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A COMPOSITE CLASS OF RATIO ESTIMATORS FOR THE MEAN OF A FINITE POPULATION IN SIMPLE RANDOM SAMPLING

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Abstract

For estimating the mean of a finite population, a composite class of ratio type estimators is formulated by considering a linear combination of the usual ratio estimator and the exponential ratio estimator in simple random sampling (SRS) design. The performance of the proposed estimator, as compared with some existing estimators, is demonstrated using the mean square error (MSE) criterion. The theoretical results are validated by incorporating an empirical analysis utilizing some real population datasets. Moreover, the percent relative efficiencies (PREs) have been obtained for the various suggested estimators of the population mean.

1. Introduction

In sample surveys, the estimation of parameters (for instance the mean and variance) of a well-defined target population is made using prior information on auxiliary variable(s). The auxiliary variable(s) should be perfectly correlated with the variable under study (i.e., the study variable) in order to furnish a precise estimator of the concerned population parameter.

The problem of estimation of mean has become indispensable in various diversified and inter-disciplinary fields of agriculture, science, technology and medicine. For instance, the estimation of average crop yield, average temperature at a place, average annual rainfall, average life span of a

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species, and much more. Most of the literature in sample surveys explores the remarkable attempts made by various authors in the estimation of mean of a finite population. Some noteworthy contributions in the development of estimators for the mean of a finite population in simple random sampling (SRS) design have been made by Cochran [8], Bahl and Tuteja [3], Upadhyaya and Singh [35], Singh and Ruiz Espejo [25], Singh and Tailor [26], Singh et al. [28], Kadilar and Cingi [15], Gupta and Shabbir [12], Gupta and Shabbir [13] and Singh et al. [28].

In recent decades, some significant contributions towards the theory of estimation of mean, under SRS design, have been made by Diana et al. [9], Lu et al. [23], Grover and Kaur [11}, Vishwakarma and Kumar [37], Kumar and Vishwakarma [19], Bhushan et al. [6], Zeeshan et al. [40] and Ayodeji et al. [2].

Some other remarkable developments relating to the theory of estimation in sample surveys have been made by Singh and Vishwakarma [29], Vishwakarma and Gangele [36], Vishwakarma and Kumar [38], Vishwakarma and Kumar [39], Kumar and Vishwakarma [20], Ngaruye et al. [24], Steland and Chang [30], Sugasawa [33], and Kumar and Vishwakarma [21] Lone et al. [22], Bhushan et al. [7], Kumar and Tiwari [18], Kumar et al. [17], Bhushan and Kumar [4], Bhushan and Kumar [5], Subzar et al. [32], and Tiwari et al. [34].

In this paper, we have developed a composite class of ratio estimators for the population mean of a study variable under SRS design. We have derived the mean square error (MSE) of the proposed class of estimators to the first order of approximation.

The necessary and sufficient conditions (NASCs) have been obtained for describing the dominance of the proposed class over the well-known existing estimators. The theoretical results have been validated with an empirical analysis using four real population datasets.

2. Some Existing Estimators of Population Mean

In order to proceed for the estimation of mean, we consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ consisting of N units. Also, we assume Y as the study variable, and X as the auxiliary variable. Let (y_i, x_i) be the values of the concerned variables, measured on the i^{th} unit $U_i = (i = 1, 2, \ldots, N)$ of the population U. Assuming that the population mean $\overline{X}$ of the auxiliary variable X is known, we estimate the population mean $\overline{Y}$ of the study variable Y by selecting a sample of size n (with $n < N$) from the population U by adopting simple random sampling without replacement (SRSWOR) scheme.

The classical ratio estimator for the population mean $\overline{Y}$ is defined by

$$\bar{y}_R = \bar{y}\left(\frac{\overline{X}}{\bar{x}}\right) \tag{1}$$

where $\bar{y} = \sum_{i=1}^{n} y_i/n$ and $\bar{x} = \sum_{i=1}^{n} x_i/n$ denote, respectively, the sample means of the variables Y and X.

Moreover, the classical linear regression estimator for population mean $\overline{Y}$ is defined by

$$\bar{y}_{REG} = \bar{y} + b_{yx}(\overline{X} - \bar{x}) \tag{2}$$

where b_{yx} is the sample regression coefficient of Y on X.

Bahl and Tuteja [3] developed an exponential-type ratio estimator for $\overline{Y}$ as follows:

$$\bar{y}_{\text{Re}} = \bar{y}\exp\left(\frac{\overline{X} - \bar{x}}{\overline{X} + \bar{x}}\right) \tag{3}$$

Upadhyaya and Singh [35] suggested the following ratio-type estimators for $\overline{Y}$:

$$\bar{y}_{US1} = \bar{y}\left(\frac{\overline{X}\beta_{2(x)} + C_x}{\bar{x}\beta_{2(x)} + C_x}\right) \tag{4}$$

$$\bar{y}_{US2} = \bar{y}\left(\frac{\overline{X}C_x + \beta_{2(x)}}{\bar{x}C_x + \beta_{2(x)}}\right) \tag{5}$$

where $C_x = S_x / \overline{X}$ denotes the population coefficient of variation of the variable *X*. The term $\beta_{2(x)} = \mu_{4(x)} / \mu_{2(x)}^2$ denotes the population coefficient of kurtosis *X*, where $\mu_{2(x)}$ and $\mu_{4(x)}$ are, respectively, the second and fourth moments about the mean of *X*.

Singh and Tailor [26] utilized the information on correlation coefficient between the variables *Y* and *X* (i.e., ρ_{yx}) and suggested the following ratio estimator for $\overline{Y}$:

$$\bar{y}_{ST} = \bar{y}\left(\frac{\overline{X} + \rho_{yx}}{\bar{x} + \rho_{yx}}\right) \tag{6}$$

Singh et al. [28] utilized the information on coefficient of kurtosis of the variable *X* and suggested the following ratio estimator for $\overline{Y}$:

$$\bar{y}_{SEA} = \bar{y}\left(\frac{\overline{X} + \beta_{2(x)}}{\bar{x} + \beta_{2(x)}}\right) \tag{7}$$

Kadilar and Cingi [15] developed the following ratio estimators for $\overline{Y}$:

$$\bar{y}_{KC1} = \bar{y}\left(\frac{\overline{X}\beta_{2(x)} + \rho_{yx}}{\bar{x}\beta_{2(x)} + \rho_{yx}}\right) \tag{8}$$

$$\bar{y}_{KC2} = \bar{y}\left(\frac{\overline{X}\rho_{yx} + \beta_{2(x)}}{\bar{x}\rho_{yx} + \beta_{2(x)}}\right) \tag{9}$$

It is well known that the sample mean $\bar{y}$ is an unbiased estimator of the population mean $\overline{Y}$, and its variance under SRSWOR scheme is given as:

$$Var(\bar{y}) = \lambda \overline{Y}^2 C_y^2 \tag{10}$$

Also, the mean square errors (MSEs), upto the first order of approximation, for the existing estimators, i.e., $\bar{y}_R$, $\bar{y}_{REG}$, $\bar{y}_{\mathrm{Re}}$, $\bar{y}_{US1}$, $\bar{y}_{US2}$, $\bar{y}_{ST}$, $\bar{y}_{SEA}$, $\bar{y}_{KC1}$, and $\bar{y}_{KC2}$, are given, respectively, by

$$MSE(\bar{y}_R) = \lambda \overline{Y}^2 (C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x) \tag{11}$$

$$MSE(\bar{y}_{REG}) = \lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2) \tag{12}$$

$$MSE(\bar{y}_{\text{Re}}) = \lambda \bar{Y}^2 \left(C_y^2 + \frac{1}{4} C_x^2 - \rho_{yx} C_y C_x \right) \tag{13}$$

$$MSE(\bar{y}_{US1}) = \lambda \bar{Y}^2 \{ C_y^2 + w_1 (w_1 C_x^2 - 2\rho_{yx} C_y C_x) \} \tag{14}$$

$$MSE(\bar{y}_{US2}) = \lambda \bar{Y}^2 \{ C_y^2 + w_2 (w_2 C_x^2 - 2\rho_{yx} C_y C_x) \} \tag{15}$$

$$MSE(\bar{y}_{ST}) = \lambda \bar{Y}^2 \{ C_y^2 + \psi (\psi C_x^2 - 2\rho_{yx} C_y C_x) \} \tag{16}$$

$$MSE(\bar{y}_{SEA}) = \lambda \bar{Y}^2 \{ C_y^2 + \xi (\xi C_x^2 - 2\rho_{yx} C_y C_x) \} \tag{17}$$

$$MSE(\bar{y}_{KC1}) = \lambda \bar{Y}^2 \{ C_y^2 + \Omega_1 (\Omega_1 C_x^2 - 2\rho_{yx} C_y C_x) \} \tag{18}$$

$$MSE(\bar{y}_{KC2}) = \lambda \bar{Y}^2 \{ C_y^2 + \Omega_2 (\Omega_2 C_x^2 - 2\rho_{yx} C_y C_x) \} \tag{19}$$

where the notations used are as follows:

$$\lambda = \frac{1-f}{n},\ f = \frac{n}{N},\ w_1 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)} + C_x},\ \omega_2 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_{2(x)}},$$

$$\psi = \frac{\bar{X}}{\bar{X} + \rho_{yx}},\ \xi = \frac{\bar{X}}{\bar{X} + \beta_{2(x)}},\ \Omega_1 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)} + \rho_{yx}},\ \Omega_2 = \frac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx} + \beta_{2(x)}},$$

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2},\ C_x^2 = \frac{S_x^2}{\bar{X}^2},\ \rho_{yx} = \frac{S_{yx}}{S_y S_x},\ S_y^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (x_i - \bar{X})^2,\ S_{yx} = \frac{1}{(N-1)} \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X}).$$

3. Proposed Class of Estimators

It is a well known fact that the linear combination of two or more estimators perform better as compared to the usual original estimators, for instance, Agarwal and Mannai [1] and Fallah and Khoshtarkib [10]. Motivated by the given fact, we use the linear combination of the usual ratio estimator and the exponential ratio estimator to develop the following composite class of ratio estimators for the population mean $\overline{Y}$ in SRS design:

$$T = \bar{y}\left[\alpha\left(\frac{\overline{X}}{\bar{x}}\right) + (1-\alpha)\exp\left(\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right)\right] \tag{20}$$

where α is a scalar quantity. The optimum value of α is obtained on minimizing the MSE of the proposed class T. Also, it is worth mentioning that for $\alpha = 1$, the proposed class T reduces to the classical ratio estimator $\bar{y}_R$, and for $\alpha = 0$, the proposed class T reduces to the Bahl and Tuteja [3] estimator $\bar{y}_{\mathrm{Re}}$.

4. MSE of the Proposed Class

The expression for MSE of the proposed class T is obtained on considering

$$\bar{y} = \overline{Y}(1+e_0),\ \bar{x} = \overline{X}(1+e_1)$$

Also, we have

$$\left.\begin{array}{l} E(e_0) = E(e_1) = 0, \\ E(e_0^2) = \lambda C_y^2,\ E(e_1^2) = \lambda C_x^2,\ E(e_0 e_1) = \lambda \rho_{yx} C_y C_x \end{array}\right\} \tag{21}$$

Now, expressing T in terms of e_0 and e_1, we have

$$T = \overline{Y}(1+e_0)\left[\alpha(1+e_1)^{-1} + (1-\alpha)\exp\left\{\left(\frac{-e_1}{2}\right)\left(1+\frac{e_1}{2}\right)^{-1}\right\}\right] \tag{22}$$

Expanding the right hand side (RHS) expression in (22), multiplying out and retaining the first order error terms, we have

$$T = \overline{Y}\left[\alpha(1+e_0-e_1) + (1-\alpha)\left(1+e_0-\frac{e_1}{2}\right)\right] \tag{23}$$

On simplifying (23), we have

$$T - \overline{Y} = \overline{Y}\left[\alpha\left(\frac{-e_1}{2}\right) + e_0 - \frac{e_1}{2}\right] \tag{24}$$

Squaring both sides of (24), taking the expectation and using results of (21), we obtain the MSE of the proposed class T to the first order of approximation (i.e., to the terms of order $O(n^{-1})$) as

$$MSE(T) = \lambda \bar{Y}^2 \left[\frac{1}{4} \alpha^2 C_x^2 + \alpha \left(\frac{C_x^2}{2} - \rho_{yx} C_y C_x \right) + C_y^2 + \frac{C_x^2}{4} - \rho_{yx} C_y C_x \right] \quad (25)$$

4.1 Optimum Value of the Scalar α. The optimum value of α is obtained on minimizing the MSE of the proposed class T. Hence, on differentiating (25) with respect to α and equating the result to zero, we obtain the optimum value of α as

$$\alpha^* = \alpha_{opt} = \frac{2\rho_{yx} C_y}{C_x} - 1 \quad (26)$$

On substituting the value of α^* from (26) in (25), the minimum attainable MSE of the proposed class T is obtained as:

$$MSE\,(T)_{\min} = \lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2) \quad (27)$$

Hence, we have the following theorem.

Theorem 4.1.1. *Up to the terms of order* $O(n^{-1})$ *we have*

$$MSE(T) \geq \lambda \bar{Y}^2 C_y^2 (1 - \rho_{yx}^2) \quad (28)$$

with equality holding for $\alpha = \alpha^*$.

Remark 4.1.1. The minimum attainable MSE of T in (27) corresponds to the MSE of asymptotic optimum estimator (AOE) T^*, which is obtained on replacing α in (20) by its optimum value, i.e., α^*. Hence, in that case, we have

$$T^* = \bar{y} \left[\alpha^* \left(\frac{\bar{X}}{\bar{x}} \right) + (1 - \alpha^*) \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right] \quad (29)$$

and $MSE(T^*) = MSE(T)_{\min}$.

Remark 4.1.2. If the variables X and Y are uncorrelated, i.e., if $\rho_{yx} = 0$, then the expression for minimum attainable MSE of T in (27) reduces to that of the variance of sample mean $\bar{y}$ as given in (10).

Remark 4.1.3. If the variables X and Y are perfectly positively correlated, i.e., if $\rho_{yx} = 1$, then the value of minimum attainable MSE of T in (27) becomes zero, and in that situation the proposed class T is regarded as the most superior estimator for the population mean $\bar{Y}$.

5. Efficiency Comparisons

In this section, we have obtained the necessary and sufficient conditions (NASCs) for the dominance of the proposed class T over the well-known existing estimators by using MSE criterion, on utilizing the equations (10) to (19), and (25), as described below:

(i) $MSE(T) < Var(\bar{y})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx}C_yC_x\right) < \rho_{yx}C_yC_x - \frac{C_x^2}{4} \tag{30}$$

(ii) $MSE(T) < MSE(\bar{y}_R)$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx}C_yC_x\right) < \frac{3}{4}C_x^2 - \rho_{yx}C_yC_x \tag{31}$$

(iii) $MSE(T) = MSE(\bar{y}_{REG})$ if

$$\alpha = \frac{2\rho_{yx}C_y}{C_x} - 1 \tag{32}$$

(iv) $MSE(T) < MSE(\bar{y}_{\text{Re}})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx}C_yC_x\right) < 0 \tag{33}$$

(v) $MSE(T) < MSE(\bar{y}_{US1})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx}C_yC_x\right) < C_x^2\left(w_1^2 - \frac{1}{4}\right) + \rho_{yx}C_yC_x(1 - 2w_1) \tag{34}$$

(vi) $MSE(T) < MSE(\bar{y}_{US2})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx} C_y C_x\right) < C_x^2\left(w_2^2 - \frac{1}{4}\right) + \rho_{yx} C_y C_x (1 - 2w_2) \quad (35)$$

(vii) $MSE(T) < MSE(\bar{y}_{ST})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx} C_y C_x\right) < C_x^2\left(\psi^2 - \frac{1}{4}\right) + \rho_{yx} C_y C_x (1 - 2\psi) \quad (36)$$

(viii) $MSE(T) < MSE(\bar{y}_{SEA})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx} C_y C_x\right) < C_x^2\left(\xi^2 - \frac{1}{4}\right) + \rho_{yx} C_y C_x (1 - 2\xi) \quad (37)$$

(ix) $MSE(T) < MSE(\bar{y}_{KC1})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx} C_y C_x\right) < C_x^2\left(\Omega_1^2 - \frac{1}{4}\right) + \rho_{yx} C_y C_x (1 - 2\Omega_1) \quad (38)$$

(x) $MSE(T) < MSE(\bar{y}_{KC2})$ if

$$\frac{1}{4}\alpha^2 C_x^2 + \alpha\left(\frac{C_x^2}{2} - \rho_{yx} C_y C_x\right) < C_x^2\left(\Omega_2^2 - \frac{1}{4}\right) + \rho_{yx} C_y C_x (1 - 2\Omega_2) \quad (39)$$

6. Empirical Study

The relative performance of the proposed class T over the well-known existing estimators has been explored by considering four real population datasets. The descriptions of populations are mentioned below, along with the values of various parameters in Table 1.

Population I- [Source: Kadilar and Cingi [14]]

Y = Amount of apple production in 1999

X = Number of apple trees in 1999

Population II- [Source: Gupta and Shabbir [12]]

Y = Area under wheat in (acres) in the year 1937

X = Area under wheat in (acres) in the year 1936

Population III- [Source: Koyuncu and Kadilar [16]]

Y = Number of teachers

X = Number of students

Population IV- [Source: Subramani [31]]

Y = Cultivated area in 1981

X = Population in 1981.

Table 1. Parameters of Populations.

Populations	N	n	$\bar{Y}$	$\bar{X}$	C_y	C_x	ρ_{yx}	$\beta_{2(x)}$
I	106	20	2212.59	27421.70	5.22	2.10	0.86	34.57
II	34	20	201.41	218.41	0.74	0.76	0.93	3.282
III	923	180	436.43	11440.49	1.71835	1.8645	0.95	18.72
IV	70	25	96.7000	175.2671	0.6254	0.8037	0.7293	7.0952

The optimum values of the scalar α are computed for the concerned populations, and the findings are depicted in Table 2.

Table 2. Optimum values of the scalar α for various populations.

Populations	I	II	III	IV
$\alpha_{opt} = \frac{2\rho_{yx}C_y}{C_x} - 1$	3.27543	0.811053	0.751067	0.135011

The percent relative efficiencies (PREs) are obtained for various estimators of $\bar{Y}$ with respect to the sample mean $\bar{y}$, and the findings are depicted in Table 3. The PREs are computed by using the formula:

$$PRE(\phi, \bar{y}) = \frac{Var(\bar{y})}{MSE(\phi)} \times 100$$

where $\phi = \bar{y}, \bar{y}_R, \bar{y}_{REG}, \bar{y}_{\text{Re}}, \bar{y}_{US1}, \bar{y}_{US2}, y_{ST}, \bar{y}_{SEA}, \bar{y}_{KC1}, \bar{y}_{KC2}, T$.

Table 3. PREs of various estimators of $\overline{Y}$ with respect to $\bar{y}$.

Estimator	Population I	Population II	Population III	Population IV
$\bar{y}$	100	100	100	100
$\bar{y}_R$	212.82	691.97	864.01	128.69
$\bar{y}_{REG}$	384.03	740.19	1025.64	213.62
$\bar{y}_{\text{Re}}$	143.99	324.09	379.46	210.24
$\bar{y}_{US1}$	212.82	692.98	864.03	128.91
$\bar{y}_{US2}$	212.72	708.99	865.93	144.89
$\bar{y}_{ST}$	212.81	695.95	864.19	130.07
$\bar{y}_{SEA}$	212.61	705.24	867.58	141.80
$\bar{y}_{KC1}$	212.82	693.20	864.02	128.89
$\bar{y}_{KC2}$	212.57	706.16	867.76	146.49
T	**384.03**	**740.19**	**1025.64**	**213.62**

Bold values indicate the maximum PREs.

7. Results

The following results are obtained from Table 3:

(i) In all the four populations, the proposed class T has maximum PREs, and hence it exhibits better performances as compared to the sample mean $\bar{y}$, and the other existing estimators, except $\bar{y}_{REG}$.

(ii) In all the four populations, the PREs of $\bar{y}_R$ and $\bar{y}_{US1}$ are nearly the same, and hence they are equally efficient for the estimation of population mean $\overline{Y}$.

(iii) In all the four populations, the PREs of $\bar{y}_{ST}$ and $\bar{y}_{KC1}$ are approximately the same, and hence they are equally efficient for the estimation of population mean $\overline{Y}$.

(iv) In all the four populations, the PREs of the sample mean $\bar{y}$ are less as compared to that of the proposed class T, and the other existing estimators.

8. Conclusion

In this paper, a composite class of ratio estimators has been developed for estimating the mean of a study variable under SRS design. It can be observed from the theoretical results that the classical ratio estimator $\bar{y}_R$, and the Bahl and Tuteja [3] estimator $\bar{y}_{\text{Re}}$, are the specific members of the proposed class T. It has been revealed from Table 3 that the estimators involving auxiliary variable are dominant over the sample mean $\bar{y}$. Hence, information on auxiliary variable is significantly used in the theory of estimation of population parameters for the study variable. Moreover, from the theoretical results of Section 4.1, we observe that if the correlation coefficient between the variables Y and X (i.e., ρ_{yx}) increases, then the MSE of the proposed class T reduces. Hence, the more highly correlated the study and auxiliary variables, the more precise is the proposed class T. This justification is also valid for the other well-known existing estimators involving auxiliary variable.

In view of the theoretical and empirical results, we conclude that the proposed class T is dominant over the sample mean $\bar{y}$, and the other well-known existing estimators, except $\bar{y}_{REG}$, for the estimation of population mean $\overline{Y}$ of the study variable Y. Moreover, the proposed class T can be regarded as an alternative to the classical linear regression estimator $\bar{y}_{REG}$, as both are equally efficient for the estimation of population mean $\overline{Y}$.

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