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APPLICATION OF NONAGONAL FUZZY NUMBER TO SOLVE SEQUENCING PROBLEM UNDER UNCERTAIN CASE

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Abstract

In the Modern Era, time management plays a vital role in every aspects. Especially in manufacturing industry, it is very important to finish the allocated schedule at a specific time frame. Otherwise the industry will face severe consequences. But handling the time becomes more difficult when vagueness arises. Because scheduling the jobs/operations in a particular order so as to minimize the time to make cost effective will become more difficult in an imprecise case. So in this paper, we make an attempt to solve the sequencing problem under uncertain case.

1. Introduction

In the Modern World, we may come across many uncertainty cases. Classical set theory fails to deal such cases, which we have come across in our day to day life. Fuzzy set theory is the most effective tool having capability to deal with imprecise and incomplete information which occurs in different disciplines including Engineering, Mathematics, Statistics, Artificial Intelligence, Medical, etc. The important concepts of Fuzzy Sets put forth by

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L. A. Zadeh in 1965 [8] has opened up keen insights and applications in a wide range of Scientific fields. Operations on Fuzzy numbers were introduced by D Dubois and H. Prade [4]. Decision Making Concept in Fuzzy Environment were introduced by Bellman and Zadeh [10]. Many researchers introduced fuzzy numbers such as triangular, trapezoidal, Pentagonal, etc. Here we arrive a solution for uncertain sequencing problem using Nonagonal Fuzzy Number.

Time management is a precious aspect in the technological world. The capitalist in the business world should be efficient in handling time to carry out the specified tasks so as to save money, human resources, power and also the time itself. Particularly, in manufacturing industries, it should be handled in a cost effective manner. Otherwise, the manufacturer could meet apparent loss because of his inefficiency in handling human resources, machines, fuels, electricity, etc. Sequencing problem plays a vital role to reduce this risk and determines the sequence in which n jobs to be processed through m machines so as to the scheduled jobs will be completed within their deadlines and gives maximum Profit. But if the Manufacturer is not familiar with the time taken by the machine to perform each of his jobs (Uncertain Case), it is hard to schedule all of his listed jobs in the specific time to reduce the cost. So in this paper, we intend to deal an imprecise case, where the vagueness arises in 9 different points by using nonagonal fuzzy number to solve sequencing problem.

2. Preliminaries

This section reviews basic definitions of fuzzy set theory, Nonagonal fuzzy number and Sequencing problem.

2.1 Definition [9]**.** A fuzzy set is characterized by its membership function, taking values from the domain, space or universe of discourse mapped into the unit interval [0, 1]. A fuzzy set A in the universal set X is defined as $A = (x, \mu(x); x \in X)$. Here $\mu_A : A \to [0, 1]$ is the grade of the membership function and $\mu_A(x)$ is the grade value of $x \in X$ in the fuzzy set A.

2.2 Definition [3]**.** The α-cut of the fuzzy set A of the Universe of discourse X is defined as,

$$A_\alpha = \{x \in X / \mu_{A(x)} \geq \alpha\}, \text{ where } \alpha \in [0, 1].$$

2.3 Definition [9]**.** A fuzzy number A is a subset of real line R, with the membership function μ_A satisfying the following properties:

(i) $\mu_A(x)$ is piecewise continuous in its domain.

(ii) A is normal, i.e., there is a $x_0 \in A$ such that $\mu_A(x_0) = 1$.

(iii) A is convex, i.e., $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$.

$\forall\, x_1, x_2$ in X.

2.4. Definition [3]**.** A Nonagonal fuzzy number A, denoted by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ and its membership function is defined as:

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{1(x-a_1)}{4(a_2-a_1)}; & a_1 \leq x \leq a_3 \\ \dfrac{1}{4} + \dfrac{1(x-a_2)}{4(a_3-a_2)}; & a_2 \leq x \leq a_3 \\ \dfrac{1}{2} + \dfrac{1(x-a_3)}{4(a_4-a_3)}; & a_3 \leq x \leq a_4 \\ \dfrac{1}{4} + \dfrac{1(x-a_4)}{4(a_5-a_4)}; & a_4 \leq x \leq a_5 \\ 1 + \dfrac{1(x-a_5)}{4(a_6-a_5)}; & a_5 \leq x \leq a_6 \\ \dfrac{3}{4} + \dfrac{1(x-a_6)}{4(a_7-a_6)}; & a_6 \leq x \leq a_7 \\ 2 + \dfrac{1(x-a_7)}{4(a_8-a_7)}; & a_7 \leq x \leq a_8 \\ \dfrac{1(a_9-x)}{4(a_9-a_8)}; & a_8 \leq x \leq a_9 \\ 0; & \text{otherwise} \end{cases}$$

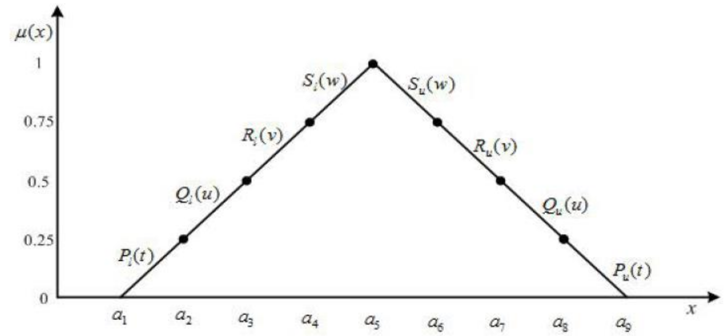

Figure A. The Nonagonal fuzzy number.

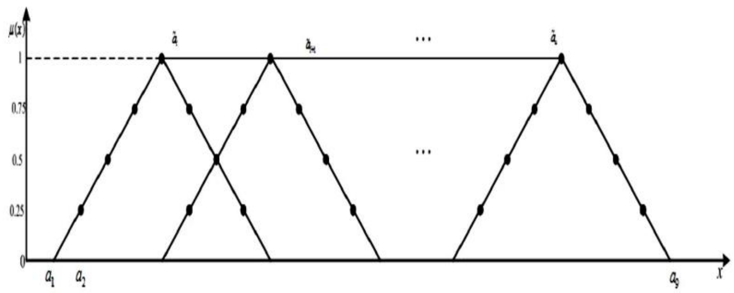

Figure B. The Nonagonal fuzzy number from uncertain linguistic term.

Linguistic values are given in the following table.

Table 1. Nonagonal Fuzzy Linguistic scale.

Linguistic terms	Linguistic vales
No influence	(0,0,0,0,0,0.04,0.08,0.12,0.16)
Very Low influence	(0,0,0.04,0.08,0.12,0.16,0.20,0.24,0.28)
Low Influence	(0.16,0.20,0.24,0.28,0.32,0.36,0.40,0.44,0.48)
Medium Influence	(0.32,0.36,0.40,0.44,0.48,0.52,0.56,0.60,0.64)
Influence	(0.48,0,52,0,56,0.60,0.64,0.68,0.72,0.76,0.80)
Very High Influence	(0.64,0.68,0.72,0.76,0.80,0.84,0.88,0.92,0.96)
Very Very High influence	(0.8,0.84,0.88,0.92,0.86,0.1,1,1,1)

Influence	(0.48,0,52,0,56,0.60,0.64,0.68,0.72,0.76,0.80)
Very High Influence	(0.64,0.68,0.72,0.76,0.80,0.84,0.88,0.92,0.96)
Very Very High influence	(0.8,0.84,0.88,0.92,0.86,0.1,1,1,1)
Influence	(0.48,0,52,0,56,0.60,0.64,0.68,0.72,0.76,0.80)
Very High Influence	(0.64,0.68,0.72,0.76,0.80,0.84,0.88,0.92,0.96)
Very Very High influence	(0.8,0.84,0.88,0.92,0.86,0.1,1,1,1)

2.5 Sequencing Problem: Sequencing is the selection of appropriate order, so as to perform the number of operations (jobs) with the available service facilities (Machines or equipment) to optimize the output in terms of cost, time or profit.

Assumptions:

1. Processing time of each operation is known and cannot be changed.

2. As operation once started, it should be completed.

3. Only one operation can be done on a machine at a time.

4. All operation should be done in the same order of machines.

5. Operation priorities cannot be changed.

Working rule:

Examine $Min(A_i) \geq Max(B_i)$, $Min(C_i) \geq Max(B_i)$. If either or both the condition satisfied we can proceed the following steps:

- For 3 machine Problem, Find $G = A_i + B_i$, $H = C_i + B_i$, and proceed as follows. Otherwise Skip this step.

- List the operations and their time at each service facility.

- Choose the job with the smallest activity time. If that activity time belongs to first service facility then schedule the job in first. If the activity time belongs to second service facility then schedule the job in last.

- If the tie occurs, then schedule the job accordingly by seeing the minimum of the smallest activity time.

- Cancel the jobs already assigned and repeat the above steps until all the operations are assigned.

2.6 Definition. Yager's ranking index is defined by,

$Y(A) = \int_0^1 0.5(A_\propto^L + A_\propto^U)$ where $(A_\propto^L + A_\propto^U)$ is the $\propto$-level cut of the fuzzy number A.

3. To solve a sequencing fuzzy problem with inadequate data using nonagonal fuzzy number

There are order ABC. The manufacturer 5 jobs. Each of the job has to be processed in three machines A, B, C in the knows the difficulties of all jobs and the rate of efficiency of the machines and it is given in the following table.

Jobs	1	2	3	4	5
Machine A	H	VH	M	H	M
Machine B	L	M	VVH	VVH	L
Machine C	M	H	L	VH	VVH

Applying Yager's methods, to find (0.48, 0.52, 0.56, 0.6, 0.64, 0.68, 0.72, 0.76, 0.8). The nonagonal fuzzy number's membership for (0.48, 0.52, 0.56, 0.6, 0.64, 0.68, 0.72, 0.76, 0.8) is given by,

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{1(x-0.48)}{4(0.52-0.48)} \\ \dfrac{1}{4} + \dfrac{1(x-0.52)}{4(0.56-0.52)} \\ \dfrac{1}{2} + \dfrac{1(x-0.56)}{4(0.6-0.56)} \\ \dfrac{3}{4} + \dfrac{1(x-0.6)}{4(0.64-0.6)} \\ 1 - \dfrac{1(x-0.64)}{4(0.68-0.64)} \\ \dfrac{3}{4} - \dfrac{1(x-0.68)}{4(0.72-0.68)} \\ \dfrac{1}{2} - \dfrac{1(x-0.72)}{4(0.76-0.72)} \\ \dfrac{1(0.8-x)}{4(0.8-0.76)} \end{cases}$$

First term implies, $\frac{1}{4}\frac{(x-0.48)}{0.04} = \alpha$

$\Rightarrow x - 0.48 = \alpha(0.16)$

$x = 0.16\alpha + 0.48$

Second term implies, $\frac{1}{4} + \frac{1}{4}\frac{(x-0.52)}{0.04} = \alpha \Rightarrow x - 0.52 = \left(\alpha - \frac{1}{4}\right)0.16$

$\Rightarrow x = 0.16\alpha + 0.48$

Proceeding,

$$Y(c_{11}) = \int_0^1 0.5(5.12)dx$$

$$= 2.56.$$

Replacing the corresponding c_{ij}'s we get

Jobs	1	2	3	4	5
Machine A	2.56	3.2	1.92	2.56	1.92
Machine B	1.28	1.92	0.81	0.81	1.28
Machine C	1.92	2.56	1.28	3.2	0.81

Since $Min(A_i) \geq Max(B_i)$, we proceed by Johnson's Algorithm,

Find $G = A_i + B_i$,

$H = C_i + B_i$

Jobs	1	2	3	4	5
Machine G	3.84	5.12	2.73	3.37	3.20
Machine H	3.20	4.48	2.09	4.01	2.09

The optimal sequence is

4	2	1	5	3

Jobs	Machine A		Machine B		Machine C	
	Time in	Time out	Time in	Time out	Time in	Time out
4	0	2.56	2.56	3.37	3.37	6.57
2	2.56	5.76	5.76	7.68	7.68	10.24
1	5.76	8.32	8.32	9.6	10.24	12.16
5	8.32	10.24	10.24	11.52	12.16	12.97
3	10.24	12.16	12.16	12.97	12.97	14.25

Total elapsed time $= 14.25$ time units

Idle time for Machine $A = 14.25 - 12.16 = 2.09$ time units

Idle time for machine $B = 2.56 + 2.39 + 0.64 + 0.64 + 0.64 + 1.28 = 8.15$ time units

Idle time for machine $C = 3.37 + 1.11 = 4.48$ time units.

Conclusion

It is concluded that the manufacturer can schedule the jobs in the sequence, $4 \rightarrow 2 \rightarrow 1 \rightarrow 5 \rightarrow 3$ with 3 available machines in 14.25 time units optimizely. The idle time for Machine A is 2.09 time units, idle time for

Machine B is 8.15 time units and idle time for Machine C is 4.48 time units. Thus we arrived the optimize solution for an uncertain sequencing problem to make cost effective by minimizing the time.

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