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PRIME CORDIAL LABELING OF FIRE CRACKER GRAPHS, DIAMOND GRAPHS AND DOUBLE ARROW GRAPHS

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Abstract

A prime cordial labeling has introduced by Sundaram et al. A prime cordial labeling of a graph G with the vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, \dots, |V(G)|\}$ such that each edge uv is assigned the label 1 if $\gcd(f(u), f(v)) = 1$ and 0 if $\gcd(f(u), f(v)) > 1$, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits a prime cordial labeling is called a prime cordial labeling. In this paper we exhibit some new constructions on prime cordial labeling graphs with respect to fire cracker graphs, diamond graphs and double arrow graphs.

Definitions: Firecracker graph:

An (n, k) firecracker is a graph obtained by the concatenation of nk-stars by linking one leaf from each [2, 5].

Double arrow graph:

A double arrow graph DA_n^t with width t and length n is obtained by joining two vertices v and w with superior vertices $P_m \times P_n$ by $m + m$ new edges from both the ends [3].

Diamond graph:

Diamond graph, denoted by d_n, is the graph obtained from the

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Mongolian tent graph Mt_n by adding a new vertex z_1 and joining each vertex u_i, $1 \leq i \leq n$ with z_1[1].

Theorem 1. *A fire cracker graph* $F_{3,n}$ *for* $n \geq 3$, *and* $F_{m,5}$, *for* $m \geq 2$ *prime cordial graphs.*

Proof. Fire cracker $F_{3,n}$ graph. Let x, y, z be the base vertices in $F_{3,n}$. From *x*, extending the path, we obtain path vertices namely $x_1, x_2, x_3, \ldots, x_n$. From *y*, extending the graph we get vertices namely $y_1, y_2, \ldots, y_n$. Similarly, from *z* extending the graph, vertices, namely z_1, z_2, …, z_n [4].

In case, *n* is odd, mid vertices are represented by $x_{\frac{n}{2}}, y_{\frac{n}{2}}, z_{\frac{n}{2}}$. In case of *n* is even, they do not exist in this way.

Labeling has to be classified into two cases, when *n* is odd, and *n* is an even.

Case (i) '*n*' is an even Let us define the function $f : V(G) \to \{1, 2, 3, \ldots, n\}$, then the labeling has to be defined as,

(i) $f(x) = 2$

(ii) $f(y) = 4$

(iii) $f(z) = 6$

(iv) $f(x_n) = 1$

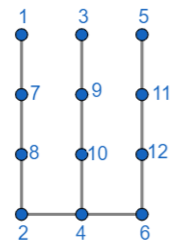
(v) $f(y_n) = 3$

(vi) $f(z_n) = 5$.

In $F_{3,n}$ '*n*' can be divided into equally half.

From the base vertices, first set of vertices can be labeled by an even number. Similarly, from the top, second set of vertices covering prior to mid vertices has to be labeled by an odd. We need not assign for mid vertices as they do not exist.

Example. $F_{3,4}$



Case (ii) 'n' is an odd. Let us define the function $f: V(G) \rightarrow \{1, 2, 3, \ldots, n\}$, then the labeling has to be given

(i) $f(x) = 2$

(ii) $f(y) = 4$

(iii) $f(z) = 6$

(iv) $f(x_n) = 1$

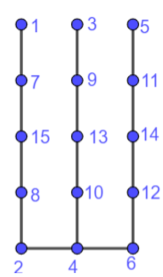
(v) $f(y_n) = 3$

(vi) $f(z_n) = 5$.

From the base vertices, first set of vertices can be labeled by an even number. Similarly, from the top, second set of vertices covering prior to mid vertices has to be labeled by an odd.

In $F_{3,n}$ mid vertices exist, namely $f(x_{\frac{n}{2}}), f(y_{\frac{n}{2}}), f(z_{\frac{n}{2}})$ can be labeled by a number which satisfy the condition $\gcd(f_{ui, ui+1,}) = 1$ or 0 [6, 2].

Example. $F_{3,5}$



Fire cracker $F_{m,5}$. Let x_{ij} be the base vertices where $0 \leq i \leq m$ and $j = 1, 2$. Labeling has to be classified into two cases.

(i) In $F_{m,5}$, when m is an even

(ii) In $F_{m,5}$, when m is an odd

Case (i). Let x_{ij} (where 'i' varies 0 to the index determined by m, and $j = 1, 2$) $y_{1,k}$ (Initial vertex of the path from the centre of the star) $y_{2,k}$, $y_{3,k}$, and $y_{4,k}$ be the star vertices, where $k = 1, 2, 3, 4$.

Let the function $f : N \times N \to N$, labeling has to be given as,

(i) $f(i, j) = Cat(i, j)$ where $Cat(i, j) = \begin{cases} ij, \text{ (Concatering } i \text{ and } j \forall i > 0) \\ j, \text{ otherwise.} \end{cases}$

(i) $f(y_1, k) = \text{odd number} > 1, k = 1, 2, 3, 4$

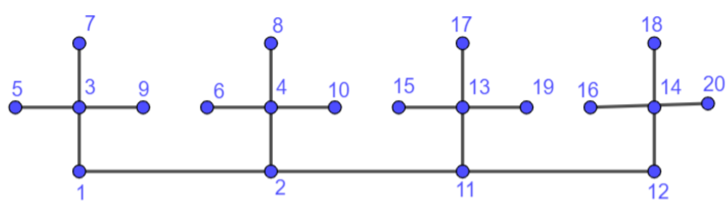
(ii) $f(y_2, k) = \text{even number} > 2, k = 1, 2, 3, 4$

(iii) $f(y_3, k) = \text{odd number} > 11, k = 1, 2, 3, 4$

(iv) $f(y_4, k) = \text{even number} > 12, k = 1, 2, 3, 4$

(v) $f(y_n, k) = \begin{cases} un \text{ assignied even number, available next to the} \\ \text{assigned number, when } n \text{ is even} \\ un \text{ assignied odd number, available next to the} \\ \text{assigned number, when } n \text{ is odd} \end{cases}$

Example. $F_{4,5}$



Case (ii). in $F_{m,5}$, where m is odd. Let x_{ij} (where 'i' varies 0 to the index determined by m, and $J = 1, 2$) $y_{1,k}$ (Initial vertex of the path from the center of the star) $y_{2,k}$, $y_{3,k}$, and $y_{4,k}$ be the star vertices, where $k = 1, 2, 3, 4$. Let the function $f : N \times N \to N$, labeling has to be given as,

(i) $f(i, j) = Cat(i, j)$

$$\text{where } Cat(i, j) = \begin{cases} ij, \text{ (Concatering } i \text{ and } j \ \forall i > 0) \\ j, \text{ otherwise.} \end{cases}$$

(i) $f(y_1, k) = \text{odd number} > 1, k = 1, 2, 3, 4$

(ii) $f(y_2, k) = \text{even number} > 2, k = 1, 2, 3, 4$

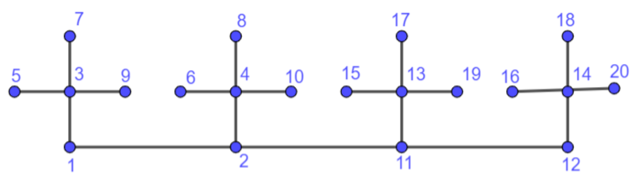
(iii) $f(y_3, k) = \text{odd number} > 11, k = 1, 2, 3, 4$

(iv) $f(y_4, k) = \text{even number} > 12, k = 1, 2, 3, 4$

(v) $f(y_n, k)$

Rest of the number (even or odd) will be given which satisfy the condition $\gcd f(f_{ui, ui+1,}) = 1$ or 0.

Example. $F_{5,5}$



Theorem 2. *A double arrow graph* DA_n^2, *for* $n \geq 2$, *which is prime cordial labeling.*

Proof. Let $G = DA_n^2$ be a double arrow graph obtained by joining two vertices x, y with $P_2 \times P_n$ by $2 + 2$ new edges on both sides.

Let $U_{ij} (i = 1, 2; j = 1, 2, \ldots, n)$ be the vertices of $P_2 \times P_n$. Join y with $U_{1,n}$ and x with $V_{1,n}$ by $2 + 2$ new edges.

Let we denote the function,

$f : V(G) \to \{1, 2, 3, \cdots n\}$

(i) $f(y) = 2$

(ii) $F(x) = 1$

(iii) $f(U_{1,1}) = 4$

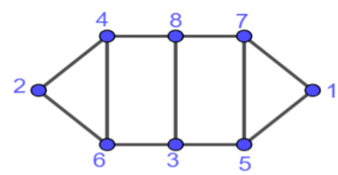
(iv) $f(U_{2,1}) = 6$

(v) $f(U_{1,n}) = 7$

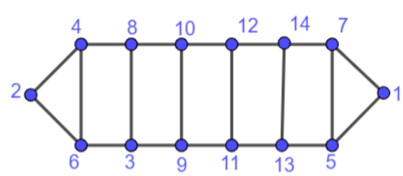
(vi) $f(U_{2,n}) = 5$.

For other $U_{1,n}$ vertices must be labeled by an even number. Similarly, for other $U_{2,n}$ vertices must be labeled by an odd, satisfy the condition $\gcd f(f_{ui,ui+1,}) = 1$ or 0.

Example. DA_3^2



Example. DA_6^2



Theorem 3. *For any integer* $m = 2$ *and* $n > 2$ *the diamond graph* $D(m, n)$ *admits prime cordial labeling.*

Proof. Let $D(m, n)$ be a diamond graph, and let $m = 2$. Consider $D(2, n)$ with the vertex set $\{u, v, x_{11}, x_{12}, \ldots x_{1n}; x_{21}, x_{22}, \ldots, x_{2n}\}$ with $V(G) = 2n + 2$ where u and v are apex vertices.

Let the function $f : V(G) \to \{1, 2, 3, \ldots, n\}$

(i) $f(u) = 5$

(ii) $f(v) = 4$

(iii) $f(x_{11}) = 1$

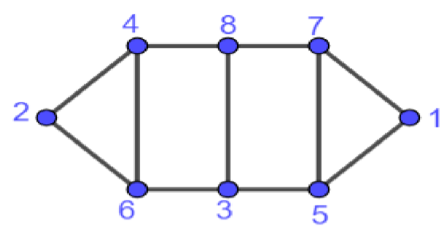
(iv) $f(x_{1n}) = 3$

(v) $f(x_{21}) = 2$

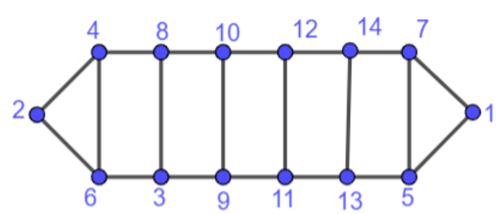
(vi) $f(x_{2n}) = 6$.

For the other vertices in x_{1n} path, labeling has to be given by an odd number, Similarly, for the other vertices in x_{2n} path, labeling has to be given by an even number, satisfy the condition $\gcd(f_{ui,ui+1},) = 1$ or 0.

Examples. d_3



Examples. d_6



Conclusion

We have established the family of fire cracker graphs, the important property for prime cordial labeling. Moreover, we included same property for diamond graph and double arrow graph.

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