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ACCURATE SPLIT (NON-SPLIT) DOMINATION IN FUZZY GRAPHS

A. MOHAMED ISMAYIL and H. SABITHA BEGUM

PG and Research Department of Mathematics
Jamal Mohamed College
(Autonomous)
Tiruchirappalli-620 020, Tamilnadu, India

Department of Mathematics
AIMAN College of Arts and Science for Women
(Affiliated to Bharathidasan University)
Tiruchirappalli-620021, Tamilnadu, India
E-mail: amismayil1973@gmail.com
Sabitha138@gmail.com

Abstract

An accurate dominating set D of a fuzzy graph is stated to be an accurate split (non-split) dominating set, if an induced fuzzy sub graph $\langle V - D\rangle$ is disconnected (connected). In this paper, accurate split (non-split) dominating set in fuzzy graph is described with appropriate illustration. Accurate minimal split (non-split) dominating set and accurate split (non-split) domination numbers in fuzzy graphs are defined. The limits on accurate split (non-split) domination numbers in fuzzy graphs are obtained for a number of standard fuzzy graphs. Connection between split (non-split) domination number and other well-known dominating parameters are acquired. Some theorems associated to accurate split (non-split) domination numbers are expressed and demonstrated.

1. Introduction

Ore [13] and Berge [2] were the first to study dominating sets in graphs. Cockayne and Hedetniemi [3] commenced domination number with independent domination number. Kulli and Kattimani [4, 5] introduced an accurate Domination, connected accurate domination. Ameenal Bibi,

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Lakshmi and Jothilakshmi [1] introduced accurate split and accurate non-split domination of a graph.

L. A. Zadeh [17] in 1965 introduced fuzzy set theory as a way for representing uncertainty. In 1975, fuzzy graphs were introduced by Rosenfeld [14] and Yeh, Bang [16] discussed about fuzzy models that can be utilized in problems handling uncertainty. In 1998, A. Somasundaram and S. Somasundaram [15] brought up the concept of domination in fuzzy graphs using effective edges. Domination in fuzzy graph using strong arcs are discussed by Nagoor Gani and Chandrasekaran [10]. Nagoor Gani and Arif Rahman [12] discussed about accurate domination and connected accurate domination in fuzzy graphs. In this paper, accurate split and non-split dominating sets and its numbers in fuzzy graphs are defined and discussed with suitable example. Here, $G(\sigma, \mu)$ is a simple connected undirected fuzzy graph.

2. Preliminaries

Definition 2.1. A fuzzy subset of a nonempty set V is a function $\sigma : V \to L$ where L is the interval [0, 1]. A fuzzy relation μ on E of a fuzzy subset of $V \times V$ is also a function $\mu : V \times V \to L$. A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions, such that $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for every $x, y \in V$. A fuzzy graph $H = (\tau, \rho)$ is defined on $G'(V', E')$ is called a fuzzy subgraph of $G = (\sigma, \mu)$ if $\tau(x) \leq \sigma(x)$ for all $x \in V'$ and $\rho(x, y) \leq \mu(x, y)$ for every $(x, y) \in E'$.

Definition 2.2. The underlying crisp graph of a fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{x \in V : \sigma(x) > 0\}$ and $\mu^* = \{(x, y) \in V \times V : \mu(x, y) > 0\}$.

Definition 2.3. The order p and size q of the fuzzy graph $G = (\sigma, \mu)$ are defined as $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{(x, y) \in E} \mu(x, y)$ respectively. If G is a fuzzy graph on V and $S \subseteq V$, then the fuzzy cardinality of S is defined to be $\sum_{x \in S} \sigma(x)$ and is denoted by $|S|$. If x is a vertex in fuzzy graph G then

$N(x) = \{y \in \sigma^* : \mu(x, y) > 0\}$ is called neighborhood of x and $N[x] = N(x) \cup \{x\}$ is called closed neighborhood of x.

Definition 2.4. The strength of the connectedness between two vertices x, y in a fuzzy graph G is $\mu^{\infty}(x, y) = \sup \{\mu_k(x, y) : k = 1, 2, 3 \ldots\}$, where $\mu_k(x, y) = \mu(x, x_1) \wedge \mu(x_1, x_2) \wedge \ldots \wedge \mu(x_{k-1}, y)$. An edge (x, y) is said to be a strong arc or strong edge, if $\mu(x, y) \geq \mu^{\infty}(x, y)$ and the node v is said to be strong neighbor of x and vice versa. $N_s(x) = \{y : (x, y)$ is a strong edge$\}$ is called strong neighborhood of x and $N_S(x) \cup \{x\}$ is called strong closed neighborhood of x.

Definition 2.5. A vertex x is said to be isolated if $\mu(x, y) = 0$ for all $x \neq y, x, y \in \sigma^*$. In a fuzzy graph, every edge is a strong edge then the fuzzy graph is called strong fuzzy graph. A path in which every edge is a strong arc then the path is called strong path.

Definition 2.6. A fuzzy graph $G = (\sigma, \mu)$ is said to be a complete fuzzy graph, if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in \sigma^*$. A fuzzy graph $G = (\sigma, \mu)$ is said to be a connected fuzzy graph, if there exists a strong path between every pair of vertices.

Definition 2.7. A fuzzy graph $G = (\sigma, \mu)$ is said to be a complete bipartite fuzzy graph, if the vertex set σ can be partitioned into two non-empty sets, σ_1 and σ_2 such that $\mu(v_1, v_2) = 0$, if $v_1, v_2 \in \sigma_1^*$ or $v_1, v_2 \in \sigma_2^*$ and $\mu(v_1, v_2) = \sigma(v_1) \wedge \sigma(v_2)$ for all $v_1 \in \sigma_1^*$ and $v_2 \in \sigma_2^*$.

Definition 2.8. A path P_σ in a fuzzy graph is a sequence of distinct vertices $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, 1 \leq i \leq n$. The path is called a cycle if $u_0 = u_n$ and $n \geq 3$.

Definition 2.9. A fuzzy star graph consists of two vertex set V and U with $|V| = 1$ and $1 \leq |U| \leq n$ such that $\mu(v, u_i) > 0$ and $\mu(u_i, u_{i+1}) = 0, 1 \leq i \leq n - 1$.

Definition 2.10. A subset D of V is said to be a dominating set of a fuzzy graph G if for each $y \in V - D$, there exists $x \in D$ such that x and $y$are strong neighbors. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating sets of G. The domination number of a fuzzy graph G is termed to be the minimum cardinality taken over all dominating sets of a fuzzy graph G and is denoted by $\gamma_f(G)$.

Definition 2.11. Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^* = (V, E)$ and D be a dominating set of V. Then D is said to be an accurate dominating set of a fuzzy graph G if $V - D$ contains no dominating set of same cardinality $|D|$. The accurate domination number of a fuzzy graph G is defined to be the minimum cardinality taken over all accurate dominating sets of a fuzzy graph G and is denoted by $\gamma_{fa}(G)$.

3. Accurate Split Domination in Fuzzy Graphs

In this section, with appropriate illustration accurate split dominating set and accurate split domination number of a fuzzy graph are introduced. Accurate split domination numbers are acquired for few standard fuzzy graphs.

3.1. Accurate split domination in fuzzy graphs:

Definition 3.1. Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^* = (V, E)$. An accurate dominating set $D \subset V$ is stated to be an accurate split dominating set of fuzzy graph G, if $\langle V - D \rangle$ is disconnected. A minimal accurate split dominating set is an accurate split dominating set D if there exists no proper accurate split dominating subset $D' \subset D$. The accurate split domination number of a fuzzy graph G is defined as the minimum cardinality taken over all accurate split dominating set of fuzzy graph G and it is denoted by $\gamma_{fas}(G)$.

Example 3.1.

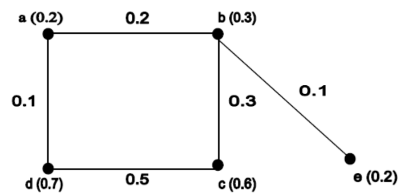

Figure 3.1.

Consider the fuzzy graph $G = (\sigma, \mu)$ given in figure 3.1, Strong arcs are $(a, b), (b, c), (c, d), (b, e)$. Minimal dominating sets of the fuzzy graph G are $\{b, d\}, \{a, c, e\}, \{a, d, e\}, \{a, b, d\}, \{b, c\}$. Some accurate dominating sets of G are $\{a, b, c\}, \{b, c, d\}, \{a, d, e\}, \{a, d, c, e\}, \{b, c, e\}, \{b, d, e\}$.

Then $\{b, c, d\}, \{a, b, c\}, \{b, d, e\}$ are accurate split dominating sets of G and its accurate split domination number is $\gamma_{fas}(G) = 1.1$.

Since, $|\{b, c, d\}| = 0.3 + 0.6 + 0.7 = 1.6$.

$$|\{a, b, c\}| = 0.2 + 0.3 + 0.6 = 1.1$$

$$|\{b, d, e\}| = 0.3 + 0.7 + 0.2 = 1.2.$$

3.2. Bounds on accurate split domination number for some standard fuzzy graphs:

In this section, the accurate split domination number for complete, cycle and star fuzzy graphs are obtained.

Theorem 3.1. *For any complete fuzzy graph* K_σ, *where* $|\sigma| = p$ *and* $|\sigma^*| = n$, $\sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_n$, *for* $n \geq 3$

$$\gamma_{fas}(K_\sigma) = \begin{Bmatrix} p - \sigma_n, & \text{if } p - \sigma_n \neq \sigma_n \\ p - \sigma_{n-1}, & \text{if } p - \sigma_n = \sigma_n \end{Bmatrix}.$$

Proof. Let $\sigma_1, \sigma_2, \ldots, \sigma_n$ be the membership value of each vertex and $\sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_n$. In a complete fuzzy graph, every singleton set is a dominating set D_1. Let us choose the vertex with minimum cardinality in D_1 but the induced graph $\langle V - D_1 \rangle$ may or may not have a dominating set with cardinality $|D_1|$ but $\langle V - D_1 \rangle$ is connected. Similarly any two-element subset D_2 is also dominating set but $\langle V - D_2 \rangle$ is connected. Proceeding like this until the accurate dominating set $D_m, 1 < m < n$ got sit $\langle V - D_m \rangle$ is disconnected. The only possible $D_{n-1} = V - \{v_i\}$ is an accurate split dominating set such that $\sigma(v_i) = \sigma_n$ and $\langle V - D_{n-1} \rangle$ is disconnected.

Case (i) If $|V - \{v_i\}| = p - \sigma_n \neq \sigma_n$, then, $\gamma_{fas}(K_\sigma) = p - \sigma_n$.

Case (ii) If $|V - \{v_i\}| = p - \sigma_n = \sigma_n$, then, $\gamma_{fas}(K_\sigma) = p - \sigma_{n-1}$.

Theorem 3.2. *For any cycle fuzzy graph* C_σ, *where* $|\sigma| = p$ *and* $|\sigma^*| = n$ *for* $n \geq 3$, $\gamma_{fas}(C_\sigma) \leq \left\lceil \frac{p}{3} \right\rceil + 2$.

Proof. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be a vertices in a cycle fuzzy graph $C_\sigma, |\sigma| = p$ and $|\sigma^*| = n$.

Case (i) If $n = 3m$, $m = 1, 2, 3 \ldots$. Then one of the split dominating set is $D = \{v_1, v_4, v_7, \ldots, v_{n-2}\}$ but $\langle V - D \rangle$ may or may not have a dominating set with cardinality $|D|$ and $\langle V - D \rangle$ is disconnected. Then add any two vertices from $\{v_2, v_3, v_5, \ldots, v_n\}$ to the split dominating set D, accurate split dominating set is obtained (say D_1). Therefore

$$|D_1| = \gamma_{fas}(C_\sigma) \leq \frac{p}{3} + 2. \tag{i}$$

Case (ii) If $n = 3m + 1$, $m = 1, 2, 3, \ldots$. Then one of the split dominating set D is $\{v_1, v_4, v_7, \ldots, v_n\}$ but $\langle V - D \rangle$ may or may not have a dominating set with cardinality $|D|$ and $\langle V - D \rangle$ is disconnected. Then add any two vertices from $\{v_2, v_3, v_5, \ldots, v_n\}$ to the split dominating set, accurate split dominating set is obtained (say D_1). Therefore

$$|D_1| = \gamma_{fas}(C_\sigma) \leq \left\lceil \frac{p}{3} \right\rceil + 2. \tag{ii}$$

Case (iii) If $n = 3m + 2$, $m = 1, 2, 3, \ldots$. Then one of the split dominating set D is $\{v_1, v_4, v_7, \ldots, v_{n-1}\}$ but $\langle V - D \rangle$ may or may not have a dominating set with cardinality $|D|$ and $\langle V - D\rangle$ is disconnected. Then, add any two vertices from $\{v_2, v_3, v_5, \ldots, v_n\}$ to the split dominating set, accurate split dominating set is obtained (say D_1). Therefore

$$|D_1| = \gamma_{fas}(C_\sigma) \leq \left\lceil \frac{p}{3} \right\rceil + 2. \qquad \text{(iii)}$$

From (i), (ii) and (iii) we obtain, $\gamma_{fas}(C_\sigma) \leq \left\lceil \frac{p}{3} \right\rceil + 2.$

Theorem 3.3. *For any star fuzzy graph* S_σ, *where* $|\sigma| = p$ *and* $|\sigma^*| = n$, $n \geq 3$,

$$\gamma_{fas}(S_\sigma) = \begin{cases} \sigma(u), & \text{if } \sigma(u) \neq p - \sigma(u) \\ \sigma(u) + \min\limits_{v \in V - \{u\}} \sigma(v), & \text{if } \sigma(u) = p - \sigma(u). \end{cases}$$

Proof. Let u be the centre of the star fuzzy graph. Then $D = \{u\}$ is the one of the minimal split dominating set but $\langle V - u\rangle$ may or may not have a dominating set with cardinality $|D|$ and $\langle V - D\rangle$ is disconnected.

Case (i) If $\langle V - u\rangle$ does not have any dominating set with cardinality $|D|$ i.e., $\sigma(u) \neq p - \sigma(u)$, then $\sigma(u)$ is the accurate split domination number.

Case (ii) If $\langle V - u\rangle$ have any dominating set with cardinality $|D|$ i.e., $\sigma(u) = p - \sigma(u)$, then $\sigma(u)$ is not the accurate split domination number. So, add a single vertex whose membership value is minimum among the vertices $V - \{u\}$ to D, accurate split dominating set is obtained. Therefore, $\gamma_{fas}(S_n) = \sigma(u) + \min\limits_{v \in V - \{u\}} \sigma(u).$

4. Accurate Non-split Domination in Fuzzy Graphs

In this section, with appropriate illustration accurate non-split dominating set along with accurate non-split domination numbers of a fuzzy graph are introduced. Accurate non-split domination numbers are acquired for few standard fuzzy graphs.

4.1. Accurate non-split domination in fuzzy graphs:

Definition 4.1. Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^* = (V, E)$. An accurate dominating set $D \subset V$ is stated to be an accurate non-split dominating set of a fuzzy graph if $\langle V - D \rangle$ is connected. A minimal accurate non split dominating set is an accurate non split dominating set D if there exists no proper accurate non split dominating subset $D' \subset D$. The accurate split domination number of a fuzzy graph G is defined as the minimum cardinality taken over all accurate split dominating set of fuzzy graph G and it is denoted by $\gamma_{fans}(G)$.

Example 4.1. Consider the fuzzy graph $G = (\sigma, \mu)$ is given in figure 3.1.

Accurate non-split dominating sets of G are $\{b, c, e\}$, $\{a, c, d, e\}$, $\{a, d, e\}$ and its accurate non-split domination number is $\gamma_{fans}(G) = 1.1$.

Since, $|\{b, c, e\}| = 0.3 + 0.6 + 0.2 = 1.1$

$$|\{a, c, d, e\}| = 0.2 + 0.6 + 0.7 + 0.2 = 1.7$$

$$|\{a, d, e\}| = 0.2 + 0.7 + 0.2 = 1.1.$$

4.2. Bounds on accurate non-split domination number for some standard fuzzy graphs.

Theorem 4.1. *For any complete fuzzy graph* K_σ *where* $|\sigma| = p$ *and* $|\sigma^*| = n$, *for* $n \geq 3$

$$\gamma_{fans}(K_\sigma) \leq \left\lfloor \frac{p}{2} \right\rfloor + 1$$

Proof. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be a vertices in a complete fuzzy graph. Every singleton set in a complete fuzzy graph is a dominating set D but the induced graph $\langle V - D \rangle$ may or may not have a dominating set with cardinality $|D|$ and $\langle V - D \rangle$ is connected. Adding vertices to the dominating set until it should have the most $\left\lfloor \frac{n}{2} \right\rfloor + 1$ vertices as well as the induced fuzzy subgraph $\langle V - D \rangle$ is connected. As a result $\gamma_{fans}(K_\sigma) \leq \left\lfloor \frac{p}{2} \right\rfloor + 1$.

Note 4.1. Equality holds if $|\sigma^*|$ is odd and $\sigma(u_i) = 1, \forall u_i \in \sigma^*$.

Theorem 4.2. *For any cycle fuzzy graph* C_σ, *where* $|\sigma| = p$ *and* $|\sigma^*| = n$, *for* $n \geq 5$,

$$\gamma_{fans}(C_\sigma) \leq p - \frac{t}{2} \text{ where } t = \max_{u \in \sigma}\{\sigma(u_i) + \sigma(u_{i+1}),\ 0 \leq i < n\}.$$

Proof. Let $\sigma(u_i) = \sigma_i,\ 0 \leq i \leq n$ and $\sigma(u_0) = \sigma(u_n) = \sigma_n$. Let D be an accurate non-split dominating set of a cycle fuzzy graph C_σ and let t be the maximum membership value of two consecutive vertices. All two consecutive vertices u_i and u_{i+1} are strong neighbors to each other. Then by deleting two vertices from $\{u_1, u_2, \ldots, u_n\}$ (i.e., $D = V - \{u_i, u_{i+1}\}$), the accurate non-split dominating set D of a cycle fuzzy graph C_σ may or may not be obtained.

Case (i) If $\gamma_{fans}(C_\sigma) = p - t \neq t$ then, the accurate non-split domination number of a fuzzy graph C_σ is $\gamma_{fans}(C_\sigma) = p - t$.

Case (ii) If $\gamma_{fans}(C_\sigma) = p - t = t$ then, the accurate non-split domination number of a fuzzy graph C_σ is $\gamma_{fans}(C_\sigma) \leq p - \frac{t}{2}$.

From case (i) and case (ii), $\gamma_{fans}(C_\sigma) \leq p - \frac{t}{2}$.

Theorem 4.3. *For any path fuzzy graph* P_σ, $|\sigma| = p$ *and* $|\sigma^*| = n$ *for* $n \geq 5$, σ_1 *and* σ_n *are end vertices,*

$$\gamma_{fans}(P_\sigma) \leq p - \frac{t}{2} \text{ where } t = \max_{u \in \sigma}\{\sigma(u_i) + \sigma(u_{i+1});\ 1 < i < n - 1\}.$$

Proof. Let $\sigma(u_i) = \sigma_i, 1 < i < n$. Let D be an accurate non-split dominating set of a path fuzzy graph P_σ and let t be the maximum membership value of two consecutive vertices except initial and terminal vertices. Then D must have exactly $n - 2$ vertices, i.e., $D = V - \{u_i, u_{i+1}\}$, where u_i and u_{i+1} are intermediate vertices in a fuzzy graph P_σ. Then, accurate non-split dominating set D of a path fuzzy graph P_σ may or may not be obtained.

Case (i) If $\gamma_{fans}(P_\sigma) = p - t \neq t$ then, the accurate non-split domination number of a fuzzy graph P_σ is $\gamma_{fans}(P_\sigma) = p - t$.

Case (ii) If $\gamma_{fans}(P_\sigma) = p - t = t$ then, the accurate non-split domination number of a fuzzy graph P_σ is $\gamma_{fans}(P_\sigma) \leq p - \frac{t}{2}$.

From case (i) and case (ii), $\gamma_{fans}(P_\sigma) \leq p - \frac{t}{2}$.

Theorem 4.4. *For any star fuzzy graph* $S_\sigma, |\sigma| = p$ *and* $|\sigma^*| = n, n \geq 3$,

$$\gamma_{fans}(S_\sigma) = \begin{cases} p - \sigma(u), & \text{if } p - \sigma(u) \neq \sigma(u) \\ p - \min\limits_{v \in V - \{u\}} \sigma(v), & \text{if } p - \sigma(u) = \sigma(u) \end{cases}$$

Proof. Let u be the centre of a star fuzzy graph. Then $D = V - u$ is the one of the minimal non-split dominating set but $\{u\}$ may or may not be a dominating set with cardinality $|D|$ and $\langle V - D \rangle$ is a trivial graph.

Case (i) If $\{u\}$ is not a dominating set with cardinality $|D|$, then D is an accurate non-split dominating set of star fuzzy graph. This implies that $\gamma_{fans}(S_\sigma) \leq p - \sigma(u)$.

Case (ii) If $\{u\}$ is a dominating set with cardinality $|D|$, then $p - \sigma(u) = \sigma(u)$. Now, adding u to D and deleting any one end vertex v having maximum membership value, then $D_1 = V - \{v\}$ is an accurate non-split dominating set of star fuzzy graph. This implies that $\gamma_{fans}(S_\sigma) = p - \max\limits_{v \in V - \{u\}} \sigma(v)$.

From cases (i) and (ii),

$$\gamma_{fans}(S_\sigma) = \begin{cases} p - \sigma(u), & \text{if } p - \sigma(u) \neq \sigma(u) \\ p - \max\limits_{v \in V - \{u\}} \sigma(v), & \text{if } p - \sigma(u) = \sigma(u) \end{cases}$$

5. Relation between Accurate Domination and Accurate Split (non-split) Domination Numbers

`**Theorem 5.1.** *For all fuzzy graph* $G(\sigma, \mu)$, (i) $\gamma_{fa}(G) \leq \gamma_{fas}(G)$ (ii) $\gamma_{fa}(G) \leq \gamma_{fans}(G)$.

Proof. (i) All accurate split dominating set of fuzzy graph G is an accurate dominating set of G. Subsequently, we have

$$\gamma_{fa}(G) \leq \gamma_{fas}(G)$$

(ii) All accurate non-split dominating set of fuzzy graph G is an accurate dominating set of G. Subsequently, we have

$$\gamma_{fa}(G) \leq \gamma_{fans}(G)$$

Theorem 5.2. *For all fuzzy graph* $G(\sigma, \mu)$, $\gamma_{fa}(G) \leq \min(\gamma_{fas}(G), \gamma_{fans}(G))$.

Proof. By Theorem 5.1(i), $\gamma_{fa}(G) \leq \gamma_{fas}(G)$ and by Theorem 5.1(ii), $\gamma_{fa}(G) \leq \gamma_{fans}(G)$

Hence, $\gamma_{fa}(G) \leq \min(\gamma_{fas}(G), \gamma_{fans}(G))$.

Theorem 5.3. *For all fuzzy graph* $G(\sigma, \mu)$, (i) $\gamma_{fs}(G) \leq \gamma_{fas}(G)$ (ii) $\gamma_{fns}(G) \leq \gamma_{fans}(G)$.

Proof. (i) All accurate split dominating set of fuzzy graph G is a split dominating set of G. Subsequently, we have $\gamma_{fs}(G) \leq \gamma_{fas}(G)$.

(ii) All accurate non-split dominating set of fuzzy graph G is non-split dominating set of G. Subsequently, we have

$$\gamma_{fns}(G) \leq \gamma_{fans}(G).$$

Theorem 5.4. *For all fuzzy graph* $G(\sigma, \mu)$, $\gamma_{fs}(G) \leq \min(\gamma_{fas}(G), \gamma_{fans}(G))$.

The proof is similar to Theorem 5.2.

Theorem 5.5. *For any star fuzzy graph* S_σ, $n \geq 3$, $\gamma_{fas}(S_\sigma) \leq \sigma(u) + 1$, *where u is central vertex of* S_σ.

Proof. Let u be the central vertex of S_σ. Then the union of central vertex of any one of the vertex of the remaining vertices is an accurate split dominating set. Therefore, $\gamma_{fas}(S_\sigma) \leq \sigma(u) + 1$.

Theorem 5.6. *Let D be an accurate non-split dominating set of a connected fuzzy graph $G(\sigma, \mu)$, if no two vertices in $V - D$ are strong neighbors of a common vertex in D, then $\gamma_{fans}(G) + \varepsilon(T) \leq n$ where $\varepsilon(T)$ is the maximum number of end vertices in any fuzzy spanning tree T of G.*

Proof. Let D be an accurate non-split dominating set of a connected fuzzy graph $G(\sigma, \mu)$.

Since for any two vertices $u, v \in V - D$, there exist two vertices $x, y \in D$ such that x is strong neighbor of u but not to v and y is neighbor of v, but not to u.

This implies that there exists a fuzzy spanning tree T_1 in $\langle V - D \rangle$ in which each vertex of $V - D$ is neighbor of a vertex of D. Therefore $\varepsilon(T) \leq |V - D|$.

Hence $\varepsilon(T) \leq n - \gamma_{fans}(G)$.

6. Conclusion

In this paper, the notions of accurate split and non-split domination in fuzzy graphs are discussed. Several limits on accurate split and non-split domination numbers are obtained. The applications and the connection between accurate split (non-split) domination numbers with different well known domination parameters can be created in future.

References

[1] K. Ameenal Bibi, A. Lakshmi and R. Jothilakshmi, Accurate split domination, accurate non-split domination of a graph, International Journal of Computational and Applied Mathematics (2017).

[2] C. Berge, Graphs and Hypergraphs, North-Holland, Amsterdam, 1973.

[3] E. J. Cockayne and S. Hedetniemi, Towards a theory of domination in graphs, Networks 7 (1977), 247- 261.

[4] V. R. Kulli and M. B. Kattimani, The accurate domination number of graph, Technical Report 2000:01, Department of Mathematics, Gulbarga University.

[5] V. R. Kulli and M. B. Kattimani, Connected accurate domination in graphs, Journal of Computer and Mathematical Sciences 6(12) (2015), 682-687.

[6] V. R. Kulli, Theory of Domination In Graphs, Vishwa International Publications, Gulbarga, India (2010).

[7] V. R. Kulli, Advances In Domination Theory, Vishwa International Publications, Gulbarga, India (2012).

[8] V. R. Kulli and B. Janakiram, The split domination number of graph, Graph Theory Notes of New York, New York Academy of Sciences, (1997), 16-19.

[9] V. R. Kulli and B. Janakiram, The non-split domination number of graph, The Journal of Pure and Applied Math. Monthly 31(5) (2000), 545-550.

[10] A. Mohamed Ismaiyil and S. Ismail Mohideen, Domination in fuzzy graph: A new approach, International Journal of Computational Science and Mathematics, ISSN 0974-3189, 2(3) (2010), 101-107.

[11] A. Nagoor Gani and V. T. Chandrasekaran, Domination in fuzzy graph, Advances in Fuzzy Sets and Systems 1(1) (2006), 17-26.

[12] A. Nagoor Gani and A. Arif Rahman, Some properties on connected accurate domination in fuzzy graphs, Global Journal of Pure and Applied Mathematics (2017).

[13] O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publi. Vol. 38 (1962), Amer. Math. Soc. Providence RI.

[14] A. Rosenseld, Fuzzy graphs, In L. A. Zadeh, K. S. Fu and M. Shimura, (Eds.), Fuzzy Sets and Their Applications, Academic Press, New York.

[15] A. Somasundaram and S. Somasundaram, Domination in fuzzy graphs-I, Pattern Recognition Letters 19(9) (1998), 787-791.

[16] R. T. Yeh and S. Y. Bang, Fuzzy Relations, fuzzy graphs and their applications to clustering analysis, In L. A. Zadeh, K. S. Fu M. Shimura (Eds.), Fuzzy Sets and Their Applications, Academic Press, New York.

[17] L. A. Zadeh, Fuzzy sets, Information and Control 8(3) (1965), 338-353.