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SINGLE GRADE MANPOWER SYSTEM WITH INTER DECISION TIMES AND WASTAGE AS A GEOMETRIC PROCESS WHEN THRESHOLD HAS THREE COMPONENTS

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Abstract

Stochastic model for time to recruitment is analyzed for a single grade manpower system using a univariate CUM policy of recruitment. Assuming that, the policy decisions and exits occur at different epochs, the inter-decision times and wastage form a geometric process and inter-exist time form an independent and identically distributed random variable. The breakdown threshold for the cumulative wastage of manpower in the system has three components which are independent exponential random variables. Employing a different probabilistic analysis, analytical results in closed form for system characteristics are derived.

1. Introduction

Wastage of personnel due to retirement, death and resignation is a common phenomenon in administrative as well as production oriented organizations. There are certain special problems associated with the organization engaged in sales and marketing. Frequent exits and recruitments are very common in such organizations. Whenever the organization announces revised polices regarding sales target, revision of

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wages, incentives and perquisites the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover of the organization. Frequent recruitments may also be expensive due to the cost of recruitments and training. As the wastage of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this wastage. The univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: Recruitment is made whenever the cumulative wastage of man hours exceeds its breakdown threshold. Several researchers have studied the problem of time to recruitment for a single grade manpower system using shock model approach. In [1] and [2] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [12] the authors have analyzed the problem of time to recruitment for the single grade manpower system which is subject to attrition with instantaneous exits, using CUM policy of recruitment when the wastage of manpower process and inter-decision time process are independent. In [11], the author has studied the problem by associating geometric process and order statistics for inter-decision times. In [8], the authors have analyzed the work in [11] with (i) exponential breakdown threshold and (ii) extended exponential threshold having shape parameter 2 using a bivariate CUM policy of recruitment when the inter-decision times form a geometric process. In [9], the authors have studied the problem of time recruitment by assuming that the attrition is generated by a geometric process of inter-decision times. In [3] the authors have considered the single grade manpower system with non-instantaneous exits and obtained variance of the time to recruitment when the wastage of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables according as the mandatory breakdown threshold is an exponential random variable or extended exponential random variable with shape parameter 2 or a continuous random variable with SCBZ property. In [6] and [7] the authors have extended the research work in [3] when the inter-decision times form (i) a geometric process and (ii) an order statistics respectively. In [4], [10] and [5] the authors have studied the work in [3], [6] and [7] using the method of [9]. The present paper extends the research work in [10] and [5] when the breakdown threshold level for the cumulative wastage in the manpower system is the sum of three components namely an exponential threshold for cumulative wastages due to exits, an exponential threshold for cumulative wastage due to frequent breaks of existing workers and an exponential threshold for cumulative wastage due to backup sources.

2. Model Description and Analysis

Consider an organization taking policy decisions at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated wastage of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For $i = 1, 2, 3, \ldots,$ let X_i be geometric process with rate $d, d > 0$ representing the amount of depletion of manpower (wastage of man hours) at the i^{th} exit epoch with probability distribution function $M(\cdot)$, and mean $\frac{1}{\alpha}(\alpha > 0)$. Let S_i be the cumulative wastage of manpower in the first i exit epochs and m_i be its probability density function. The inter-policy decision times form a geometric process with rate $c, c > 0$. It is assumed that distribution of A_1 is hyper-exponential with parameters a_1, λ_1 and λ_2. Let B_i be the time between $(i+1)^{\text{th}}$ and i^{th} exit epochs, forming a sequence of independent and identically distributed random variables with probability distribution function $G(\cdot)$ and density function $g(\cdot)$. Let D_{i+1} be the waiting time upto $(i+1)^{\text{th}}$ exit epoch. Let Y be the breakdown threshold for the cumulative wastage of manpower in the organization with probability density function $h(\cdot)$. It is assumed that sum of the exponential breakdown threshold level Y_1 for cumulative wastage due to exits with mean $\frac{1}{\theta_1}(\theta_1 > 0)$ and exponential threshold Y_2 for cumulative wastage due to frequent breaks of existing workers with mean $\frac{1}{\theta_2}(\theta_2 > 0)$. and Y_3 backup sources with mean $\frac{1}{\theta_3}(\theta_3 > 0)$. Let $q(q \neq 0)$ be the probability that every policy decision has exit of personnel. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean $E(T)$ and variance $V(T)$.

Time to recruitment is,

$$T = \begin{cases} D_1, & \text{if } S_1 > Y_1 + Y_2 + Y_3 \\ D_2, & \text{if } S_1 \leq Y_1 + Y_2 + Y_3 < S_2 \\ D_{i+1}, & \text{if } S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1} \end{cases}$$

In terms of indicator function of an event, we can write T as

$$T = \sum_{i=0}^{\infty} D_{i+1} \chi(S_i \leq Y_1 + Y_2, Y_3 < S_{i+1}) \tag{1}$$

Taking expectation on both sides of (1) and using the result (for any event L)

$E(X\,\chi(L)) = E(X)P(L)$, we get

$$E(T) = \sum_{i=0}^{\infty} E(D_{i+1}) P(S_i \leq Y_1 + Y_2, Y_3 < S_{i+1}) \tag{2}$$

From (1) and from the definition of D_{i+1}, we get

$$E(T) = \sum_{i=0}^{\infty} E\left(\sum_{i=0}^{i+1} B_k\right) P(S_i \leq Y_1 + Y_2, Y_3 < S_{i+1}) \tag{3}$$

Since the inter-exit times B_i's are identically distributed, we write $E(B_i) = E(B)$, $i = 1, 2, \ldots$. Therefore from (3) we get

$$E(T) = \sum_{i=0}^{\infty} (i+1) E(B) P(s_i \leq Y_1 + Y_2 + Y_3 \leq S_{i+1}) \tag{4}$$

By using law of total probability we get

$P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1}) = P(0 \leq (Y_1, Y_2, Y_3) - S_i < X_{i+1})$

$$= \int_0^\infty \int_0^t P(0 \leq (Y_1 + Y_2 + Y_3) - S_i < X_{i+1} / S_i = x,\ Y_1 + Y_2 + Y_3 = t)$$

$$(e_{S_i,\ Y_1 + Y_2 + Y_3}(x,\ t)) dx dt$$

$$= \int_0^\infty \int_0^t \widetilde{M}(t - x) m_i(x) h(t) dx dt \tag{5}$$

By hypothesis,

$$\widetilde{M}(t - x) = e^{-\alpha d^i (t - x)},\ m_i(x) = d^{i-1} m(d^{i-1} x)$$

and

$$h_1(t) = e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1 + \theta_2)t} - e^{-(\theta_2 + \theta_3)t}$$

Therefore

$$P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1})$$

$$= d^{i-1} \int_0^\infty \int_0^t e^{-\alpha d^i (t-x)} [m(d^{i-1} x)] \frac{\theta_1 \theta_2}{\theta_1 - \theta_2} [e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1 + \theta_2)t}$$

$$- e^{-(\theta_1 + \theta_2)t}] dx dt$$

$$= \frac{\alpha \theta_1 \theta_2}{\theta_1 - \theta_2} d^{i-1} \int_0^\infty \left[\int_0^t [e^{-\alpha d^{i-1} \overline{d} x}] dx \right] [e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1 + \theta_2)t}$$

$$- e^{-(\theta_1 + \theta_2)t}] e^{-\alpha d^i t} dt$$

where $\overline{d} = 1 - d$

$$= \frac{\alpha \theta_1 \theta_2}{\theta_1 - \theta_2} d^{i-1} \int_0^\infty \left[\left[\frac{e^{-\alpha d^{i-1} \overline{d} x}}{-\alpha d^{i-1} \overline{d}} \right]_0^t \right] [e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1 + \theta_2)t} e^{-(\theta_2 + \theta_3)t}] e^{-\alpha d^i t} dt$$

$$P(S_i \leq Y_1 + Y_2 + Y_3 < S_{i+1}) = \frac{\alpha \theta_1 \theta_2}{\theta_1 - \theta_2} \int_0^\infty [1 - e^{-\alpha d^{i-1} \overline{d} t}]$$

$$[e^{-\theta_2 t} - e^{-\theta_1 t} + e^{-(\theta_1+\theta_2)t} - e^{-(\theta_2+\theta_3)t}]e^{-\alpha d^i} dt \tag{6}$$

Consider,

$$\int_0^\infty [1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-\theta_2 t}]e^{-\alpha d^i t} dt$$

$$= \int_0^\infty e^{-(\alpha d^i + \theta_2)t} - e^{-\alpha d^{i-1}}\bar{d}t[e^{-\theta_2 t}]e^{-\alpha d^i t} dt$$

$$\int_0^\infty [1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-\theta_2 t}]e^{-\alpha d^i} dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_2)(\alpha d^{i-1} + \theta_2)} \tag{7}$$

Similarly,

$$\int_0^\infty [1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-\theta_2 t}]e^{-\alpha d^i} dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_1)(\alpha d^{i-1} + \theta_1)} \tag{8}$$

$$\int_0^\infty [1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-(\theta_1+\theta_2)t}]e^{-\alpha d^i t} dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)} \tag{9}$$

$$\int_0^\infty [1 - e^{-\alpha d^{i-1}\bar{d}t}][e^{-(\theta_1+\theta_3)t}]e^{-\alpha d^i t} dt = \frac{\alpha d^{i-1}\bar{d}}{(\alpha d^i + \theta_2 + \theta_3)(\alpha d^{i-1} + \theta_2 + \theta_3)} \tag{10}$$

Sub's (7), (8), (9) and (10) in (6) we get,

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}) = \frac{\alpha\theta_1\theta_2}{\theta_1 - \theta_2}\alpha d^{i-1}\bar{d}\left[\frac{1}{(\alpha d^i + \theta_2)(\alpha d^{i-1} + \theta_2)}\right.$$

$$- \frac{1}{(\alpha d^i + \theta_1)(\alpha d^{i-1} + \theta_1)} + \frac{1}{(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)}$$

$$\left. - \frac{1}{(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)}\right]$$

$$P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})$$

$$= \frac{\alpha\theta_1\theta_2}{\theta_1 - \theta_2}\alpha d^{i-1}\left[\frac{(d^i + d^{i-1})(\alpha\theta_1 - \alpha\theta_2) + (\theta_1^2 - \theta_2^2)}{(\alpha d^i + \theta_2)(\alpha d^{i-1} + \theta_2)(\alpha d^i + \theta_1)(\alpha d^{i-1} + \theta_1)}\right.$$

$$\left. + \frac{(d^i + d^{i-1})(\alpha\theta_1 - \alpha\theta_3) + (\theta_1^2 - \theta_2^2) + (\theta_1^2 - \theta_3^2) + 2\theta_1\theta_2 - 2\theta_2\theta_3}{(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)}\right] \quad (11)$$

From (4) and (11)

$$E(T) = E(B)\sum_{i=0}^{\infty}(i+1)\frac{\alpha\theta_1\theta_2}{\theta_1 - \theta_2}\alpha d^{i-1}$$

$$\left[\frac{(d^i + d^{i-1})(\alpha\theta_1 - \alpha\theta_2) + (\theta_1^2 - \theta_2^2)}{(\alpha d^i + \theta_2)(\alpha d^{i-1} + \theta_2)(\alpha d^i + \theta_1)(\alpha d^{i-1} + \theta_1)}\right.$$

$$\left. + \frac{(d^i + d^{i-1})(\alpha\theta_1 - \alpha\theta_3) + (\theta_1^2 - \theta_2^2) + (\theta_2^2 - \theta_3^2) + 2\theta_1\theta_2 - 2\theta_2\theta_3}{(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)(\alpha d^i + \theta_1 + \theta_2)(\alpha d^{i-1} + \theta_1 + \theta_2)}\right]$$

$$E(T) = K_1 E(B) \quad (12)$$

We now find $E(B)$ for the present model.

The distribution function $G(\cdot)$ of the inter-exit times B_i, $i = 1, 2, \ldots$ satisfy the relation

$$G(x) = q\sum_{n=1}^{\infty}(1-q)^{n-1}F_n(X) \quad (13)$$

$$E(B) = q\sum_{n=1}^{\infty}(1-q)^{n-1}E(A_1 + A_2 + \ldots + A_n)$$

$$= q\sum_{n=1}^{\infty}(1-q)^{n-1}\left[\frac{\alpha_1\lambda_2 + \alpha_2\lambda_1}{\lambda_1\lambda_2} + \frac{\alpha_1\lambda_2 + \alpha_2\lambda_1}{\lambda_1\lambda_2 c}\right.$$

$$\left. + \frac{\alpha_1\lambda_2 + \alpha_2\lambda_1}{\lambda_1\lambda_2 c^2} + \ldots + \frac{\alpha_1\lambda_2 + \alpha_2\lambda_1}{\lambda_1\lambda_2 c^{n-1}}\right]$$

i.e. $E(B) = \dfrac{c[a_1\lambda_2 + a_2\lambda_1]}{\lambda_1\lambda_2(c-1+q)}$ (14)

Using (11) in (9), we get

$$E(T) = \frac{c[a_1\lambda_2 + a_2\lambda_1]}{\lambda_1\lambda_2(c-1+q)}\left[\frac{\alpha(\theta_1 - \theta_2) + \theta_1\theta_2}{\theta_1\theta_2} - \frac{\theta_1\theta_2\alpha}{(\theta_1 - \theta_3)(\theta_1 - \theta_3)}\right]$$

$$E(T) = N_1 E(B) \quad (15)$$

We now evaluate $E(T^2)$ for the present model.

Squaring both sides on (2), we get

$$E(T^2) = \sum_{i=0}^{\infty} E(D_{i+1}^2)P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1}). \quad (16)$$

Now

i.e. $E(D_{i+1}^2) = (i+1)V(B) + [(i+1)E(B)]^2$ (17)

Using (15) in (14), we get

$$E(T^2) = \sum_{i=0}^{\infty}(i+1)V(B)[P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})]$$

$$+ \sum_{i=0}^{\infty}(i+1)^2 E^2(B)[P(S_i \le Y_1 + Y_2 + Y_3 < S_{i+1})] \quad (18)$$

We now evaluate $E(B^2)$ for the present model

From (10) we get

$$E(B^2) = \int_0^{\infty} x^2 dG(x) = \int_0^{\infty} x^2\left[q\sum_{n=1}^{\infty}(1-q)^{n-1}f_n(x)\right]dx$$

i.e. $E(B^2) = q\sum_{n=1}^{\infty}(1-q)^{n-1}\int_0^{\infty} x^2 f_n(x)dx$ (19)

We now evaluate $\int_0^\infty x^2 f_n(x)dx$ in (17).

$$\int_0^\infty x^2 f_n\, dx = \frac{c^2[2(a_1\lambda_2^2 + a_2\lambda_1^2) - (a_1\lambda_2 + a_2\lambda_1)^2}{(c^2-1)(\lambda_1\lambda_2)^2}\left[\frac{c^{2n}-1}{c^{2n}}\right]$$

$$+\left\{\frac{c[a_1\lambda_2 + a_2\lambda_1]}{(c-1)\lambda_1\lambda_2}\left[\frac{c^n-1}{c^n}\right]\right\}^2 \tag{20}$$

Therefore from (18) we get

$$E(B^2) = \frac{c^2 q[2(a_1\lambda_2^2 + a_2\lambda_1^2) - (a_1\lambda_2 + a_2\lambda_1)^2]}{(c^2-1)(\lambda_1 - \lambda_2)^2}\sum_{n=1}^{\infty}(1-q)^{n-1}\left[\frac{c^{2n}-1}{c^{2n}}\right]$$

$$+\frac{c^2 q[a_1\lambda_2 + a_2\lambda_1]^2}{(c-1)^2(\lambda_1\lambda_2)^2}\sum_{n=1}^{\infty}(1-q)^{n-1}\left[\frac{c^n-1}{c^n}\right]^2$$

i.e. $$E(B^2) = \frac{2c^2(a_1\lambda_2^2 + a_2\lambda_1^2)}{(c^2-1+q)(\lambda_1\lambda_2)^2} \tag{21}$$

From (21) and (18) we get

$$E(T^2) = K_1 V(B) + K_2 E^2(B)$$

$$= K_1 E(B^2) - E^2(B)[K_1 - K_2]$$

$$V(T) = K_1 E(B^2) - E^2(B)[K_1^2 - K_1 - K_2] \tag{22}$$

3. Conclusion

The problem of time to recruitment is studied in a more general setting on policy decision and breakdown threshold. Consequently the process of planning of recruitment is more enlighted. Data analysis can be used to test the suitability of distributions assumed in the present work.

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