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ORDERING AND RANKING GENERALIZED INTERVAL VALUED PENTAGONAL FUZZY NUMBERS TO SOLVE FUZZY ASSIGNMENT PROBLEM

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Abstract

In all the fields like science, engineering and management, decision makers come across incomplete and imprecise data. Many real life environments with vague information are difficult to deal by already existing conventional methods. These situations are effectively dealt by using fuzzy numbers, generalized fuzzy numbers, intuitionistic fuzzy numbers, *L-R* fuzzy numbers, interval valued fuzzy numbers, neutrosophic numbers, etc., depending on the situation, the apt model is applied. This paper deals with the situation where the vague information is represented as generalized interval valued pentagonal fuzzy numbers. The main objective of this paper is solving the Fuzzy assignment problem whose costs are generalized interval valued pentagonal fuzzy numbers, by using an efficient ranking technique.

1. Introduction

Uncertainty in the real life situations is modelled mathematically as fuzzy set by L. A. Zadeh [12] in 1965. The fuzzy set theory is explored by

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many researchers and dealt with varied dimensions in recent years. Due to the different forms of applications, the fuzzy set theory has become a topic of boundless interest. Depending upon the geometrical representation, fuzzy numbers are organised, to solve the problems of management, engineering, medical diagnosis, networking, etc.

Atanassov [1] introduced prominent form by combining the concept of intuitionistic fuzzy sets and interval valued fuzzy sets. In 1975, Sambuc [7] presented in his doctoral research the concept of interval valued fuzzy sets. Interval valued fuzzy sets were suggested by Gorzlczany [5] and Turksen [10]. Wang and Li [11] defined interval valued fuzzy numbers with their extended operations. Pentagonal fuzzy numbers was first used by Rajkumar and T. Pathinathan [6] to study the fuzzy term poverty. The situations, in which experts give their opinions in intervals, should be considered without generalizing. [8] Sankar Prasad Mondal et al. has explored pentagonal intuitionistic fuzzy numbers, its properties and applications. S. Shumnugapriya and G. Uthra [9] used centroid ranking method. A. Ebrahimnejad [4] discussed a method for solving linear programming with interval valued fuzzy variables. The membership function of an interval valued fuzzy number is also an interval and not crisp as of any other fuzzy number.

This structure of the paper is as follows: Section 2, recalls the basic notions related to interval valued fuzzy numbers, interval valued pentagonal fuzzy number, generalized interval valued pentagonal fuzzy number and define a new methodology to rank generalized interval valued pentagonal fuzzy numbers. In section 3, the fuzzy assignment problem and its mathematical formulation displayed. Section 4 presents the proposed algorithm. In section 5 the numerical example is illustrated to apply the proposed ranking. Section 6 exhibits the ranking comparison and concluding notes.

2. Basic Definition

Definition 2.1. Fuzzy set

A fuzzy set $\widetilde{A}$ is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval

$[0, 1].(i.e)\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)); x \in X\}$, here $\mu_{\widetilde{A}} : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set $\widetilde{A}$ and $\mu_{\widetilde{A}}(x)$ called the membership function value of $x \in X$ in the fuzzy set. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2. Fuzzy number

A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

- $\mu_{\widetilde{A}}(x) : R \to [0, 1]$ is continuous.
- $\mu_{\widetilde{A}}(x) = 0$ for all $\in (-\infty, a] \cup [d, \infty)$.
- $\mu_{\widetilde{A}}(x)$ strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
- $\mu_{\widetilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

Definition 2.3. Generalized Fuzzy Number

A fuzzy set $\widetilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:

- $\mu_{\widetilde{A}} : R \to [0, \omega]$ is continuous.
- $\mu_{\widetilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
- $\mu_{\widetilde{A}}(x)$ strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
- $\mu_{\widetilde{A}}(x) = \omega$, for all $x \in [b, c]$, where $0 < \omega \leq 1$.

Definition 2.4. Interval valued fuzzy set

An Interval Valued Fuzzy set (IVFS) $\widetilde{A}$ on R is defined by $\widetilde{A}^{IV} = [\{x, (\mu^{U}_{\widetilde{A}^{IV}}(x), \mu^{L}_{\widetilde{A}^{IV}}(x))\} : x \in R]$ where $x \in R$ and $\mu^{U}_{\widetilde{A}^{IV}}(x)$ maps R into $[0, 1]$, $\mu^{L}_{\widetilde{A}^{IV}}(x)$ maps R into $[0, 1] \forall x \in R$, $\mu^{L}_{\widetilde{A}^{IV}}(x)$.

Definition 2.5. Interval valued pentagonal fuzzy number [5]

An Interval valued pentagonal fuzzy number is written as $\widetilde{A}^{IV} = \{(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); \omega^U_{\widetilde{A}^{IV}}, \omega^U_{\widetilde{A}^{IV}}\}$, where $b_1 \le a_1 \le b_2 \le a_2 \le b_3 \le a_3 \le a_4 \le b_4 \le a_5 \le b_5, 0 \le k_1 \le k_2 \le \omega^L_{\widetilde{A}^{IV}} \le \omega^U_{\widetilde{A}^{IV}} \le 1$, whose corresponding membership function is

$$\mu^L_{\widetilde{A}^{IV}}(x) = \begin{Bmatrix} 0, & x < a_1 \\ k_1 - \dfrac{k_1(x - a_2)}{a_1 - a_2}, & a_1 \le x \le a_2 \\ \omega^L_{\widetilde{A}^{IV}} + \dfrac{(k_1 - \omega^L_{\widetilde{A}^{IV}})(x - a_3)}{a_2 - a_3}, & a_2 \le x \le a_3 \\ \omega^L_{\widetilde{A}^{IV}} + \dfrac{(k_1 - \omega^L_{\widetilde{A}^{IV}})(x - a_3)}{a_4 - a_3}, & a_3 \le x \le a_4 \\ k_1 - \dfrac{k_1(x - a_4)}{a_5 - a_4}, & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{Bmatrix}$$

$$\mu^U_{\widetilde{A}^{IV}}(x) = \begin{Bmatrix} 0, & x < b_1 \\ k_2 - \dfrac{k_2(x - b_2)}{b_1 - b_2}, & b_1 \le x \le b_2 \\ \omega^U_{\widetilde{A}^{IV}} + \dfrac{(k_2 - \omega^U_{\widetilde{A}^{IV}})(x - b_3)}{b_2 - b_3}, & b_2 \le x \le b_3 \\ \omega^U_{\widetilde{A}^{IV}} + \dfrac{(k_2 - \omega^U_{\widetilde{A}^{IV}})(x - b_3)}{a_4 - a_3}, & b_3 \le x \le b_4 \\ k_2 - \dfrac{k_2(x - b_4)}{b_5 - b_4}, & b_4 \le x \le b_5 \\ 0, & x > b_5 \end{Bmatrix}.$$

Definition 2.6. Arithmetic Operations on Generalized Interval valued pentagonal fuzzy number [5]

Let $\widetilde{A}^{IV} = \{(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); \omega^L_{\widetilde{A}^{IV}}, \omega^L_{\widetilde{A}^{IV}}\}$, and $\widetilde{B}^{IV} = \{(p_1, p_2, p_3, p_4, p_5), (q_1, q_2, q_3, q_4, q_5); u^L_{\widetilde{A}^{IV}}, u^U_{\widetilde{A}^{IV}}\}$, be two GIVPFNs. Then

Addition:

$$\widetilde{A}^{IV} + \widetilde{B}^{IV} = [[(a_1, a_2, a_3, a_4, a_5), \omega^L_{\widetilde{A}^{IV}}(b_1, b_2, b_3, b_4, b_5), \omega^U_{\widetilde{A}^{IV}}]]$$

$$+ [[(p_1, p_2, p_3, p_4, p_5), u^L_{\widetilde{A}^{IV}}], [(q_1, q_2, q_3, q_4, q_5), u^U_{\widetilde{A}^{IV}}]]$$

$$= [[(a_1 + p_1, a_2 + p_2, a_3 + p_3, a_4 + p_4, a_5 + p_5, \omega)],$$

$$[(b_1 + q_1, b_2 + q_2, b_3 + q_3, b_4 + q_4, b_5 + q_5), u]],$$

where $\omega = \min\{\omega^L_{\widetilde{A}^{IV}}, \omega^U_{\widetilde{A}^{IV}}\}$ and $u = \max\{u^L_{\widetilde{A}^{IV}}, u^U_{\widetilde{A}^{IV}}\}$.

Difference:

$$\widetilde{A}^{IV} - \widetilde{B}^{IV} = [[(a_1, a_2, a_3, a_4, a_5), \omega^L_{\widetilde{A}}], [(b_1, b_2, b_3, b_4, b_5), \omega^U_{\widetilde{A}}]]$$

$$- [[(p_1, p_2, p_3, p_4, p_5), u^L_{\widetilde{A}^{IV}}], [(q_1, q_2, q_3, q_4, q_5), u^U_{\widetilde{A}^{IV}}]]$$

$$= [(\max(a_1 - p_5, 0), \max(a_2 - p_4, 0), \max(a_3 - p_3, 0), \max(a_4 - p_2, 0)$$

$$\max(a_5 - p_1, 0), (\max(b_1 - q_5, 0), \max(b_2 - q_4, 0), \max(b_3 - q_3, 0),$$

$$\max(b_4 - q_2, 0), \max(b_5 - q_1, 0), u],$$ where $\omega = \min\{\omega^L_{\widetilde{A}^{IV}}, \omega^U_{\widetilde{A}^{IV}}\}$

and $u = \max\{u^L_{\widetilde{A}^{IV}}, u^U_{\widetilde{A}^{IV}}\}$.

Scalar Multiplication:

$$\lambda\widetilde{A}^{IV} = \lambda[(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5), \omega^L_{\widetilde{A}^{IV}}, \omega^U_{\widetilde{A}^{IV}}]$$

$$= [(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5), (\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4, \lambda b_5); \omega^L_{\widetilde{A}^{IV}}, \omega^U_{\widetilde{A}^{IV}}], \lambda \geq 0.$$

Definition 2.7. Let $\widetilde{A}^{IV}$ and $\widetilde{B}^{IV}$ be two IVPFNs. The ranking of $\widetilde{A}^{IV}$ and $\widetilde{B}^{IV}$ by the $R(\cdot)$ on the set of IVPFNs is defined as follows:

i. $R(\widetilde{A}^{IV}) > R(\widetilde{B}^{IV})$ *iff* $\widetilde{A}^{IV} > \widetilde{B}^{IV}$

ii. $R(\widetilde{A}^{IV}) < R(\widetilde{B}^{IV})$ *iff* $\widetilde{A}^{IV} < \widetilde{B}^{IV}$

iii. $R(\widetilde{A}^{IV}) = R(\widetilde{B}^{IV})$ *iff* $\widetilde{A}^{IV} = \widetilde{B}^{IV}$.

Definition 2.8. The ordering $\geq$ and $\leq$ between n any two IVPFNs $\widetilde{A}^{IV}$ and $\widetilde{B}^{IV}$ are defined as follows:

i. $\widetilde{A}^{IV} \geq \widetilde{B}^{IV}$ *iff* $\widetilde{A}^{IV} > \widetilde{B}^{IV}$ or $\widetilde{A}^{IV} = \widetilde{B}^{IV}$ and

ii. $\widetilde{A}^{IV} \leq \widetilde{B}^{IV}$ *iff* $\widetilde{A}^{IV} < \widetilde{B}^{IV}$ or $\widetilde{A}^{IV} = \widetilde{B}^{IV}$.

Definition 2.9. Let $\{\widetilde{A}_i^{IV}, i = 1, 2, \cdots, n\}$ be a set of IVPFNs. If $R(\widetilde{A}_k^{IV}) \leq R(\widetilde{A}_i^{IV})$ for all i, then the IVPFNs $\widetilde{A}_k^{IV}$ is the minimum of $\{\widetilde{A}_i^{IV}, i = 1, 2, \cdots, n\}$.

Definition 2.10. Let $\{\widetilde{A}_i^{IV}, i = 1, 2, \cdots, n\}$ be a set of IVPFNs. If $R(\widetilde{A}_k^{IV}) \geq R(\widetilde{A}_i^{IV})$ for all i, then the IVPFNs $\widetilde{A}_t^{IV}$ is the minimum of A $\{\widetilde{A}_i^{I}, i = 1, 2, \cdots, n\}$.

Definition 2.11. Ranking Technique

Average of the α-cut of a generalized interval valued pentagonal fuzzy number:

$$A(\mu^L_{\widetilde{A}^{IV}}(\alpha_1)) = \frac{1}{16}\left[1 - \frac{\alpha_1}{k_1}\right)(a_1 + a_5 - a_2 - a_4) + \frac{(\alpha_1 - \omega^L_{\widetilde{A}^{IV}})}{(k_1 - \omega^L_{\widetilde{A}^{IV}})}(a_2 + 2a_3 + a_4)$$

$$+a_2 + 2a_3 + a_4] \text{ and}$$

$$A(\mu^U_{\widetilde{A}^{IV}}(\alpha_2)) = \frac{1}{16}\left[1 - \frac{\alpha_2}{k_2}\right)(b_1 + b_5 - b_2 - b_4) + \frac{(\alpha_2 - \omega^U_{\widetilde{A}^{IV}})}{(k_1 - \omega^U_{\widetilde{A}^{IV}})}$$

$$(b_2 - 2b_3 + b_4) + (b_2 + 2b_3 + b_4).$$

Let $\widetilde{A}^{IV} = [(a_1, a_2, a_3, a_4, a_5), (b_1, b_2, b_3, b_4, b_5); \omega^L_{\widetilde{A}^{IV}}, \omega^U_{\widetilde{A}^{IV}}]$, where $b_1 \leq a_1 \leq b_2 \leq a_2 \leq b_3 \leq a_3 \leq a_4 \leq b_4 \leq a_5 \leq b_5$, $0 \leq k_1 \leq k_2 \leq \omega^L_{\widetilde{A}^{IV}} \leq \omega^U_{\widetilde{A}^{IV}} \leq 1$, be the generalized interval valued pentagonal fuzzy number, the ranking function is defined as follows:

$$R(\widetilde{A}^{IV}) = \frac{A(\mu^L_{\widetilde{A}^{IV}}(\alpha_1))\omega^L_{\widetilde{A}^{IV}} + A(\mu^U_{\widetilde{A}^{IV}}(\alpha_2))\omega^U_{\widetilde{A}^{IV}}}{2}.$$

3. Mathematical Formulation of Fuzzy Assignment Problem

The generalized interval valued pentagonal fuzzy assignment problem can be represented in the form of $n \times n$ fuzzy cost table $[\widetilde{C}_{ij}]$ is given below.

		Jobs					
		1	2	...	J	...	n
Persons	1	$\widetilde{C}_{11}$	$\widetilde{C}_{12}$	...	$\widetilde{C}_{1j}$	...	$\widetilde{C}_{1n}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	i	$\widetilde{C}_{i1}$	$\widetilde{C}_{i2}$	...	$\widetilde{C}_{ij}$	...	$\widetilde{C}_{in}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	n	$\widetilde{C}_{n1}$	$\widetilde{C}_{n2}$	...	$\widetilde{C}_{nj}$	...	$\widetilde{C}_{nn}$

The costs or time $\widetilde{C}_{ij}$ are generalized interval valued pentagonal fuzzy numbers, $\widetilde{C}_{ij} = ([C_p^L, C_q^U], [C_l^L, C_m^U])$. The goal is, effective way of assigning the j^{th} job to the i^{th} person (all jobs to available persons) by minimizing the total cost with minimum time.

The fuzzy assignment problem can be mathematically stated as

Minimize $\widetilde{Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{C}_{ij}\, x_{ij},\ i = 1, 2, \ldots, n.$

Subject to $x_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ resource is assigned } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases}$

$\sum_{i=1}^{n} x_{ij} = 1$ (i^{th} resources does one job $i = 1, 2, \ldots, n$) and $\sum_{i=1}^{n} x_{ij} = 1$ (Only one resource should be assigned i^{th} job, $j = 1, 2, \ldots, n$). Where X_{ij} specifies that j^{th} job is assigned to the i^{th} resource.

4. Proposed approach of Fuzzy Assignment Problem [FAP]

Step 1. Express the real time cost values to generalized interval valued pentagonal fuzzy numbers.

Step 2. Test whether the given Fuzzy Assignment Problem is balanced or not.

(i) If it is a balanced one (i.e., the number of resource are equal to the number of jobs) then go to step 3.

(ii) If it is an unbalanced one (i.e., the number of resources are not equal to the number of jobs) then go to step 2.

Step 3. Introduce dummy rows and /or dummy columns with zero fuzzy costs to form a balanced one.

Step 4. Find the rank of each cell $\widetilde{C}_{ij}$ of the chosen fuzzy cost matrix by using the ranking function as mentioned in section 2.

Step 5. Proceed by the Hungarian method to solve fuzzy cost table to get optimal fuzzy assignment.

Step 6. Add the optimal fuzzy assignment using fuzzy addition mentioned in section 2, to optimize cost within minimum time.

5. Numerical example

Consider an assignment problem of assigning 3 jobs to 3 machines, whose costs is considered as generalized interval valued pentagonal fuzzy numbers in '000 lakhs. The problem is to find the optimal allocation in an efficient way.

Jobs/ Persons	P	Q	R
A	(0, 1, 2, 3, 4; 0.5) (–1, 0, 1, 2, 1; 0.8)	(2, 3, 4, 5, 6; 0.4) (–2, –1, 0, 1, 2; 0.7)	(– 2, –1, 0, 1, 2; 0.6) (2, 3, 4, 5, 6; 0.8)
B	(1, 2, 3, 4, 5; 0.6) (2, 3, 4, 5, 6; 0.7)	(0, 1, 2, 3, 4; 0.5) (2, 3, 4, 5, 6; 0.9)	(–2, –1, 0, 2, 2; 0.4) (6, 7, 8, 9, 10; 0.5)
C	(2, 3, 4, 5, 6; 0.4) (0, 1, 2, 3, 4; 0.6)	(2, 3, 4, 5, 6; 0.3) (–2, –1, 0, 1, 2; 0.5)	(–2, –1, 0, 2, 3; 0.5) (–1, –2, –1, 3, 3; 0.7)

Solution:

Using step 4, the rank of generalized interval valued pentagonal fuzzy cost matrix is

Rank Table:

Jobs / Persons	P	Q	R
A	0.225	0.2	0.4
B	0.575	0.575	0.0281
C	0.35	0.15	0.0375

Proceeding by Hungarian method, the optimal allocations are:

Jobs/Persons	P	Q	R
A	0	0	0.3125
B	0.4094	0.5444	0
C	0.175	0	0

Therefore, the assignment is $A \to P$, $B \to R$ and $C \to Q$.

By fuzzy addition, the Minimum Cost: $0.225 + 0.0281 + 0.15 = 0.4031$.

Comparison Table

Ranking Methods	Example	Ranking Results
The centroid method	(1,2,3,4,5;0.8), (–1,0,2,4,5;0.8)	0.6389
	(–1,0,0.2,0.3,0.4;0.6), (–1,–0.5,0,0.4,0.5;0.6)	– 0.0182
The proposed ranking method	(1,2,3,4,5;0.8), (–1,0,2,4,5;0.8)	0.5
	(–1,0,0.2,0.3,0.4;0.6), (–1,–0.5,0,0.4,0.5;0.6)	0.0075

6. Conclusion

This paper proposes a new ranking for generalized interval valued pentagonal fuzzy numbers. The proposed average ranking is applied to elucidate the generalized interval valued pentagonal fuzzy assignment problem. Further, a numerical example is illustrated whose costs are taken as generalized interval valued pentagonal fuzzy numbers. The efficiency of the ranking technique is shown in the comparison table. As a future extension, the proposed algorithm may be used to solve, generalized interval valued pentagonal fuzzy transportation problem and generalized interval valued octagonal fuzzy assignment and fuzzy transportation problems.

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