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ON PAIRWISE WEAK FUZZY P-SPACES

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Abstract

In this paper, the notions of pairwise fuzzy regular G_δ-sets and pairwise fuzzy regular F_σ-sets are introduced and by means of pairwise fuzzy regular G_δ-sets, the concept of pairwise weak fuzzy P-spaces, is introduced and studied. Several characterizations of pair wise weak fuzzy P-spaces are obtained and the conditions under which pairwise weak fuzzy P-spaces become pair wise fuzzy Baire spaces and pair wise fuzzy second category spaces, are established.

1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by L. A. Zadeh [17] in his classical paper in the year 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. The concepts of fuzzy topology was defined by C. L. Chang [3] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then, much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In 1989, the concepts of regular G_δ-sets was introduced by J. Mack [5]. A.

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Kandil [4] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. The concept of pairwise fuzzy P-spaces was introduced by G. Thangaraj and V. Chandiran in [12]. In this paper the notions of pairwise fuzzy regular G_δ-sets and pairwise fuzzy regular F_σ-sets are introduced and by means of these concepts, the notion of pairwise weak fuzzy P-spaces, is introduced and studied. In this paper several characterizations of pairwise weak fuzzy P-spaces, are studied and the conditions under which pairwise weak fuzzy P-spaces become pair wise fuzzy Baire spaces and pair wise fuzzy second category spaces, are established.

It is also established that the pair wise fuzzy regular G_δ-sets in pair wise weak fuzzy P-spaces are pair wise fuzzy somewhere dense sets. It is established that if the fuzzy bitopological space is the pair wise weak fuzzy P-space, then the pairwise fuzzy weakly regular Lindelofness of the bitopological space implies the pairwise fuzzy almost regular Lindelofness of the bitopological space and vice-versa.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0, 1]. A fuzzy set λ in X is a function from X into I. The null set 0_X is the function from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the function from X into I which takes 1 only. By a fuzzy bitopological space (Kandil, [4]) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1[3]**.** Let λ and μ be fuzzy sets in X. Then, for all $x \in X$,

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,

(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,

(iii) $\Psi = \lambda \vee \mu \Leftrightarrow \Psi(x) = \max\{\lambda(x), \mu(x)\}$,

(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \Psi(x) = \min\{\lambda(x), \mu(x)\}$,

(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i(\lambda_i)$ and intersection $\delta = \wedge_i(\lambda_i)$, are defined respectively as

(vi) $\Psi(x) = \sup_i\{\lambda_i(x) / x \in X\}$,

(vii) $\delta(x) = \inf_i\{\lambda_i(x) / x \in X\}$.

Definition 2.2[1]**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior and the closure of λ are defined respectively as follows:

(i) $\mathrm{int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$ and

(ii) $cl(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1[1]**.** *For a fuzzy set λ of a fuzzy topological space X,*

(i) $1 - Int(\lambda) = Cl(1 - \lambda)$,

(ii) $1 - Cl(\lambda) = Int(1 - \lambda)$.

Definition 2.3[12]**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i (i = 1, 2)$. The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.4[6]**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$, in (X, T_1, T_2).

Definition 2.5[6]**.** A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $\mathrm{int}_{T_1} cl_{T_2}(\lambda) = \mathrm{int}_{T_2} cl_{T_1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.6[8]**.** Let (X, T_1, T_2) be the fuzzy bitopological space. A fuzzy set λ defined on X is called a pairwise fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set in (X, T_1, T_2).

Definition 2.7[8]**.** If λ is a pairwise fuzzy first category set in the fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $(1 - \lambda)$ is called a pairwise fuzzy residual set in (X, T_1, T_2).

Definition 2.8[12]**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.9[12]**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X, T_1, T_2).

Definition 2.10[2]**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy regular open set in (X, T_1, T_2), if $\text{int}_{T_1} cl_{T_2}(\lambda) = \lambda = \text{int}_{T_2} cl_{T_1}(\lambda)$, in (X, T_1, T_2). That is, $\text{int}_{T_i} cl_{T_j}(\lambda) = \lambda$, $(i \neq j$ and $i, j = 1, 2)$.

Definition 2.11[2]**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy regular closed set in (X, T_1, T_2), if $cl_{T_1} \text{int}_{T_2}(\lambda) = \lambda = cl_{T_2} \text{int}_{T_1}(\lambda)$, in (X, T_1, T_2). That is, $cl_{T_i} cl_{T_j}(\lambda) = \lambda$, $(i \neq j$ and $i, j = 1, 2)$.

Definition 2.12[14]**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy regular G_δ-set in (X, T_1, T_2), if $\lambda = \wedge_{k=1}^{\infty}(\text{int}_{T_i} cl_{T_j}(\lambda_k))(i \neq j$ and $i, j = 1, 2)$ where (λ_i)'s are fuzzy sets in (X, T_1, T_2).

Definition 2.13[14]**.** A fuzzy set μ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy regular F_σ-set in (X, T_1, T_2), if

$\lambda = \vee_{k=1}^{\infty}(cl_{T_i}\ \mathrm{int}_{T_j}(\mu_k))(i \neq j$ and $i, j = 1, 2)$ where (μ_k)'s are fuzzy sets in (X, T_1, T_2).

Definition 2.14[12]**.** A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy somewhere dense set in (X, T_1, T_2), if $(\mathrm{int}_{T_i}\ cl_{T_j}(\lambda) \neq 0\,(i \neq j$ and $i, j = 1, 2)$. That is, λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2), if $\mathrm{int}_{T_1}\ cl_{T_2}(\lambda) \neq 0$ and $\mathrm{int}_{T_2}\ cl_{T_1}(\lambda) \neq 0$, in (X, T_1, T_2).

Definition 2.15[9]**.** A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy strongly irresolvable space if $cl_{T_1}\ \mathrm{int}_{T_2}(\lambda) = 1 = cl_{T_2}\ \mathrm{int}_{T_1}(\lambda)$ for each pair wise fuzzy dense set λ in (X, T_1, T_2).

Definition 2.16[8]**.** A fuzzy bitopological (X, T_1, T_2) is called a pairwise fuzzy Baire space if $\mathrm{int}_{T_i}[\vee_{k=1}^{\infty}\ (\lambda_k)] = 0$, $(i = 1, 2)$, where (λ_k)'s are pair wise fuzzy nowhere dense sets in (X, T_1, T_2).

Definition 2.17[8]**.** A fuzzy bitopological (X, T_1, T_2) is called a pairwise first category space if the fuzzy set 1_X is a pairwise fuzzy first category set in (X, T_1, T_2). That is, $X = \vee_{k=1}^{\infty}\ (\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called a pairwise fuzzy second category space.

Definition 2.18[12]**.** A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy P-space if every non-zero pairwise fuzzy G_δ-set in (X, T_1, T_2), is a pairwise fuzzy open set in (X, T_1, T_2). That is, if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy open sets in (X, T_1, T_2), then λ is a pairwise fuzzy open set in (X, T_1, T_2).

Definition 2.19[13]**.** A fuzzy set λ in the fuzzy bitopological space (X, T_1, T_2) is called a pair wise fuzzy σ-nowhere dense set if λ is a pair wise fuzzy F_σ-set in (X, T_1, T_2) such that $\mathrm{int}_{T_1}\ \mathrm{int}_{T_2}(\lambda) = \mathrm{int}_{T_2}\ \mathrm{int}_{T_1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.20[16]**.** A fuzzy bitopological (X, T_1, T_2) is called a pair wise fuzzy σ-Baire space if $\text{int}_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = 0, (i = 1, 2)$, where (λ_k)'s are pair wise fuzzy σ-nowhere dense sets in (X, T_1, T_2).

Definition 2.20[11]**.** A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy D-Baire space if $\text{int}_{T_1} cl_{T_2}(\vee_{k=1}^{\infty}(\lambda_k)) = \text{int}_{T_2} cl_{T_1}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, where (λ_k)'s are pair wise fuzzy nowhere dense sets in (X, T_1, T_2).

Definition 2.20[9]**.** A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy almost resolvable space, if $\vee_{k=1}^{\infty}(\lambda_k) = 1$, where the fuzzy sets (λ_k)'s in (X, T_1, T_2) are such that $\text{int}_{T_2}(\lambda_k) = 0, (i = 1, 2)$. Otherwise (X, T_1, T_2) is called a pairwise fuzzy almost irresolvable space.

Theorem 2.1[14]**.** *Let (X, T_1, T_2) be the fuzzy bitopological space.*

(a) *If λ is a pair wise fuzzy open set in (X, T_1, T_2), then $cl_{T_i}(\lambda)$ $(i = 1, 2)$ is a pair wise fuzzy regular closed set in (X, T_1, T_2).*

(b) *If μ is a pair wise fuzzy closed set in (X, T_1, T_2), then $\text{int}_{T_i}(\mu)$ $(i = 1, 2)$ is a pair wise fuzzy regular open set in (X, T_1, T_2).*

Theorem 2.2[14]**.** If λ is a pairwise fuzzy regular G_δ-set in the fuzzy bitopological space (X, T_1, T_2) if and only if $1 - \lambda$ is a pairwise fuzzy regular F_σ-set in (X, T_1, T_2).

Theorem 2.3[14]**.** *Let (X, T_1, T_2) is a fuzzy bitopological space.*

(1) *If λ is a pairwise fuzzy regular G_δ-set in (X, T_1, T_2), then $\lambda = \wedge_{k=1}^{\infty}(\delta_k)$, where (δ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2).*

(2) *If μ is a pairwise fuzzy regular F_σ-set in (X, T_1, T_2), then $\mu = \vee_{k=1}^{\infty}(\mu_k)$, where (μ_k)'s are pair wise fuzzy regular closed sets in (X, T_1, T_2).*

Theorem 2.4[10]. *If* $cl_{T_1}cl_{T_2}(\lambda) = 1$ *and* $cl_{T_2}cl_{T_1}(\lambda) = 1$, *for a fuzzy set* λ *in the pair wise fuzzy strongly irresolvable space* (X, T_1, T_2), *then* $cl_{T_1}(\lambda) = 1$ *and* $cl_{T_2}(\lambda) = 1$ *in* (X, T_1, T_2).

Theorem 2.5[14]. *If* λ *is a pairwise fuzzy regular* G_δ*-set in the fuzzy bitopological space* (X, T_1, T_2), *then* λ *is a pair wise fuzzy* G_δ*-set in* (X, T_1, T_2).

Theorem 2.6[14]. *If* λ *is a pairwise fuzzy regular* F_σ*-set in the fuzzy bitopological space* (X, T_1, T_2), *then* λ *is a pair wise fuzzy* F_σ*-set in* (X, T_1, T_2).

Theorem 2.7[9]. *If* λ *is a pair wise fuzzy dense and pair wise fuzzy* G_δ*-set in the pair wise fuzzy strongly irresolvable space* (X, T_1, T_2), *then* $1 - \lambda$ *is a pairwise fuzzy first category set in* (X, T_1, T_2).

Theorem 2.8[8]. *If the fuzzy bitopological space* (X, T_1, T_2) *is a pair wise fuzzy Baire space, then* (X, T_1, T_2) *is a pair wise fuzzy second category space.*

Theorem 2.9[13]. *If* λ *is a pair wise fuzzy* G_δ*-set such that* $cl_{T_i}(\lambda) = 1, (i = 1, 2)$, *in the fuzzy bitopological space* (X, T_1, T_2), *then* $1 - \lambda$ *is a pair wise fuzzy first category set in* (X, T_1, T_2).

Theorem 2.10[13]. In the fuzzy bitopological space (X, T_1, T_2), the fuzzy set λ is a pair wise fuzzy σ-nowhere dense set in (X, T_1, T_2) if and only if $(1 - \lambda)$ is a pair wise fuzzy dense and pair wise fuzzy G_δ-set in (X, T_1, T_2).

Theorem 2.11[9]. *If the fuzzy bitopological space* (X, T_1, T_2) *is the pairwise fuzzy second category space, then* (X, T_1, T_2) *is the pairwise fuzzy almost irresolvable space.*

Theorem 2.12[11]. *If the pairwise fuzzy first category set* λ, *is a pairwise fuzzy closed set, in the pairwise fuzzy Baire space* (X, T_1, T_2), *then* (X, T_1, T_2), *is the pairwise fuzzy D-Baire space.*

Theorem 2.13[15]. *If* λ *is a pair wise fuzzy somewhere dense set in the* *fuzzy bitopological space* (X, T_1, T_2), *then* $cl_{T_i} \operatorname{int}_{T_j}(1-\lambda) \neq 1 (i \neq j$ *and* $i, j = 1, 2)$, *in* (X, T_1, T_2).

Theorem 2.14[7]. *If* $\operatorname{int}_{T_i} \operatorname{int}_{T_j}(\lambda) = 0, (i, j = 1, 2$ *and* $i \neq j)$ *where* λ *is a pairwise fuzzy* F_σ*-set in the pairwise fuzzy submaximal space* (X, T_1, T_2), *then* (X, T_1, T_2) *is the pairwise fuzzy P-space.*

3. Pairwise Weak Fuzzy P-Spaces

Definition 3.1. A fuzzy bitopological space (X, T_1, T_2) is called a pair wise weak fuzzy P-space if $\wedge_{k=1}^{\infty} (\lambda_k)$ is a pair wise fuzzy regular open set in (X, T_1, T_2), where (λ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2).

That is, $\operatorname{int}_{T_i} cl_{T_j}[\wedge_{k=1}^{\infty} (\lambda_k)] = [\wedge_{k=1}^{\infty} (\lambda_k)] (i \neq j$ and $i, j = 1, 2)$.

Example 3.1. Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ, γ and β defined on X as follows

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.4, \lambda(b) = 0.5, \lambda(c) = 0.6$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.6, \mu(b) = 0.4, \mu(c) = 0.5$,

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.5, \gamma(b) = 0.6, \gamma(c) = 0.4$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.8, \gamma(b) = 0.5, \gamma(c) = 0.4$.

Then,

$T_1 = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, [\gamma \wedge (\lambda \vee \mu)]$

$[\mu \wedge (\lambda \vee \gamma)], [\lambda \wedge (\mu \vee \gamma)], [\gamma \vee (\lambda \wedge \mu)], [\mu \vee (\lambda \wedge \gamma)], [\lambda \vee (\mu \wedge \gamma)]$,

$[\lambda \wedge \gamma \wedge \gamma], [\lambda \vee \mu \vee \gamma], 1\}$ and

$T_2 = \{0, \lambda, \gamma, \beta, \lambda \vee \gamma, \lambda \vee \beta, \gamma \vee \beta, \lambda \wedge \gamma, \gamma \wedge \beta, [\lambda \vee (\gamma \wedge \beta)], [\lambda \vee \gamma \vee \beta], 1\}$

are fuzzy topologies on X. On computation, $\lambda, \gamma, \lambda \vee \gamma, \lambda \wedge \gamma, [\gamma \wedge (\lambda \vee \mu)]$, $[\lambda \vee (\mu \wedge \gamma)], \gamma \wedge \beta, [\lambda \vee (\gamma \wedge \beta)]$, are pair wise fuzzy open sets in (X, T_1, T_2).

Also,

$$\operatorname{int}_{T_1} cl_{T_2}[\gamma \wedge (\lambda \vee \mu)] = \operatorname{int}_{T_2} cl_{T_1}[\gamma \wedge (\lambda \vee \mu)] = \gamma \wedge (\lambda \vee \mu),$$

$$\operatorname{int}_{T_1} cl_{T_2}[\lambda \vee (\mu \wedge \gamma)] = \operatorname{int}_{T_2} cl_{T_1}[\lambda \vee (\mu \wedge \gamma)] = \lambda \vee (\mu \wedge \gamma),$$

$$\operatorname{int}_{T_1} cl_{T_2}[\gamma \wedge \beta] = \operatorname{int}_{T_2} cl_{T_1}[\gamma \wedge \beta] = \gamma \wedge \beta,$$

$$\operatorname{int}_{T_1} cl_{T_2}[\lambda \vee (\gamma \wedge \beta)] = \operatorname{int}_{T_2} cl_{T_1}[\lambda \vee (\lambda \wedge \beta)] = \lambda \vee (\gamma \wedge \beta).$$

Thus $[\gamma \wedge (\lambda \vee \mu)], [\lambda \vee (\mu \wedge \gamma)], (\gamma \wedge \beta), [\lambda \vee (\gamma \wedge \beta)]$ are pair wise fuzzy regular open sets in (X, T_1, T_2). Also on computation one can find that $[\gamma \wedge (\lambda \vee \mu)] \wedge [\lambda \vee (\mu \wedge \gamma)] \wedge (\gamma \wedge \beta \wedge [\lambda \vee (\gamma \wedge \beta)])$ and $\gamma \wedge (\lambda \vee \mu)$ is a pair wise fuzzy regular open set in (X, T_1, T_2). Thus the fuzzy bitopological space (X, T_1, T_2) is a pair wise weak fuzzy P-space.

Proposition 3.1. *A fuzzy bitopological space* (X, T_1, T_2) *is a pair wise weak fuzzy P-space if* $\vee_{k=1}^{\infty} (\mu_k)$ *is a pair wise fuzzy regular closed set in* (X, T_1, T_2), *where* (μ_k)*'s are pair wise fuzzy regular closed sets in* (X, T_1, T_2).

Proof. Let the fuzzy bitopological space (X, T_1, T_2) be a pair wise weak fuzzy P-space. Then, $\operatorname{int}_{T_i} cl_{T_j}[\wedge_{k=1}^{\infty} (\lambda_k)] = [\wedge_{k=1}^{\infty} (\lambda_k)] (i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2).

Now, $1 - \operatorname{int}_{T_i} cl_{T_j}[\wedge_{k=1}^{\infty} (\lambda_k)] = [\wedge_{k=1}^{\infty} (\lambda_k)]$, in (X, T_1, T_2) and this implies that $cl_{T_i} \operatorname{int}_{T_j}(1 - [\wedge_{k=1}^{\infty} (\lambda_k)]) = 1 - [\wedge_{k=1}^{\infty} (\lambda_k)] (i \neq j$ and $i, j = 1, 2)$. Then, $cl_{T_i} \operatorname{int}_{T_j}[\vee_{k=1}^{\infty} (1 - \lambda_k)] = [\vee_{k=1}^{\infty} (1 - \lambda_k)] (i \neq j$ and $i, j = 1, 2)$. Let $\mu_k = 1 - \lambda_k$. Since (λ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2), (μ_k)'s are pair wise fuzzy regular closed sets in (X, T_1, T_2). Thus, one can find that $cl_{T_i} \operatorname{int}_{T_j}[\vee_{k=1}^{\infty} (\lambda_k)] = [\vee_{k=1}^{\infty} (\lambda_k)] (i \neq j$ and $i, j = 1, 2)$, where (μ_k)'s are pair wise fuzzy regular closed sets in (X, T_1, T_2), and this implies that $\vee_{k=1}^{\infty} (\mu_k)$ is a pair wise fuzzy regular closed set in (X, T_1, T_2).

Conversely, suppose that $\vee_{k=1}^{\infty}(\mu_k)$ is a pair wise fuzzy regular closed set in (X, T_1, T_2), where (μ_k)'s are pair wise fuzzy regular closed sets in (X, T_1, T_2). Then, $cl_{T_i}\operatorname{int}_{T_j}[\vee_{k=1}^{\infty}(\mu_k)] = [\vee_{k=1}^{\infty}(\mu_k)](i \neq j$ and $i, j = 1, 2)$. This implies that $1 - cl_{T_i}\operatorname{int}_{T_j}[\vee_{k=1}^{\infty}(\mu_k)] = 1 - [\vee_{k=1}^{\infty}(\mu_k)](i \neq j$ and $i, j = 1, 2)$ and then, $\operatorname{int}_{T_i} cl_{T_j}[1 - [\vee_{k=1}^{\infty}(\mu_k)]] = 1 - [\vee_{k=1}^{\infty}(\mu_k)]$, in (X, T_1, T_2). By the Lemma 2.1, $\operatorname{int}_{T_i} cl_{T_j}[\wedge_{k=1}^{\infty}(1 - \mu_k)] = \wedge_{k=1}^{\infty}(1 - \mu_k)$, in (X, T_1, T_2). Let $\lambda_k = 1 - \mu_k$. Since (μ_k)'s are pair wise fuzzy regular closed sets in (X, T_1, T_2), (λ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2). Thus, one can find that $\operatorname{int}_{T_i} cl_{T_j}[\wedge_{k=1}^{\infty}(\lambda_k)] = [\wedge_{k=1}^{\infty}(\lambda_k)](i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2).

Hence the fuzzy bitopological space (X, T_1, T_2) is a pair wise weak fuzzy P-space.

Proposition 3.2. *If the fuzzy bitopological space (X, T_1, T_2) is a pairwise weak fuzzy P-space, then $cl_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(\lambda_k)]$, $(i = 1, 2)$ where the fuzzy sets (λ_k)'s are pair wise fuzzy open sets in (X, T_1, T_2).*

Proof. Let (λ_k)'s be pair wise fuzzy open sets in (X, T_1, T_2). Then, by the Theorem 2.1, $[cl_{T_i}(\lambda_k)]$, $(i = 1, 2)$ are pair wise fuzzy regular closed sets in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise weak fuzzy P-space, by the Proposition 3.1, $\vee_{k=1}^{\infty}[cl_{T_i}(\lambda_k)]$ is a pair wise fuzzy regular closed set in (X, T_1, T_2). Then, $cl_{T_i}\operatorname{int}_{T_j}[\vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))] = \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))(i \neq j$ and $i, j = 1, 2)$, in (X, T_1, T_2). Now $cl_{T_i}\operatorname{int}_{T_j}[\vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))]$ $\leq cl_{T_i}\operatorname{int}_{T_j}[\vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))] = \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))$ and then $cl_{T_i}\operatorname{int}_{T_j}[\vee_{k=1}^{\infty}((\lambda_k))]$ $\leq \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))$. Since (λ_k)'s are pairwise fuzzy open sets in (X, T_1, T_2),

$\text{int}_{T_i}[\lambda_k] = \lambda_k (i = 1, 2)$ and $\text{int}_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] \geq \vee_{k=1}^{\infty}[\text{int}_{T_i}(\lambda_k)] = \vee_{k=1}^{\infty}(\lambda_k)$ and then $\vee_{k=1}^{\infty} (\lambda_k) \leq \text{int}_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)]$. But $[\vee_{k=1}^{\infty} (\lambda_k)] \geq \text{int}_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)]$. This implies that, $\text{int}_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}(\lambda_k)$. Thus, $cl_{T_i}\, \text{int}_{T_j}[\vee_{k=1}^{\infty} (\lambda_k)] = cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)]$ and $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] \leq \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))$. But $\vee_{k=1}^{\infty} (cl_{T_i}(\lambda_k)) \leq cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)]$.

Hence in the pairwise weak fuzzy P-space (X, T_1, T_2), $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}(cl_{T_i}(\lambda_k))$, where $(\lambda_k)'k$ are pair wise fuzzy open sets in (X, T_1, T_2).

Proposition 3.3. *If the fuzzy bitopological space* (X, T_1, T_2) *is the pairwise weak fuzzy P-space, then* $\text{int}_{T_i}[\wedge_{k=1}^{\infty} (\mu_k)] = \wedge_{k=1}^{\infty}[\text{int}_{T_i}(\mu_k)]$, $(i = 1, 2)$ *where the fuzzy sets* (μ_k)*'s are pair wise fuzzy closed sets in* (X, T_1, T_2).

Proof. Let (μ_k)'s be the pair wise fuzzy closed sets in (X, T_1, T_2).

Then, $[1 - \mu_k]$'s are pair wise fuzzy open sets in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise weak fuzzy P-space, by the Proposition 3.2, $cl_{T_i}[\vee_{k=1}^{\infty} (1 - \mu_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(1 - \mu_k)]$, $(i = 1, 2)$. This implies that $cl_{T_i}[\wedge_{k=1}^{\infty} (1 - \mu_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(1 - \mu_k)]$ and then $1 - \text{int}_{T_i}[\wedge_{k=1}^{\infty} (\mu_k)] = 1 - \wedge_{k=1}^{\infty}(\text{int}_{T_i}(\mu_k))$. Hence $\text{int}_{T_i}[\wedge_{k=1}^{\infty} (\mu_k)] = \wedge_{k=1}^{\infty} \text{int}_{T_i}(\mu_k)$, $(i = 1, 2)$ where the fuzzy sets (μ_k)'s are pair wise fuzzy closed sets in (X, T_1, T_2).

Proposition 3.4. *If the fuzzy bitopological space* (X, T_1, T_2) *is a pair wise fuzzy P-space, then* (X, T_1, T_2) *is a pair wise weak fuzzy P-space.*

Proof. Let (λ_k)'s $(k = 1 \text{ to } \infty)$ be pair wise fuzzy regular closed sets in the fuzzy bitopological space (X, T_1, T_2). Since the pair wise fuzzy regular closed sets in (X, T_1, T_2) are pair wise fuzzy closed sets, (λ_k)'s $(k = 1 \text{ to } \infty)$ are pair wise fuzzy closed sets in (X, T_1, T_2). Let $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$. Then, λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy P-space, by Theorem 3.1, λ is a pairwise fuzzy closed set in (X, T_1, T_2). Then, $cl_{T_i}(\lambda) = \lambda (i = 1, 2)$ and then $cl_{T_i} = (\vee_{k=1}^{\infty} (\lambda_k)) = \vee_{k=1}^{\infty}(\lambda_k)$. Now $cl_{T_i} \operatorname{int}_{T_j}[\vee_{k=1}^{\infty} (\lambda_k)] \leq cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}(\lambda_k)$, $(i = 1, 2)$. implies that $cl_{T_i} \operatorname{int}_{T_j}[\vee_{k=1}^{\infty} (\lambda_k)] \leq \vee_{k=1}^{\infty}(\lambda_k)$ (1). Since (λ_k)'s are pair wise fuzzy regular closed sets in (X, T_1, T_2), $cl_{T_i} \operatorname{int}_{T_j}(\lambda_k) = \lambda_k$, $(i \neq j$ and $i, j = 1, 2)$. This implies that $\vee_{k=1}^{\infty} cl_{T_i} \operatorname{int}_{T_j}(\vee_{k=1}^{\infty} (\lambda_k)) = \vee_{k=1}^{\infty}(\lambda_k)$ in (X, T_1, T_2). Now $\vee_{k=1}^{\infty} (\lambda_k) = \vee_{k=1}^{\infty} cl_{T_i} \operatorname{int}_{T_i}(\lambda_k) \leq cl_{T_i} \operatorname{int}_{T_j}[\vee_{k=1}^{\infty} (\lambda_k)]$, implies that $\vee_{k=1}^{\infty} (\lambda_k) \leq cl_{T_i} \operatorname{int}_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)]$ (2). From (1) and (2), one will have that $cl_{T_i} \operatorname{int}_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s ($k = 1$ to ∞) are pair wise fuzzy regular closed sets in (X, T_1, T_2).

Hence, by the Proposition 3.1, (X, T_1, T_2) is a pair wise weak fuzzy P-space.

Remark. The converse of the above proposition need not be true. That is, pair wise weak fuzzy P-spaces need not be pair wise fuzzy P-spaces. For consider the following example.

Example 3.2. Let $X = \{a, b, c\}$. Consider the fuzzy sets $\lambda, \mu, \gamma, \alpha$ and β defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.6, \lambda(b) = 0.4, \lambda(c) = 0.5$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4, \mu(b) = 0.7, \mu(c) = 0.6$,

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.5, \gamma(b) = 0.3, \gamma(c) = 0.7$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.5, \beta(b) = 0.2, \beta(c) = 0.7$.

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.5, \alpha(b) = 0.4, \alpha(c) = 0.6$.

Then, $T_1 = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, [\gamma \wedge (\lambda \vee \mu)]$

$[\mu \wedge (\lambda \vee \gamma)], [\lambda \wedge (\mu \vee \gamma)], [\gamma \vee (\lambda \wedge \mu)], [\mu \vee (\lambda \wedge \gamma)], [\lambda \vee (\mu \wedge \gamma)],$

$[\lambda \wedge \mu \wedge \gamma], [\lambda \vee \mu \vee \gamma], 1\}$ and

$T_2 = \{0, \lambda, \mu, \beta, \lambda \vee \mu, \lambda \vee \beta, \mu \vee \beta, \lambda \wedge \mu, \lambda \wedge \beta, \mu \wedge \beta, [\beta \wedge (\lambda \vee \mu)]$

$[\mu \wedge (\lambda \vee \beta)], [\lambda \wedge (\mu \vee \beta)], [\beta \vee (\lambda \wedge \mu)], [\mu \vee (\lambda \wedge \beta)], [\lambda \vee (\mu \wedge \beta)],$

$[\lambda \wedge \mu \wedge \beta], [\lambda \vee \mu \vee \beta], 1\}$

are fuzzy topologies on X. On computation, $\lambda, \mu, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu,$ $[\mu \wedge (\lambda \wedge \gamma)], [\lambda \wedge (\mu \vee \gamma)], [\gamma \vee (\lambda \wedge \mu)], [\mu \vee (\lambda \wedge \gamma)], [\lambda \vee (\mu \wedge \gamma)], [\lambda \vee \mu \vee \gamma],$ $\lambda \vee \beta, \mu \vee \beta, [\mu \wedge (\lambda \vee \beta)], [\lambda \wedge (\mu \vee \beta)], [\beta \vee (\lambda \wedge \mu)], [\mu \vee (\lambda \wedge \beta)], [\lambda \vee (\mu \wedge \beta)],$ $[\lambda \vee \mu \vee \beta]$, are pair wise fuzzy open sets in (X, T_1, T_2).

Also, $\text{int}_{T_1} cl_{T_2}[\lambda] = \text{int}_{T_2} cl_{T_1}[\lambda] = \lambda,$

$\text{int}_{T_1} cl_{T_2}[\lambda \wedge (\mu \vee \gamma)] = \text{int}_{T_2} cl_{T_1}[\lambda \wedge (\mu \vee \gamma)] = \lambda \wedge (\mu \vee \gamma),$

$\text{int}_{T_1} cl_{T_2}[\lambda \wedge (\mu \vee \beta)] = \text{int}_{T_2} cl_{T_1}[\lambda \wedge (\mu \vee \beta)] = \lambda \wedge (\mu \vee \beta).$

Thus $\lambda, [\lambda \wedge (\mu \vee \gamma)], [\lambda \wedge (\mu \vee \beta)]$ are pair wise fuzzy regular open sets in (X, T_1, T_2). Also on computation $\lambda \wedge [\lambda \wedge (\mu \vee \gamma)] \wedge [\lambda \wedge (\mu \vee \beta)] = [\lambda \wedge (\mu \vee \beta)]$ and $[\lambda \wedge (\mu \vee \beta)]$ is the pair wise fuzzy regular open set in (X, T_1, T_2) implies that the fuzzy bitopological space (X, T_1, T_2) is the pair wise weak fuzzy P-space. Now $\alpha = [\lambda \vee (\mu \wedge \gamma)] \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)],$ implies that α is the pair wise fuzzy G_δ-set in (X, T_1, T_2). But α is not the pairwise fuzzy open set in (X, T_1, T_2) and hence (X, T_1, T_2) is not the pair wise fuzzy P-space.

Proposition 3.5. *If λ is the non-zero pair wise fuzzy regular G_δ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), then λ is the pair wise fuzzy regular open set in (X, T_1, T_2).*

Proof. Let (X, T_1, T_2) be the pair wise weak fuzzy P-space and λ is the pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Then, $\lambda = \wedge_{k=1}^{\infty}(\text{int}_{T_i} cl_{T_j}(\lambda_k))$ $(i \neq j$ and $i, j = 1, 2)$ where (λ_k)'s are fuzzy sets in (X, T_1, T_2). Since $cl_{T_j}(\lambda_k)(j = 1, 2)$ is the fuzzy closed set in (X, T_1, T_2), by the Theorem 2.1,

$(\text{int}_{T_i}[cl_{T_j}(\lambda_k)])$'s are pair wise fuzzy regular open sets in (X, T_1, T_2).

Let $\delta_k = \text{int}_{T_i}[cl_{T_j}(\lambda_k)]$. Then (δ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2). Then $\lambda = \wedge_{k=1}^{\infty}(\delta_k)$, where (δ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise weak fuzzy P-space, λ is the pair wise fuzzy regular open set in (X, T_1, T_2).

Proposition 3.6. *If* μ *is the non-zero pair wise fuzzy regular* F_σ*-set in the pair wise weak fuzzy P-space* (X, T_1, T_2), *then* μ *is the pair wise fuzzy regular closed set in* (X, T_1, T_2).

Proof. Let μ be the pair wise fuzzy regular F_σ-set in (X, T_1, T_2). Then, by the Theorem 2.2, $1 - \mu$ is the pair wise fuzzy regular G_δ-set (X, T_1, T_2).

Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.5, $1 - \mu$ is the pair wise fuzzy regular open set (X, T_1, T_2). Then, μ is the pair wise fuzzy regular closed set in (X, T_1, T_2).

Proposition 3.7. *Let* (X, T_1, T_2) *be the fuzzy bitopological space. Then,* (X, T_1, T_2) *is a pair wise weak fuzzy P-space if and only if each pair wise fuzzy regular* G_δ*-set in* (X, T_1, T_2) *is the pair wise fuzzy regular open set in* (X, T_1, T_2).

Proof. Let (X, T_1, T_2) be the pair wise weak fuzzy P-space and λ is a pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Then, by the Proposition 3.5, λ is the pair wise fuzzy regular open set in (X, T_1, T_2).

Conversely suppose that each pair wise fuzzy regular G_δ-set in (X, T_1, T_2) is a pair wise fuzzy regular open set in (X, T_1, T_2). Let λ be a pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Then, by the Theorem 2.4, $\lambda = \wedge_{k=1}^{\infty}(\delta_k)$, where (δ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2).

By the hypothesis, λ is a pair wise fuzzy regular open set in (X, T_1, T_2).

That is, $\text{int}_{T_i} cl_{T_j}(\lambda) = \lambda (i \neq j$ and $i, j = 1, 2)$. This implies that, $\text{int}_{T_i} cl_{T_j}(\wedge_{k=1}^{\infty} (\delta_k)) = \wedge_{k=1}^{\infty} (\delta_k)$, where (δ_k)'s are pair wise fuzzy regular open sets in (X, T_1, T_2) and hence (X, T_1, T_2) is a pair wise weak fuzzy P-space.

Proposition 3.8. *Let (X, T_1, T_2) be the fuzzy bitopological space. Then, (X, T_1, T_2) is a pair wise weak fuzzy P-space if and only if each pair wise fuzzy regular F_σ-set in (X, T_1, T_2) is the pair wise fuzzy regular closed set in (X, T_1, T_2).*

Proof. The proof follows from the Proposition 3.6 and the Theorem 2.2.

Proposition 3.9. *If λ is the non-zero pair wise fuzzy regular G_δ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), then λ is the pair wise fuzzy somewhere dense set in (X, T_1, T_2).*

Proof. Let λ be the non-zero pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, the pair wise fuzzy regular G_δ-set λ in (X, T_1, T_2) is the pair wise fuzzy regular open set in (X, T_1, T_2) and then $\text{int}_{T_1} cl_{T_2}(\lambda) = \lambda = \text{int}_{T_2} cl_{T_1}(\lambda)$, in (X, T_1, T_2). This implies that $\text{int}_{T_1} cl_{T_2}(\lambda) \neq 0$ and $\text{int}_{T_1} cl_{T_2}(\lambda) \neq 0$, in (X, T_1, T_2) and hence λ is the pairwise fuzzy somewhere dense set in (X, T_1, T_2).

Definition 3.2. A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy cs dense set in (X, T_1, T_2), if $1 - \lambda$ is the pair wise fuzzy somewhere dense set in (X, T_1, T_2).

Proposition 3.10. *If μ is the non-zero pair wise fuzzy regular F_σ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), then μ is the pair wise fuzzy cs dense set in (X, T_1, T_2).*

Proof. Let μ be the non-zero pair wise fuzzy regular F_σ-set in (X, T_1, T_2). Then, by the Theorem 2.2, $1 - \mu$ is the pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.8, $1-\mu$ is the pair wise fuzzy somewhere dense set in (X, T_1, T_2) and hence μ is the pair wise fuzzy cs dense set in (X, T_1, T_2).

Proposition 3.11. *If λ is the non-zero pair wise fuzzy regular G_δ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), then λ is the pair wise fuzzy regular open and pair wise fuzzy G_δ-set in (X, T_1, T_2).*

Proof. Let λ be the non-zero pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.6, the pair wise fuzzy regular G_δ-set λ in (X, T_1, T_2) is the pair wise fuzzy regular open set in (X, T_1, T_2). By the Theorem 2.5, the pair wise fuzzy regular G_δ-set λ in (X, T_1, T_2) is the pair wise fuzzy G_δ-set in (X, T_1, T_2).

Hence λ is the pair wise fuzzy regular open and pair wise fuzzy G_δ-set in (X, T_1, T_2).

Proposition 3.12. *If λ is the non-zero pair wise fuzzy regular F_σ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), then λ is the pair wise fuzzy regular closed and pair wise fuzzy F_σ-set in (X, T_1, T_2).*

Proof. Let λ be the non-zero pair wise fuzzy regular F_σ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.6, the pair wise fuzzy regular F_σ-set λ in (X, T_1, T_2) is the pair wise fuzzy regular closed set in (X, T_1, T_2). By the Theorem 2.6, the pair wise fuzzy regular F_σ-set λ in (X, T_1, T_2) is the pair wise fuzzy F_σ-set in (X, T_1, T_2).

Hence λ is the pair wise fuzzy regular closed and pair wise fuzzy F_σ-set in (X, T_1, T_2).

Proposition 3.13. *If the pair wise fuzzy regular G_δ-set λ is the pair wise fuzzy dense set in the fuzzy bitopological space (X, T_1, T_2), then $\lambda = \wedge_{k=1}^{\infty}(\mathrm{int}_{T_i}\, cl_{T_j}(\lambda_k))(i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are fuzzy the pair* *wise dense sets in* (X, T_1, T_2).

Proof. Let λ be the pair wise fuzzy regular G_δ-set in the fuzzy bitopological space (X, T_1, T_2). Then, $\lambda = \wedge_{k=1}^{\infty}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k))(i \neq j$ and $i, j = 1, 2)$ where (λ_k)'s are fuzzy sets defined on X in (X, T_1, T_2). By the hypothesis, λ is the pairwise fuzzy dense set in (X, T_1, T_2). Then, $cl_{T_i} cl_{T_j}(\lambda) = 1, (i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). This implies that $cl_{T_i} cl_{T_j}(\wedge_{k=1}^{\infty}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k))) = 1$. But $cl_{T_i} cl_{T_j}(\wedge_{k=1}^{\infty}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k))) \leq (\wedge_{k=1}^{\infty} cl_{T_i} cl_{T_j}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k)))$ and then $\wedge_{k=1}^{\infty} cl_{T_i} cl_{T_j}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k)) = 1$ and then $cl_{T_i} cl_{T_j}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k)) = 1$, in (X, T_1, T_2). Now $cl_{T_i} cl_{T_j}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k)) \leq cl_{T_i} cl_{T_j}(l_{T_j}(\lambda_k)) = cl_{T_i} cl_{T_j}(\lambda_k)$, and this implies that $cl_{T_i} cl_{T_j}((\lambda_k)) = 1$. Hence (λ_k)'s are the fuzzy pair wise dense sets in (X, T_1, T_2). Thus, $\lambda = \wedge_{k=1}^{\infty}(\text{int}_{T_i}\, cl_{T_j}(\lambda_k))(i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are fuzzy the pair wise dense sets in (X, T_1, T_2).

Proposition 3.14. *If λ is the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the fuzzy pairwise weak fuzzy P-space (X, T_1, T_2), then $(1 - \lambda)$ is the pair wise fuzzy σ-nowhere dense set in (X, T_1, T_2).*

Proof. Let λ be the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 2.5, the pair wise fuzzy regular G_δ-set λ is the pair wise fuzzy G_δ-set in (X, T_1, T_2). Thus λ is the pair wise fuzzy dense and pair wise fuzzy G_δ-set in (X, T_1, T_2). Then, by the Theorem 2.10, $1 - \lambda$ is the pair wise fuzzy σ-nowhere dense set in (X, T_1, T_2).

Proposition 3.15. *If λ is the non-zero pair wise fuzzy regular G_δ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), then λ is the pair wise fuzzy open set in (X, T_1, T_2).*

Proof. Let λ be the pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.5, λ is the pair wise fuzzy regular open set in (X, T_1, T_2). Since the pair wise fuzzy regular open set is the pair wise fuzzy open set in fuzzy bitopological spaces, λ is the pair wise fuzzy open set in (X, T_1, T_2).

Proposition 3.16. *If λ is the non-zero pair wise fuzzy regular G_δ-set in the pair wise weak fuzzy P-space (X, T_1, T_2), $cl_{T_i} \operatorname{int}_{T_j}(1-\lambda) \neq 1 \, (1 \neq j$ and $i, j = 1, 2)$, in (X, T_1, T_2).*

Proof. The proof follows from the Proposition 3.9, and the Theorem 2.13.

4. Pairwise Weak Fuzzy P-Spaces and Other Fuzzy Topological Spaces

Proposition 4.1. *If the pair wise weak fuzzy P-space (X, T_1, T_2) is the pair wise fuzzy hyper connected space and pair wise fuzzy strongly irresolvable space and (λ_k)'s are the pair wise fuzzy open sets in (X, T_1, T_2), then $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = 1$ in (X, T_1, T_2).*

Proof. Let (λ_k)'s be the pair wise fuzzy open sets in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.2, $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(\lambda_k)]$, where the fuzzy sets (λ_k)'s are pair wise fuzzy open sets in (X, T_1, T_2). By the hypothesis, (X, T_1, T_2) is the pair wise fuzzy hyper connected space, and then the pair wise fuzzy open sets (λ_k)'s in (X, T_1, T_2) are pair wise fuzzy dense sets in (X, T_1, T_2) and $cl_{T_1} cl_{T_2}(\lambda_k) = cl_{T_2} cl_{T_1}(\lambda_k) = 1$, in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable space, by the Theorem 2.4, $cl_{T_1}(\lambda_k) = 1$ and $cl_{T_2}(\lambda_k) = 1$, in (X, T_1, T_2). That is, $cl_{T_i}(\lambda_k) = 1$, in (X, T_1, T_2). Thus, $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(\lambda_k)] = \vee_{k=1}^{\infty}(1) = 1$ and hence $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = 1$ in (X, T_1, T_2).

Proposition 4.2. *If λ is a pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then $1-\lambda$ is a pair wise fuzzy first category set in*

(X, T_1, T_2).

Proof. Let λ be a pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Theorem 2.5, the pair wise fuzzy regular G_δ-set λ in (X, T_1, T_2) is the pair wise fuzzy G_δ-set in (X, T_1, T_2).

Thus λ is a pair wise fuzzy dense and pair wise fuzzy G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2). Then, by the Theorem 2.7, $1 - \lambda$ is the pair wise fuzzy first category set in (X, T_1, T_2).

Proposition 4.3. *If λ is a pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then λ is the pair wise fuzzy residual set in (X, T_1, T_2).*

Proof. Let λ be the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space, by the Proposition 4.2, $1 - \lambda$ is a pair wise fuzzy first category set in (X, T_1, T_2). Then, $1 - [1 - \lambda]$ is the pairwise fuzzy residual set in (X, T_1, T_2). Hence λ is the pair wise fuzzy residual set in (X, T_1, T_2).

The following Propositions give the conditions for the pairwise fuzzy strongly irresolvable and pairwise weak fuzzy P-spaces to become pairwise fuzzy Baire spaces.

Proposition 4.4. *If $\operatorname{int}_{T_i}(1 - \lambda) = 0\, (i = 1, 2)$, where λ is the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then the fuzzy bitopological space (X, T_1, T_2) is the pair wise fuzzy Baire space.*

Proof. Suppose that λ is the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space, by the Proposition

4.2, $1-\lambda$ is the pair wise fuzzy first category set in (X, T_1, T_2). Then, $1-\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). From the hypothesis, $\operatorname{int}_{T_i}(1-\lambda) = 0\,(i = 1, 2)$. Thus, $\operatorname{int}_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0\,(i = 1, 2)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) implies that (X, T_1, T_2) is the pair wise fuzzy Baire space.

Proposition 4.5. *If there exists a pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy Baire space.*

Proof. Let λ be the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable space, for the pairwise dense set λ, by the Theorem 2.4, $cl_{T_i}(\lambda) = 1\,(i = 1, 2)$. This implies that $\operatorname{int}_{T_i}(1-\lambda) = 0\,(i = 1, 2)$. Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space, $\operatorname{int}_{T_i}(1-\lambda) = 0\,(i = 1, 2)$ implies, by the Proposition 4.4, that (X, T_1, T_2) is the pairwise fuzzy Baire space.

The following proposition give the conditions for the pairwise fuzzy strongly irresolvable and pairwise weak fuzzy P-spaces to become pairwise fuzzy second category spaces and pairwise fuzzy almost irresolvable spaces.

Proposition 4.6. *If there exists a pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy second category space.*

Proof. Let λ be the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable space, for the pairwise dense set λ, by the Theorem 2.4, $cl_{T_i}(\lambda) = 1\,(i = 1, 2)$. This implies that $\operatorname{int}_{T_i}(1-\lambda) = 0\,(i = 1, 2)$. Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space, $\text{int}_{T_i}(1-\lambda) = 0\,(i = 1, 2)$ implies, by the Proposition 4.4, that (X, T_1, T_2) is the pairwise fuzzy Baire space. Then, by the Theorem 2.8, (X, T_1, T_2) is the pair wise fuzzy second category space.

Proposition 4.7. *If there exists a pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy almost irresolvable space.*

Proof. The proof follows from the Proposition 4.6 and the Theorem 2.11.

The following propositions give the conditions for the pairwise weak fuzzy P-spaces to become pairwise fuzzy σ-Baire spaces and pairwise fuzzy D-Baire spaces.

Proposition 4.8. *If $cl_{T_i}(\wedge_{k=1}^{\infty}(\lambda_k)) = 1\,(i = 1, 2)$, where (λ_k)'s $(k = 1$ to $\infty)$ are pair wise fuzzy dense and pair wise fuzzy regular G_δ-sets in the fuzzy pair wise weak fuzzy P-space (X, T_1, T_2), then (X, T_1, T_2) is the pair wise fuzzy σ-Baire space.*

Proof. Let (λ_k)'s $(k = 1$ to $\infty)$ be the pair wise fuzzy dense and pair wise fuzzy regular G_δ-sets in (X, T_1, T_2). Now $cl_{T_i}[\wedge_{k=1}^{\infty}(\lambda_k)] = 1$, $(i = 1, 2)$, implies that $1 - cl_{T_i}[\wedge_{k=1}^{\infty}(\lambda_k)] = 0$. Then, $\text{int}_{T_i}(1 - [\wedge_{k=1}^{\infty}(\lambda_k)]) = 0$. This implies that $\text{int}_{T_i}[\vee_{k=1}^{\infty}(1-\lambda_k)] = 0$. Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.14, for the pair wise fuzzy dense and pair wise fuzzy regular G_δ-sets (λ_k)'s in $(X, T_1, T_2)(1-\lambda_k)$'s are the pair wise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Thus, $\text{int}_{T_i}[\vee_{k=1}^{\infty}(1-\lambda_k)] = 0$, where $(1-\lambda_k)$'s are the pair wise fuzzy σ-nowhere dense sets in (X, T_1, T_2), implies that (X, T_1, T_2) is the pair wise fuzzy σ-Baire space.

Proposition 4.9. *If there exists a pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space (X, T_1, T_2), then the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy D-Baire space.*

Proof. Suppose that λ is the pair wise fuzzy dense and pair wise fuzzy regular G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise fuzzy strongly irresolvable and pair wise weak fuzzy P-space, by the Proposition 4.2, $1-\lambda$ is the pair wise fuzzy first category set in (X, T_1, T_2). By the Proposition 3.15, the pair wise fuzzy regular G_δ-set λ in the pair wise weak fuzzy P-space (X, T_1, T_2), is the pair wise fuzzy open set in (X, T_1, T_2) and then $1-\lambda$ is the pair wise fuzzy closed set in (X, T_1, T_2). By the Proposition 4.5, (X, T_1, T_2) is the pairwise fuzzy Baire space. Since the pairwise fuzzy first category set $1-\lambda$ is the pair wise fuzzy closed set in the pairwise fuzzy Baire space (X, T_1, T_2), by the Theorem 2.12, (X, T_1, T_2) is the pairwise fuzzy D-Baire space.

Proposition 4.10. *If* $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0$, $(i, j = 1, 2$ *and* $i \neq j)$ *where* λ *is a pair wise fuzzy regular* F_σ*-set in the pairwise fuzzy submaximal then* (X, T_1, T_2) *is the pairwise fuzzy P-space.*

Proof. Suppose that λ is the pair wise fuzzy regular F_σ-set in (X, T_1, T_2). By the Theorem 2.6, λ is the pair wise fuzzy F_σ-set in (X, T_1, T_2). Thus, $\text{int}_{T_i}\text{int}_{T_j}(\lambda) = 0$, $(i, j = 1, 2$ and $i \neq j)$ where λ is the pair wise fuzzy F_σ-set in the pairwise fuzzy submaximal space (X, T_1, T_2). Then, by the Theorem 2.14, (X, T_1, T_2) is the pair wise fuzzy P-space.

5. Pairwise Fuzzy Regular Lindelof Spaces, Pairwise Fuzzy Weakly Lindelof Spaces and Pairwise Fuzzy Weak P-Spaces

Definition 5.1. A collection $\{\lambda_\alpha : \alpha \in \Delta\}$ of pairwise fuzzy regular open sets of the fuzzy bitopological space (X, T_1, T_2) is said to be pairwise fuzzy regular cover of X if $\vee_{\alpha\in\Delta}\{\lambda_a\} = 1$.

Definition 5.2. A fuzzy bitopological space (X, T_1, T_2) is called the pairwise fuzzy weakly regular Lindelof space if for each cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy regular open sets in (X, T_1, T_2), there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n\in N}\{\lambda_{\alpha_n}\}] = 1$, $(i = 1, 2)$ in

(X, T_1, T_2).

That is, if $\vee_{\alpha \in \Delta}\{\lambda_a\} = 1$, where (λ_α)'s are pairwise regular open sets in (X, T_1, T_2), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n \in N} \{\lambda_{\alpha_n}\}] = 1, (i = 1, 2)$ in (X, T_1, T_2).

Definition 5.3. A fuzzy bitopological space (X, T_1, T_2) is called the pairwise fuzzy almost regular Lindelof space if is each cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy regular open sets in (X, T_1, T_2), there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\underset{n \in N}{\vee}(cl_{T_i}[\lambda_{\alpha_n}]) = 1, (i = 1, 2)$ in (X, T_1, T_2).

That is, if $\vee_{\alpha \in \Delta}\{\lambda_a\} = 1$, where (λ_α)'s are pairwise regular open sets in (X, T_1, T_2), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N}(cl_{T_i}[\lambda_{\alpha_n}]) = 1, (i = 1, 2)$ in (X, T_1, T_2).

Definition 5.4. A fuzzy bitopological space (X, T_1, T_2) is called the pairwise fuzzy nearly Lindelof space if for each cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy open sets in (X, T_1, T_2), there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\underset{n \in N}{\vee}[\text{int}_{T_i}\, cl_{T_j}\{\lambda_{\alpha_n}\}] = 1, (i, j = 1, 2 \text{ and } i \neq j)$ in (X, T_1, T_2). That is, if $\vee_{\alpha \in \Delta}\{\lambda_a\} = 1$, where (λ_α)'s are pairwise open sets in (X, T_1, T_2), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N} cl_{T_i} cl_{T_j}\{\lambda_{\alpha_n}\} = 1, (i, j = 1, 2 \text{ and } i \neq j)$ in (X, T_1, T_2).

Definition 5.5. A fuzzy bitopological space (X, T_1, T_2) is called the pairwise fuzzy almost Lindelof space if for each cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy open sets in (X, T_1, T_2), there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\underset{n \in N}{\vee}[cl_{T_i}\{\lambda_{\alpha_n}\}] = 1, (i = 1, 2)$ in (X, T_1, T_2). That is, if $\vee_{\alpha \in \Delta}\{\lambda_a\} = 1$, where (λ_α)'s are pairwise open sets in (X, T_1, T_2), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N} cl_{T_i}\{\lambda_{\alpha_n}\} = 1, (i = 1, 2)$ in (X, T_1, T_2).

Definition 5.6. A fuzzy bitopological space (X, T_1, T_2) is called the pairwise fuzzy weakly Lindelof space if for each cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy open sets in (X, T_1, T_2), there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n \in N}\{\lambda_{\alpha_n}\}] = 1, (i = 1, 2)$ in (X, T_1, T_2). That is, if $\vee_{\alpha \in \Delta}\{\lambda_a\} = 1$, where (λ_α)'s are pairwise open sets in (X, T_1, T_2), then there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n \in N}\{\lambda_{\alpha_n}\}] = 1, (i = 1, 2)$ in (X, T_1, T_2).

Proposition 5.1. *If the fuzzy bitopological space* (X, T_1, T_2) *is the pair wise weak fuzzy P-space, then* (X, T_1, T_2) *is the pair wise fuzzy weakly regular Lindelof space if and only if* (X, T_1, T_2) *is the pair wise fuzzy almost regular Lindelof space.*

Proof. Let (X, T_1, T_2) be the pair wise fuzzy weakly regular Lindelof space.

Then, for the cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy regular open sets, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n \in N} \{\lambda_{\alpha_n}\}] = 1$, in (X, T_1, T_2). Since the pair wise fuzzy regular open sets are pair wise fuzzy open sets in fuzzy bitopological spaces, (λ_α)'s are pairwise open sets in (X, T_1, T_2).

Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.2, $cl_{T_i}[\vee_{k=1}^{\infty} (\lambda_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(\lambda_k)], (i = 1, 2)$, in (X, T_1, T_2). Hence, for the cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy regular open sets, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N}(cl_{T_i}[\lambda_{\alpha_n}]) = 1$, in (X, T_1, T_2).

Hence (X, T_1, T_2) is the pair wise fuzzy (X, T_1, T_2) is the pair wise fuzzy almost regular Lindelof space.

Conversely, suppose that (X, T_1, T_2) be the pair wise fuzzy almost regular Lindelof space. Then, for the cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy regular open sets, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$

such that $\vee_{n \in N} cl_{T_i}[\lambda_{\alpha_n}] = 1,$ in (X, T_1, T_2). Since the pair wise fuzzy regular open sets are pair wise fuzzy open sets in fuzzy bitopological spaces, (λ_α)'s are pairwise open sets in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.2, $cl_{T_i}[\vee_{k=1}^{\infty} \lambda_{\alpha_n}] = \vee_{k=1}^{\infty}[cl_{T_i}\lambda_{\alpha_n}],$ $(i = 1, 2)$, in (X, T_1, T_2). Hence, for the cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy regular open sets, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n \in N}\{\lambda_{\alpha_n}\}] = 1,$ in (X, T_1, T_2). Hence (X, T_1, T_2) is the pair wise fuzzy weakly regular Lindelof space.

Proposition 5.2. *If the fuzzy bitopological space* (X, T_1, T_2) *is the pair wise weak fuzzy P-space, and pair wise fuzzy nearly Lindelof space, then* (X, T_1, T_2) *is the pair wise fuzzy weakly Lindelof space.*

Proof. Let (X, T_1, T_2) be the pair wise fuzzy nearly Lindelof space. Then, for the cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy open sets in (X, T_1, T_2), there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $\vee_{n \in N}[cl_{T_i}cl_{T_j}\{\lambda_{\alpha_n}\}] = 1, (i, j = 1, 2$ and $i \neq j)$ in (X, T_1, T_2). Now $\text{int}_{T_i}\, cl_{T_j}\{\lambda_{\alpha_n}\} \leq cl_{T_j}\{\lambda_{\alpha_n}\},$ implies that $\vee_{n \in N} cl_{T_i}cl_{T_j}\{\lambda_{\alpha_n}\} \leq \vee_{n \in N} cl_{T_j}\{\lambda_{\alpha_n}\}$ and then $1 \leq \vee_{n \in N} cl_{T_j}\{\lambda_{\alpha_n}\}$. Thus, $\vee_{n \in N} cl_{T_j}\{\lambda_{\alpha_n}\} = 1,$ in (X, T_1, T_2). Since (X, T_1, T_2) is the pair wise weak fuzzy P-space, by the Proposition 3.2, $cl_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = \vee_{k=1}^{\infty}[cl_{T_i}(\lambda_k)], (i = 1, 2),$ in (X, T_1, T_2) and hence $cl_{T_i}[\vee_{k=1}^{\infty}(\lambda_k)] = 1,$ in (X, T_1, T_2). Hence, for the cover $\{\lambda_\alpha : \alpha \in \Delta\}$ of X by pairwise fuzzy open sets, there exists a countable subset $\{\alpha_n : n \in N\} \subseteq \Delta$ such that $cl_{T_i}[\vee_{n \in N}(\lambda_{\alpha_n})] = 1,$ in (X, T_1, T_2).

Hence (X, T_1, T_2) is the pair wise fuzzy weakly Lindelof space.

Conclusions

In this paper the notions of pairwise fuzzy regular G_δ-sets and pair wise fuzzy regular F_σ-sets are introduced and by means of pairwise fuzzy regular G_δ-sets, the notion of pair wise weak fuzzy P-spaces, is introduced and studied. Several characterizations of pair wise weak fuzzy P-spaces, are obtained. It is shown that pairwise fuzzy P-spaces are pair wise weak fuzzy P-spaces. The conditions for the pairwise fuzzy strongly irresolvable and pairwise weak fuzzy P-spaces to become pair wise fuzzy Baire spaces, pair wise fuzzy second category spaces and pair wise fuzzy almost irresolvable spaces, are established. It is also established that pair wise fuzzy regular G_δ-sets in pair wise weak fuzzy P-spaces are pair wise fuzzy somewhere dense sets and the pair wise fuzzy regular F_σ-are pair wise fuzzy cs dense sets. The conditions for the pair wise weak fuzzy P-spaces to become pair wise fuzzy σ-Baire spaces and pairwise fuzzy D-Baire spaces, are also obtained in this paper. It is established that if the fuzzy bitopological space is the pair wise weak fuzzy P-space, then the pair wise fuzzy weakly regular Lindelof ness of the bitopological space implies the pair wise fuzzy almost regular Lindelofness of the bitopological space and vice-versa and the pair wise fuzzy nearly Lindelof spaces are the pair wise fuzzy weakly Lindelof spaces.

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