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FINDING CRITICAL PATH IN A FUZZY PROJECT NETWORK USING NEUTROSOPHIC FUZZY NUMBER

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Abstract

Fuzzy systems and Intuitionistic fuzzy systems cannot successfully deal with a situation where the conclusion is adequate, unacceptable and decision-makers declaration is uncertain. Neutrosophic sets are more practical and adequate than fuzzy and intuitionistic fuzzy systems. In this paper, we have used neutrosophic fuzzy numbers to solve the fuzzy critical path problem. A numerical example is given to illustrate the method.

1. Introduction

In the 1950s, the Critical Path Method (CPM) was recognised as a useful tool for planning and scheduling undertakings. It is a useful strategy for efficiently managing projects [1]. In many circumstances, activity times are not always accurate. Program evaluation and review method (PERT) [2, 3] and Monte Carlo simulation based on probability were used to deal with such inaccurate data. The concept of fuzziness [4] was used to deal with inaccurate

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data, and activity periods were represented by fuzzy numbers. Prade [6] suggested the use of fuzzy sets in scheduling problems because the cost and execution time do not have to be exact all of the time. Chanas Kumburowsi [5] proposed the PERT, the first fuzzy approach for scheduling problems.

Wang and Qingli [7] created a method to complete the fuzzy slack time and identify the important path based on fuzzy network analysis, which was an extension of deterministic network concepts.

As a generalisation of the notions of fuzzy sets [3], intuitionistic fuzzy sets [4], Smarandache [1] created the concept of neutrosophic set (NS) and neutrosophic logic. Neutrosophic sets can deal with certain types of uncertain information that cannot be dealt with by fuzzy sets or intuitionistic fuzzy sets, such as incomplete, indeterminate, and inconsistent information that exists in the actual world.

Truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F) are the three independent membership degrees that define the concept of neutrosophic set (F). However, the truth-membership, indeterminacy-membership, and falsity-membership of SVNS cannot be expressed with accurate real numbers or interval numbers in uncertain and complex situations. In addition, rather than interval numbers, triangular fuzzy numbers may successfully handle fuzzy data. Biswas et al. [16] suggested the notion of Triangular Fuzzy Number Neutrosophic sets (TFNNS) for this purpose, which combines triangular fuzzy numbers (TFNs) and single valued neutrosophic sets (SVNS). To solve multi-attribute decision making, Ye [17] created trapezoidal fuzzy neutrosophic number weighted arithmetic averaging and trapezoidal fuzzy neutrosophic number weighted arithmetic geometric averaging operators.

In this paper, basic concepts of neutrosophic sets are briefly reviewed. Then the mathematical model of neutrosophic CPM and a method to calculate Critical path in project network is proposed.

2. Preliminaries

In this section basic definition existing representations of triangular neutrosophic numbers, arithmetic operations some gives a brief overview of concepts of some neutrosophic sets and trapezoidal neutrosophic set.

2.1. Basic Definitions

Definition 2.1.1. A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1.

Definition 2.1.2. Let X denote a Universal Set. Then the membership function $\mu_{\tilde{A}}$ by which a fuzzy set A is usually defined has the form

$$\mu_{\tilde{A}}(x) : X \to [0, 1]$$

where $[0, 1]$ denotes the interval of real numbers from 0 to 1.

Definition 2.1.3 [8]**.** Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form

$$A = \{\langle x : T_A(x),\ I_A(x),\ F_A(x)\rangle x; x \in X\}$$

where the functions $T, I, F : X \to]0^-, 1^+[$ define respectively the truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with, condition

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$$

The functions $T_A(x), I_A(x), F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$.

Definition 2.1.4[9]**.** Let X be a space of points with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$ is an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$

A SVNS A can be written as

$$A = \{\langle x : T_A(x),\ I_A(x),\ F_A(x)\rangle x; x \in X\}$$

and for every $x \in X$

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Definition 2.1.5 [10]**.** A trapezoidal fuzzy neutrosophic number $\widetilde{A} = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ is said to be zero triangular fuzzy number neutrosophic number if $(a_1, a_2, a_3, a_4) = (0, 0, 0, 0), (b_1, b_2, b_3, b_4) = (1, 1, 1, 1), (c_1, c_2, c_3, c_4) = (1, 1, 1, 1)$.

3. Single Valued Non-Linear Triangular Neutrosophic Number

A Single Valued Non-linear triangular Neutrosophic number with nine components is defined as $\widetilde{A}_{Neu} = (p_1, p_2, p_3; q_1, q_2, q_3; r_1, r_2, r_3)$ whose truth membership, indeterminacy and falsity membership is defined as follows:

$$T_{\widetilde{A}_{Neu}}(x) = \begin{cases} \left(\dfrac{x - p_1}{p_2 - p_1}\right)^{a_1} & \text{when } p_1 \leq x \leq p_2 \\ 1 & \text{when } x = p_2 \\ \left(\dfrac{p_3 - x}{p_3 - p_2}\right)^{a_2} & \text{when } p_2 \leq x \leq p_3 \\ 0 & \text{otherwise} \end{cases}$$

and

$$I_{\widetilde{A}_{Neu}}(x) = \begin{cases} \left(\dfrac{x - q_1}{q_2 - q_1}\right)^{b_1} & \text{when } q_1 \leq x \leq q_2 \\ 1 & \text{when } x = q_2 \\ \left(\dfrac{q_3 - x}{q_3 - q_2}\right)^{b_2} & \text{when } q_2 \leq x \leq q_3 \\ 0 & \text{otherwise} \end{cases}$$

$$F_{\widetilde{A}_{Neu}}(x) = \begin{cases} \left(\dfrac{x - r_1}{q_2 - q_1}\right)^{c_1} & \text{when } r_1 \leq x \leq r_2 \\ 1 & \text{when } x = p_2 \\ \left(\dfrac{r_3 - x}{r_3 - r_2}\right)^{r_2} & \text{when } r_2 \leq x \leq r_3 \\ 0 & \text{otherwise} \end{cases}$$

where $0 \leq T_{\widetilde{A}_{Neu}}(x) + I_{\widetilde{A}_{Neu}}(x) + F_{\widetilde{A}_{Neu}}(x) \leq 3,\ x \in \widetilde{A}_{Neu}$.

3.1. Arithmetic Operations of Triangular Neutrosophic Numbers

In this section arithmetic operations between two triangular Neutrosophic numbers are given.

If $\widetilde{A}_{Neu}$ and $\widetilde{B}_{Neu}$ are two single valued neutrosophic numbers given as $\widetilde{A}_{Neu} = (a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3); \widetilde{B}_{Neu} = (a_4, a_5, a_6; b_4, b_5, b_6; c_4, c_5, c_6)$ where a, b and c are the scores given by the decision maker. Then the arithmetic operations are defined as

- Addition

$$\widetilde{C}_{Neu} = \widetilde{A}_{Neu} + \widetilde{B}_{Neu} = \{(a_1 + a_4,\, a_2 + a_5,\, a_3 + a_6);$$

$$(a_1 + a_4,\, a_2 + a_5,\, a_3 + a_6); (a_1 + a_4,\, a_2 + a_5,\, a_3 + a_6)\}$$

- Subtraction

$$\widetilde{D}_{Neu} = \widetilde{A}_{Neu} - \widetilde{B}_{Neu} = \widetilde{A}_{Neu} + (-\widetilde{B}_{Neu})$$

$$= \{(a_1 - a_6,\, a_2 - a_5,\, a_3 - a_4); (b_1 - b_6,\, b_2 - b_5,\, b_3 - b_4);$$

$$(c_1 - c_6,\, c_2 - c_5,\, c_3 - c_4)\}$$

- Deneutrosophication

If $\widetilde{A}_{Neu} = (a_1, a_2, a_3; b_1, b_2, b_3; c_1, c_2, c_3)$ is a triangular neutrosophic number then the deneutrosophication is defined as $R(\widetilde{D}, 0) = \dfrac{a_1 + 2a_2 + a_3 + b_1 + 2b_2 + b_3 + c_1 + 2c_2 + c_3}{2}$.

4. Calculating Critical Path in a Project Network

In our proposed model, the three different times (optimistic, Pessimistic, most likely) are interpreted as triangular neutrosophic number in project network.

The expected time is calculated by the formula $E_{jk} = \dfrac{t_0 + 4t_m + t_p}{6}$. We use CPM method for further calculation of earliest/latest time critical path, and float.

A fuzzy project network is an acyclic digraph, where the vertices represent events, and the directed edges represent the activities. Formally, a fuzzy project network is represented by $N = (V, A, T)$.

Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ be a set of vertices, where v_1 and v_n are the start and final events of the project and event v_i belongs to some path from v_1 to v_n.

Let $A \subset V \times V$ be a set of directed edges $a_{ij} = (v_i, v_j)$ that represents the activities to be performed in the project. Activity a_{ij} is presented by one and only one arrow starting with a event v_i and ending with event v_j. For each activity a_{ij}, a fuzzy number $t_{ij} \in T$ is defined, where t_{ij} is the fuzzy time required for the completion of a_{ij}. A critical path is a longest path from v_1 to v_n and an activity a_{ij} on the critical path is called critical activity. Let E_i and L_i be the earliest event time, and the latest event time for event i, respectively.

Let $D_j = \{i/i \in V \text{ and } a_{ij} \in A\}$ be a set of events obtained from event $j \in V$ and $i < j$.

We then obtain E_j using the following equations

$$E_j = \max\{E_i \oplus t_{ij}\}, \; i \in D, \text{ where } E_1 = 0.$$

Similarly, let $H_i = \{j/j \in V \text{ and } a_{ij} \in A\}$ be a set of event obtained from event $i \in V$ and $i < j$. Then we obtain

$$L_i = \min\{L_j - t_{ij}\}, \; j \in H, \text{ where } L_n = E_n.$$

The interval $[E_i, L_j]$ is the time during which a_{ij} must be completed. When the earliest fuzzy event time and latest fuzzy event time have been obtained, we can calculate the total float of each activity. For activity $i - j$ in a fuzzy network, the total of the activity $i - j$ can be computed as follows

$$T_{ij} = L_j - E_i - t_{ij}.$$

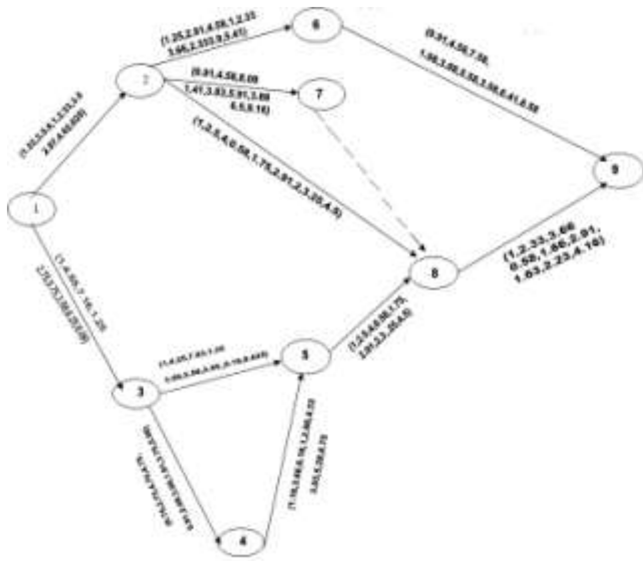
Using the foregoing formulae, we can calculate the earliest fuzzy event time, the latest fuzzy event time, and the total float of each activity. We defuzzify each activity's total float and locate the critical path where the sum of the total floats of the activities in the path equals zero.

5. Numerical Example

To explain the proposed approach in a better way, we have solved a numerical example and steps of solution are determined clearly. Input data for neutrosophic cpm.

Activity	Predecessor	Optimistic	Pessimistic time	Most Likely time
A	–	(1,2,3,0.5,1.5,2.5,1.2,2.7,3.5)	(1.5,9,1.5,4.5,6.5,4,7,10)	(1.5,3.5,5.5,1,2.3,3,4.5,6)
B	–	(1,5,8,1.3,6,4,7,9)	(1,2,3,0.5,1.5,2.5,1.5,2.5,3.5)	(1.5,8,1.5,3,6.5,4,7,9)
C	A	(1,4,7,1,3,5,3.5,6,7.5)	(1,1.5,4,0.5,1,2.5,1.25,3,4.25)	(1,5,9,1.5,4.5,6.5,4,7,10)
D	B	(1,3,5,0.5,2.5,3.5,2.5,4,6)	(1.5,3,4,5,5,1,2,3,3,4,5,6)	(0.5,2.5,4.5,1,2,3,1.5,3.5,5.5)
E	B	(0.5,2.5,4.5,5,1.5,3.5,2,4,6)	(1.5,9,1.5,4.5,6.5,4,7.5,10.5)	(1.5,2.5,3.5,1,1.5,3,2,3,4)
F	A	(2,4,6,1.5,2.5,3.5,3,5,7)	(1,2.3,0.5,1.5,2.5,1.2,2.7,3.5)	(1.4,7,1,3,5,3.5,6,7.5)
G	C	(0.5,2.5,4.5,1,2.3,2,4,6)	(1,5,8,1.5,3,6.5,4,7, 9)	(1,5,9,1.5,4.5,6.5,4,7,10)
H	D	(1.5,3.5,5.5,1.2,3.3,4.5,6)	(0.5,3.5,6.5,0.5,2.5,4.5,3,5,7)	(1,2,3,0.5,1.5,2.5,1.5,2.5,3.5)
I	A	(1.5,9,1.5,4.5,6.5,4,7,10)	(0.5,2.5,4.5,1,2.3,1.5,3.5,5.5)	(1,5,8,1,3,6,4,7,9)

J	E, G, H	(0.5,3.5,6.5,0.5,2.5,4.5,3,5,7)	(1.5,2.5,3.5,11.5,3,2,3,4)	(1,2,3,0.5,1.5,2.5,5,1.2,2.7,3.5)
K	F, I, J	(1,5,8,1.5,3.5,6.5,46,5.5)	(1,4,7,1,3,5,3.5,6,7.5)	(1,1.5,4,0.5,1.2,5,1,2.5,1.25,3,4.25)



Start and Finish Calculations

Activity	Duration	Earliest Start	Latest finish
1-2	(1.33, 3.5, 5.6, 1, 2.33, 3.5, 2.87, 4.62, 6.25)	E_{s1} = (0, 0, 0, 0, 0, 0, 0, 0, 0)	L_{F9} = (4.91, 15.74, 25.73, 4.32, 10.9, 18.98, 12.15, 20.76, 29.15)
1-3	(1, 4.5, 7.16, 1.25, 2.75, 5.75, 3.58, 6.25, 8.08)	E_{s2} = (1.33, 3.5, 5.6, 1, 2.33, 3.5, 2.87, 4.62, 6.25)	L_{F8} = (1.25, 13.41, 24.73, 1.41, 9.24, 18.4, 7.99, 18.53, 27.52)
3-5	(1, 4.5, 7.83, 1.25, 3.66, 5.58, 3.45, 6.16, 8.625)	E_{s3} = (1, 4.5, 7.16, 1.25, 2.75, 5.75, 3.58, 6.25, 8.08)	L_{F7} = (1.25, 13.41, 24.73, 1.41, 9.24, 18.4, 7.99, 18.53, 27.52)

3-4	(0.75, 2.75, 4.75, 0.91, 2.08, 3.08, 1.91, 3.75, 5.66)	E_{s4} = (1.75, 7.25, 11.91, 2.16, 4.83, 8.83, 5.49, 10, 13.74)	L_{F6} = (–2.67, 11.16, 24.82, 1.26, 7.82, 17.9, 3.57, 14.35, 25.57)
4-5	(1.25, 2.91, 4.58, 1, 2.33, 3.66, 2.33, 3.91, 5.41)	E_{s5} = (2.91, 10.91, 18.07, 3.16, 7.49, 13.16, 8.52, 15.28, 20.49)	L_{F5} = (–2.67, 10.91, 23.73, –1.5, 7.49, 17.72, 3.49, 15.28, 25.52)
2-6	(1.16, 3.66, 6.16, 1, 2.66, 4.33, 3.03, 5.28, 6.75)	E_{s6} = (2.58, 6.41, 4.66, 7.16, 10.15, 2, 5.2, 1053, 11.66)	L_{F4} = (–8.91, 7.25, 22.57, –5.83, 4.83, 16.72, –3.2, 10, 22.49)
2-7	(0.91, 4.58, 8.08, 1.41, 3.83, 5.91, 3.66, 6.5, 8.16)	E_{s7} = (2.24, 8.08, 13.68, 2.41, 6.16, 9.41, 6.53, 11.12, 15.41)	L_{F3} = (–13.66, 4.5, 21.82, –8.91, 2.75, 15.81, –8.92, 6.25, 20.58)
7-8	0	E_{s8} = (3.91, 13.41, 22.07, 3.4, 9.24, 16.07, 10.52, 18.53, 24.99)	L_{F2} = (–7.25, 8.25, 23.57, – 4.92, 5.49, 16.9, –1.84, 10.44, 23.24)
6-9	(0.91, 4.58, 7.58, 1.08, 3.08, 5.58, 3.58, 6.41, 8.48)	E_{s9} = (4.91, 15.74, 25.73, 4.32, 10.91, 8.98, 12.15, 2.76, 29.15)	L_{F1} = (–20.82, 0, 20.82, – 14.66, 0, 14.56, –17, 0, 12.5)
8-9	(1, 2.33, 3.66, 0.58, 1.66, 2.9, 1, 1.63, 2.23, 4.16)	-	-

Calculation of Total Float

Path	Total float	Deneutrosophic Value
$1 \rightarrow 2 \rightarrow 6 \rightarrow 9$	{-38.55,14.25,68.34; -25.27,9.48,45.19; -24.27,15.46,61.11}	13.7525
$1 \rightarrow 2 \rightarrow 7 \rightarrow 8 \rightarrow 9$	{-40.34,20.74,89.76; -32.43,12.4,61.36;	21.13

	-30.35,27.97,83.34}	
$1 \to 2 \to 8 \to 9$	{-33.63,17.49,67.13; -21.43,11.4,46.2; -18.27,23.81,64.01}	17.4508
$1 \to 2 \to 8 \to 9$	{-33.25,5.33,43.3; -12.66,3.08,30.65; -24.42,7.33,37.99}	6.0916
$1 \to 3 \to 4 \to 5 \to 8 \to 9$	{-95.71,5.33,105.77; -56.64,3.08,74.33; -75.42,7.33,87.07}	5.9066

The critical path is $1 \to 3 \to 4 \to 5 \to 8 \to 9$.

Conclusion

The concept of a neutrosophic fuzzy number was used to identify the critical path in a fuzzy project network in this paper. A numerical example is used to demonstrate the method.

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