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A GRAPH THEORETICAL APPROACH ON CONSTELLATION OF STARS

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Abstract

In our solar system, there are 88 constellations. In this paper all 88 constellations are considered as graphs. For that graphs, graph theoretical invariants such as independence number, chromatic number, clique number are studied.

1. Introduction

Constellations are a part of the vault of heaven. These are simply a group of stars that are believed to be fashioned into prominent layout of insentient articles or creatures. In the earlier times when constellations were discovered, they helped in locating peculiar stars among the humongous number of stars in the skies, relating with the real-life experiences and beliefs. The International Astronomical Union has formally recognised 88 constellations that divide up the entire sky as seen from the Earth. These constellations have been used as graphs in this paper. Graph theory is a mathematical field that is used to monitor the properties of bonds of nature. Anyone with the basic knowledge in graph theory can quickly get to know that constellations can be deciphered as graphs. A graph G consists of a pair $(V(G), X(G))$ where $V(G)$ is a non-empty finite set whose elements are called points or vertices and $X(G)$ is a set of unordered pair of distinct elements of

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$V(G)$ are called the lines or edges. Constellations contain a set of stars which are considered to be the vertices and non-existent lines that form objects are considered to be the edges of the graph. This paper involves in identification, analysis and classification of all the 88 constellations and their patterns in a graph theoretical way. Comprehending these constellations in mathematical way, this paper classifies graph theoretical invariants such as independence number, chromatic number, clique number of the constellations.

2. Motivation and Main Results

Definition 2.1. A graph G consists of a pair $(V(G), X(G))$ where $V(G)$ is a non-empty finite set whose elements are called points or vertices and $X(G)$ is a set of unordered pairs of distinct elements of $V(G)$. The elements of $X(G)$ are called lines or edges of the graph G. If $x = \{u, v\} \in X(G)$, the line x is said to join u and v. If $x = uv$ then the points u and v are adjacent. This also means the line x is incident with the point u. If two distinct lines x and y are incident with a common point then they are called adjacent lines.

Definition 2.2. A graph in which any two distinct points are adjacent is called a complete graph.

Definition 2.3. A graph $H = (V_1, X_1)$ is called a subgraph of $G = (V, X)$ if $V_1 \subseteq V$ and $X_1 \subseteq X$.

Definition 2.4. An Assignment of colours to the vertices of a graph so that no two adjacent edges or vertices get the same colour is called colouring. Colouring each vertex such that adjacent vertices are coloured differently is called vertex colouring. Colouring each edge such that adjacent edges are coloured differently is called edge colouring.

Definition 2.5. The minimum number of colours used in colouring the vertices of a graph is called chromatic number $\chi(G)$ and is defined as $\chi(G) = \min\{\chi(c) : c \text{ is a colouring of } G\}$.

Definition 2.6. The minimum number of colours used in colouring the edges of a graph is called edge chromatic number. Edge chromatic number is also called the chromatic index which is denoted by $\chi'(G)$.

Definition 2.7. A graph G is called n-colourable if $\chi(G) \leq n$.

Definition 2.8. A graph G is called n-edge colourable if $\chi'(G) \leq n$.

Definition 2.9. A set of vertices in G is independent if no two vertices are adjacent. The maximum independent set is an independent set of maximum cardinality. The number of vertices in a maximum independent set of G is known as the vertex independence number and is denoted by $\beta(G)$.

Definition 2.10. A clique in a graph G is a complete subgraph of G. The order of the largest clique in a graph G is its clique number, which is denoted by $\omega(G)$.

Theorem 2.11. *The following statements are equivalent for any graph G*

i. *G is 2-colourable*

ii. *G is bipartite*

iii. *Every cycle of G has even length.*

Theorem 2.12. *For any graph G, the edge chromatic number is either* Δ or $\Delta + 1$ *where Δ is the maximum degree of any vertex in the graph G.*

The table below shows vertex chromatic number, edge chromatic number, vertex independence number and clique number of all the 88 constellations.

Constellations	Tag	Chromatic number		Vertex independence number $\beta(G)$	Clique number $\Omega(G)$
		Vertex chromatic number $\chi(G)$	Edge chromatic number $\chi'(G)$		
Andromeda	Anr	2	4	13	2
Antlia	Ata	3	3	2	3
Apus	Aps	2	2	2	2
Aquarius	Aqs	2	3	13	2
Aquila	Aqu	3	4	5	3

Ara	Ara	3	3	4	2
Aries	Ars	2	4	5	2
Auriga	Arg	3	4	7	3
Bootes	Bts	3	4	13	3
Caelum	Clm	2	2	12	2
Camelopardalis	Cpl	2	3	4	2
Cancer	Ccr	2	3	4	2
Canes Venatici	CVci	2	2	3	2
Canis Major	CMj	3	3	5	3
Canis Minor	CMn	2	1	3	2
Capricornus	Cpc	2	2	5	2
Carina	Cna	2	2	7	2
Cassiopeia	Csp	2	2	9	2
Centaurus	Cnt	3	4	10	3
Cepheus	Cph	3	3	9	3
Cetus	Cts	3	3	7	2
Chameleon	Cml	2	2	3	2
Circinus	Crn	2	2	2	2
Columba	Clb	2	3	7	2
Coma Berenices	CBr	2	2	2	2
Corona Australis	CAs	2	2	3	2
Corona Borealis	CBl	2	2	4	2
Corvus	Crv	2	3	3	2
Crater	Crt	2	3	4	2

Crux	Crx	2	1	3	2
Cygnus	Cyn	3	4	12	2
Delphinus	Dpn	2	3	3	2
Dorado	Drd	3	4	3	3
Draco	Drc	2	3	12	2
Equuleus	Eql	2	2	2	2
Eridanus	Edn	2	2	17	2
Fornax	Frx	2	2	2	2
Gemini	Gmn	2	4	10	2
Grus	Grs	3	3	6	2
Hercules	Hrc	2	3	12	2
Horologium	Hlg	2	2	4	2
Hydra	Hya	2	2	10	2
Hydrus	Hys	2	2	5	2
Indus	Ins	3	3	2	2
Lacerta	Lrt	2	2	5	2
Leo	Leo	3	5	10	2
Leo Minor	LMn	2	3	3	2
Lepus	Lps	3	4	6	2
Libra	Lbr	3	3	5	3
Lupus	Lpu	3	4	6	3
Lynx	Lyx	2	2	4	2
Lyra	Lya	3	4	4	3
Mensa	Mns	2	2	2	2
Microscopium	Mcr	3	3	2	2

Monoceros	Mcs	2	3	4	2
Musca	Msc	2	3	3	2
Norma	Nrm	3	3	2	2
Octans	Otn	2	3	4	2
Ophiuchus	Opi	2	3	5	2
Orion	Orn	2	3	15	2
Pavo	Pvo	3	6	6	2
Pegasus	Pgs	2	4	8	2
Perseus	Prs	3	4	11	2
Phoenix	Phx	2	3	8	2
Pictor	Ptr	2	2	2	2
Pisces	Psc	3	3	10	3
Piscis Austrinus	PAs	3	4	5	3
Puppis	Pps	3	3	4	2
Pyxis	Pys	2	2	7	2
Reticulum	Rtl	2	2	4	2
Sagitta	Sgt	2	3	3	2
Sagittarius	Sgs	3	4	10	3
Scorpius	Sps	2	3	9	2
Sculptor	Spt	3	3	2	2
Scutum	Stm	3	3	3	2
Serpens	Spn	2	3	8	2
Sextans	Sxt	2	2	2	2
Taurus	Trs	2	3	7	2

Telescopium	Tls	2	2	3	2
Triangulum	Tgm	3	3	2	3
Triangulum Australe	TAl	2	2	3	2
Tucana	Tcn	2	2	5	2
Ursa Major	UMj	3	4	9	3
Ursa Minor	UMn	2	3	4	2
Vela	Vla	2	2	4	2
Virgo	Vrg	2	4	7	2
Volans	Vln	3	3	3	2
Vulpecula	Vlp	2	2	3	2

Classification of constellations

Predicated on outcome of the above table, the subsequent 4 main classifications are procured. The numbers in the brackets denotes the number of constellations possessing the property.

1. Chromatic number

2. Chromatic index

3. Vertex independence number

4. Clique number

Further, the constellations are classified under chromatic number. There are two classifications with chromatic number 2(57) and chromatic number 3 (31).

Next, the constellations are classified based on chromatic index. There are six classifications with chromatic index 1(2), chromatic index 2(29), chromatic index 3(37), chromatic index 4(18), chromatic index 5(1) and chromatic index 6(1).

Next, the constellations are classified under the vertex independence number. There are fourteen with vertex independence number 2(14), vertex independence number 3(16), vertex independence number 4(14), vertex independence number 5(10), vertex independence number 6(4), vertex independence number 7(7), vertex independence number 8(3), vertex independence number 9(4), vertex independence number 10(6), vertex independence number 11(1), vertex independence number 12(4), vertex independence number 13(3), vertex independence number 15(1) and vertex independence number 17(1).

Finally, the constellations are classified based on clique number. Again, there are two classifications with clique number 2(72) and clique number 3(16).

3. Observations and Discussions

The following observations have been made from the above table.

1. All the constellations have chromatic number either 2 or 3.

2. All the constellations are 3-colourable.

3. By Theorem 2.11 every constellation which is 2 colourable is bipartite and every cycle of the graph has even length.

4. Among 57 graphs which are 2-colourable, 26 constellations have even cycles and all these constellations have at most one cycle.

5. All the constellations are 6-edge colourable.

6. By Theorem 2.12, all the constellations have edge chromatic number as Δ or $\Delta + 1$.

7. Under the classification of chromatic index with chromatic index 3, barely 6 constellations have edge chromatic number as $\Delta + 1$. All these 6 constellations are cycles of odd length.

8. The chromatic index of the constellations ranges from 1 to 6.

9. Pavo has the highest chromatic index with order 6.

10. The vertex independence number of the constellations lies between 2 and 17 excluding 14 and 16.

11. Eridanus has the highest vertex independence number 17.

12. All the constellations have 2 or 3 as clique number.

13. Every constellation with clique number 2 has at least one complete graph of order $2(K_2)$.

14. Every constellation with clique number 3 has at least one complete subgraph of order $3(K_3)$.

4. Applications

Graph theory has applications in various fields. The graph theoretical concepts that are discussed in this paper are used in the field of astronomy. The vertices and edges that are defined in graph theory relate to the constellations with stars and the lines that connect are the link to the nearest neighbour stars. The biggest dream for astronomers for centuries is to find a way to establish colonies in stars. Now with advancing technology, research is going on for stellar colonisation. Imagining the benefits of colonizing the stars and locating the human habitation in the stars, E. A. C. T. Sandamali and G. H. J. Lanel have proposed a paper titled "A use of graph theory properties of constellations in stellar colonization" [3] which aims in developing a scale free networking for stellar colonization applying the concepts of graph theory is one of the main applications. Also, Haruhiko Ueda and Makoto Itoh have proposed a paper titled "A graph theoretical approach for quantifying the large-scale structure of the universe" [6] where constellation graphs and clustering maps were constructed by calculating the order, degree and eigenvalue of adjacency matrix for the constellations. It plays an important in providing an explanation of the dynamics of universe allowing to inspect about the origin, evolution etc.

5. Conclusion

In this study, vertex chromatic number, edge chromatic number, vertex independence number, clique number found for 88 constellations. In future, other graph theoretical invariants will be studied for 88 constellations.

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