_{kl} = 1,\ k = 1 \text{ to } n$$

$$\sum_{k=1}^{n} x_{kl} = 1,\ l = 1 \text{ to } n$$

$x_{kl} \in \{0, 1\}$, where

$$x_{kl} = \begin{cases} 1, & \text{if the kth individual is allocated the lth job} \\ 0, & \text{otherwise} \end{cases}$$

Where $f_i : R^n \to R^i$

Then the MOLPP can be written using the weighting factor as

$$Min_{x \in X} \sum_{m=1}^{r} w_m f_m(x)$$

Subject to (1)

Step 5. Convert the "MOLPP with Intuitionistic fuzzy coefficients" obtained from step 4 to the "crisp LPP", by giving weights achieved from "Singular Perturbation Problem" given in section 5 to obtain the optimal solution.

7. Numerical Example ["Sagaya and Henry (2015)"]

The following is the cost matrix of an IFAP with three rows demonstrating three persons P_1, P_2, P_3 and three columns demonstrating three jobs J_1, J_2, J_3. The manufacturer desires to conclude the Intuitionistic fuzzy optimal assignment of jobs to persons with the aim of the over-all approximate assignment cost is minimized, using Singular Perturbation problem specified in section 5 for obtaining the weights.

$$\begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{array}{c} \begin{array}{ccc} J_1 & J_2 & J_3 \end{array} \\ \begin{pmatrix} (1,2,4,5,7,8,10,12) & (2,3,5,6,9,10,12,14) & (1,3,5,6,9,10,11,13) \\ (3,4,6,7,9,11,13,15) & (1,2,4,6,10,12,14,16) & (2,3,5,7,10,12,14,15) \\ (2,4,8,10,11,12,14) & (2,3,4,5,8,9,10,11) & (3,4,5,7,11,12,13,14) \end{pmatrix} \end{array}$$

This IFAP is a balanced one.

By the Steps of the suggested method, the above balanced IFAP can be transformed to crisp linear programming problem by giving weights $(0.40866, 0.29704, 0.19521, 0.0991)^T$ obtained from the Singular Perturbation Problem in section 5.

As a result, with the given IFAP, we obtain the IF optimal assignment as,

$$P_1 \to J_1,\ P_2 \to J_3,\ P_3 \to J_2$$

and IF optimal cost is $\widetilde{C}_{11} + \widetilde{C}_{23} + \widetilde{C}_{32} = [5, 8, 13, 17, 25, 29, 34, 38]$.

8. Comparative Study

In the table below, the solution obtained from the numerical example is compared to the solution obtained from Sagaya and Henry (2015).

Table 3. Comparison of the numerical solution of the proposed approach wit the existing approach.

Persons	Approach	Jobs		
		1	2	3
1	The proposed approach	(1, 2, 4, 5, 7, 8, 10 , 12)		
	Sagaya and Henry (2015)	(1, 2, 4, 5, 7, 8, 10 , 12)		
2	The proposed approach			(2, 3, 5, 7, 10 12, 14, 15)
	Sagaya and Henry (2015)			(2, 3, 5, 7, 10 12, 14, 15)
3	The proposed approach		(2, 3, 4, 5, 8, 9, 10, 11)	
	Sagaya and Henry (2015)		(2, 3, 4, 5, 8, 9, 10, 11)	

The table shows that the solutions obtained by the proposed method and by the existing method are the same. The suggested method, on the other hand, is simpler and provides the optimal solution faster than the existing approach.

9. Conclusion

Here, assignment problem under fuzzy environment is examined. The proposed approach converts the IFAP to MOLPP with Intuitionistic fuzzy coefficients and the weights are calculated according to the manufacturer decisions and then the optimal solution is achieved. An illustration is provided to validate the suggested approach and then the solutions are compared to the existing method and found that the method discussed produces outcomes faster than the existing method.

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