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# DEMONSTRATION OF TWO DISPARATE STRUCTURES OF INTEGER TRIPLES CONCERNING PAN-SAN AND PAN-SAN COMRADE NUMBERS

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## Abstract

In this calligraphy, two disparate arrangements of triples $\{\alpha, \beta, \gamma\}, \{\beta, \gamma, \delta\}, \{\gamma, \delta, \varepsilon\}$ etc. where in one of each module is a Pan-San number and in the other it is a Pan-San Comrade number composed with the condition that the multiplication of any two modules added with $k^2 k \in N - \{1\}$ is again a square of an integer are explored.

## 1. Introduction

A Diophantine $m$-tuple with property $D(n)$ is a set of positive integers $\{a_1, a_2, \ldots, a_m\}$ such that $a_i a_j + n$ is a perfect square for all $i \neq j$ belonging to the set $\{1, 2, \ldots, m\}$. In [1], Diophantus has previously investigated how to trace these and he create the rational quadruple with

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property $D(1)$. In [2,3], the authors Fermat and Euler have exposed two separate integer quadruples with the similar property. For across-the-board appraisal of a variability of articles, one may refer [3-16]. In this communication, the measures of triples $\{\alpha, \beta, \gamma\}, \{\beta, \gamma, \delta\}, \{\gamma, \delta, \varepsilon\}$ etc where each segment is Pan-San and Pan-San Comrade numbers serene with the condition that the product of any two segments enhanced by $k^2, k \in N - \{1\}$ also a square of an integer are dissected.

Hypothesize that

$\alpha = C_{n-1, k}, \beta = C_{n+1, k}, n \in N$ where

$C_{n, k} = 2(k^2 + 1)C_{n-1, k} - C_{n-2, k}, k, n \in N - \{1\}$

be two conflicting Pan-San numbers such that $\alpha\beta + k^2$ is a number with power raised to two.

Let $\gamma$ be an additional positive integer that accomplishes the ensuing consequences

$$\alpha\gamma + k^2 = a^2 \quad (1)$$

$$\beta\gamma + k^2 = b^2 \quad (2)$$

The resolution of (1) and (2) provides the possibility of $\gamma$ by

$$\gamma = \frac{a^2 - b^2}{\alpha - \beta} \quad (3)$$

The collaboration of (3) in (2) interprets the relationship in terms of $\alpha$ and $\beta$ as

$$\beta a^2 - \alpha b^2 = -k^2(\alpha - \beta) \quad (4)$$

To achieve the necessary condition, let us generate the following linear expansions

$$a = X - \alpha T \quad (5)$$

$$b = X - \beta T \quad (6)$$

The standard quadratic equation in $X$ and $T$ is projected by restoring the overhead values of $a$ and $b$ in (4) as below

$$X^2 - \alpha\beta T^2 = k^2 \tag{7}$$

Making a choice for the necessity of establishing (7) in the codes

$$X_0 = C_{n,k},\ T_0 = 1$$

and enforcing these cryptographs in the previous held equations (5) and (6) vintages that

$$a = C_{n,k} - \alpha$$

$$b = C_{n,k} - \beta$$

The third element of conventional triple which pledge the assertion by swap the above appropriate outcomes of and in (3) is identified by

$$\gamma = 2k^2 C_{n,k}$$

Hence, it is clinched that

$\{C_{n-1,k}, C_{n-1,k}, 2k^2 C_{n,k}\}$ is an integer triple with the property $D(k^2)$, $k \in N - \{1\}$. Let $\delta$ be the next positive integer together with the statements that

$$\beta\delta + k^2 = c^2 \tag{8}$$

$$\gamma\delta + k^2 = d^2 \tag{9}$$

Deducting (9) from (8), the substantial value of is determined by

$$\delta = \frac{c^2 - d^2}{\beta - \gamma} \tag{10}$$

The partnership of (9) and (10) construes the succeeding bond as

$$\gamma c^2 - \beta d^2 = -k^2(\gamma - \delta) \tag{11}$$

Contemplate the fresh rectilinear modifications for $c$ and $d$ as

$$c = X + \beta T \tag{12}$$

$$d = X + \gamma T \tag{13}$$

Reestablishing the above values of $c$ and $d$ in (11), the orthodox second degree equation is appraised by

$$X^2 - \gamma\delta T^2 = k^2 \tag{14}$$

Captivating $X_0 = C_{n+1,k} - C_{n,k}$, $T_0 = 1$ and imposing them in the expressions (12) and (13) produces the selections of $c$ and $d$ as

$$c = C_{n+1,k} - C_{n,k} + \beta$$

$$d = C_{n+1,k} - C_{n,k} + \gamma$$

In sight of (10), the essential option of is calculated by

$$\delta = 3C_{n+1,k} + 2C_{n,k}(k^2 - 1)$$

Hence, $\{C_{n+1,k}, 2k^2C_{n,k}, 3C_{n+1,k} + 2C_{n,k}(k^2 - 1)\}$ is a required triple in Pan-San numbers in which the product of two elements in the set added with a square is a number with exponent two.

Now, pick to be some other integer that meets the following requirements

$$\gamma\varepsilon + k^2 = e^2 \tag{15}$$

$$\delta\varepsilon + k^2 = f^2 \tag{16}$$

By implementing a simple analysis in (15) and (16), it is interesting to emphasize that

$$\varepsilon = \frac{e^2 - f^2}{\gamma - \delta} \tag{17}$$

Suppose that

$$\delta\zeta + k^2 = g^2 \tag{18}$$

$$\varepsilon\zeta + k^2 = h^2 \tag{19}$$

where $\zeta \in Z - \{0\}$

Ensuing the erstwhile course in (18) and (19), the corresponding value of the factor $\zeta$ in the sequence is predicted by

$$\zeta = \frac{g^2 - h^2}{\delta - \varepsilon} \tag{20}$$

Since the mission is to deliver the exact integer values for the criteria in the vital patterns, let us use the equivalent conversions

$$e = C_{n-1,k} + 2C_{n+1,k} - 3C_{n,k} - \gamma$$

$$f = C_{n-1,k} + 2C_{n+1,k} - 3C_{n,k} - \delta$$

$$g = 2C_{n-1,k} + 6C_{n+1,k} - 7C_{n,k} + \gamma$$

$$h = 2C_{n-1,k} + 6C_{n+1,k} - 7C_{n,k} + \varepsilon$$

and subsequently the elements with the requisite forms of triples by the relevant resource in the same structure as outlined above are analyzed by

$$\varepsilon = 2C_{n-1,k} + 7C_{n+1,k} - 4C_{n,k} - \gamma$$

$$\zeta = 6C_{n-1,k} + 22C_{n+1,k} - 6C_{n,k}(k^2 - 4)$$

Accordingly,

$$\{C_{n-k}, C_{n+1,k}, 2k^2C_{n,k}\}, \{C_{n-1,k}, 2k^2C_{n,k}, 3C_{n+1,k} + 2C_{n,k}(k^2 - 1)\},$$

$$\{2k^2C_{n,k}, 3C_{n+1,k} + 2C_{n,k}(k^2 - 1), 2C_{n-1,k} + 7C_{n,1,k} + 4C_{n,k}(k^2 - 2)\},$$

$$\{3C_{n+1}, 2C_{n,k}(k^2 - 1), 2C_{n-1} + 7C_{n+1,k} + 4C_{n,k}(k^2 - 2), 6C_{n-1,k}$$

$$+ 22C_{n+1,k} + 6C_{n,k}(k^2 - 4)\}$$

etc are shapes of triples concerning Pan-San sequence whereas the multiplication of two barebones upgraded by $k^2$ is a perfect square where is a natural number other than 1. Hence, the patterns of integer triples $\{\alpha, \beta, \gamma\}, \{\beta, \gamma, \delta\}, \{\gamma, \delta, \varepsilon\}$ etc in which the factors filling the above proclamation are assessed.

**Elucidations in tabular form for the numerical replacements of the above patterns of triples are demarcated below.**

| k | n | {α, β, γ} | {β, γ, δ} | {γ, δ, ε} | {δ, ε, ζ} |
|---|---|---|---|---|---|
| 2 | 2 | {20, 1960, 1584} | {1960, 1584, 7068} | {1584, 7068, 15344} | {7068, 15344, 43240} |
| 3 | 1 | {3, 1197, 1080} | {1197, 1080, 4551} | {1080, 4551, 10065} | {4551, 10065, 28152} |
| 1 | 1 | {4, 4620, 4352} | {4620, 4352, 17940} | {4352, 17940, 39964} | {17940, 39964, 111456} |

**Remark.** By smearing the identical technique as above, the following proposals of triples in which every component belong to Pan-San Comrade sequence such that the product of any two components enlarged by $k^2$ is a number with exponent two are designated.

$$\{R_{n-k},\ R_{n+1,k},\ 2k^2R_{n,k}\},\ \{R_{n+1,k}2k^2R_{n,k},\ 3R_{n+1,k}+2R_{n,k}(k^2+1)\},$$

$$\{2k^2R_{n,k},\ 3R_{n+1,k}+2R_{n,k}(k^2+1),\ 2R_{n+1,k}+7R_{n+1,\,k}+4R_{n,k}(k^2+2)\},$$

$$\{3R_{n+1,k}+2R_{n,k}(k^2+1),\ 2R_{n-1,k}+7R_{n+1,k}+4R_{n,k}(k^2+2),\ 6R_{n-1,k}$$

$$+22R_{n+1,k}+6R_{n,k}(k^2+4)\}$$

where $R_{n,k}=2(k^2+1)R_{n-1,k}R_{n-2,k}$ and $R_{0,k}=0,\ R_{1,k}=k,$ $k\in N-\{1\}$.

**A limited number of numerical cases for the above sequences of triples are offered below.**

| k | n | {α, β, γ} | {β, γ, δ} | {γ, δ, ε} | {δ, ε, ζ} |
|---|---|---|---|---|---|
| 2 | 2 | {12, 408, 560} | {408, 560, 1924} | {560, 1924, 4560} | {1924, 4560, 12408} |
| 3 | 1 | {3, 765, 864} | {765, 864, 3255} | {864, 3255, 7473} | {3255, 7473, 20592} |
| 1 | 1 | {4, 3596, 3840} | {3596, 3840, 14868} | {3840, 14868, 33820} | {1468, 33820, 93536} |

**Substantiation of the numerical examples is unveiled by the subsequent C program.**

```
#include<stdio.h>

#include<conio.h>

#include<math.h>
```

```
void main()
{
int ca,n,k;
char ch;
long long int C(int n, int k),a,b,c,d,e,f,int R(int n, int k);
clrscr();
do {
printf("\nEnter the value of k and n\n");
scanf("%d%d",&k,&n);
printf("\nEnter your choice 1 or 2 for Pan-San or Pan-San Comrade
Sequence\n");
scanf("%d",&ca);
switch (ca)
{
case 1:
a=C(n-1,k);
b=C(n+1,k);
c=2*k*k*C(n,k);
d=3*b+2*C(n,k)*(k*k-1);
e=7*b+2*a+4*C(n,k)*(k*k*-2);
f=6*a+22*b+6*C(n,k)*(k*k-4);
break;
case 2:
a=R(n-1,k);
b=R(n+1,k);
c=2*k*k*R(n,k);
```

```
d=3*b+2*R(n,k)*(k*k+1);
e=7*b+2*a+4*R(n,k)*(k*k+2);
f=6*a+22*b+6*R(n,k)*(k*k*+4);
break;
}
printf("\n(%lld,%lld,%lld),(%lld,%lld,%lld),(%lld,%lld,%lld),(%lld,%lld,%ll
d),... ",a,b,c,b,c,d,c,d,e,d,e,f);
printf("\nDo you want to continue for different n and k (y/n)?\n");
 ch=getche();
}while (ch=='y'||ch=='Y');
getch();
}
long long int C(int n,int k)
{ long long C[50],y;
C[0]=0;
C[1]=k;
int i;
for(i=2;i<=n;i++)
C[i]=2*(k*k+1)*C[i-1]-C[i-2];
y=C[i-1];
return y;
} long long R(int n,int k)
{
long long R[50],y;
R[0]=0;
R[1]=k;
```

```
int i;
for(i=2,i<=n;i++)
R[i]=2*(k*k-1)*R[i-1]-R[i-2];
y=R[i-1];
return y;
}
```

## 3. Conclusion

In this script, two different patterns of triples entailing Pan-San and Pan-San Comrade sequences whereas the multiplication of two basics raised by is a perfect square where $k \in N - \{1\}$ are engendered. It is concluded that one can study with some other features to look at various trends of Diophantine triples, quadruples, quintuples, etc.

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