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DEDUCTIVE GROUP INVARIANT ANALYSIS OF MAGNETO HYDRODYNAMICS FLOW IN NON-NEWTONIAN POWER-LAW FLUID OVER A STRETCHING SHEET

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Abstract

This paper deals with the Deductive Group invariant analysis of magneto hydrodynamics flow in non-Newtonian power-law fluid over a stretching sheet. The Partial differential equation are reduced into an ordinary differential equation using Deductive Group Invariant analysis of similarity analysis. The numerical solutions in terms of graphs are obtained using fourth order Runge-Kutta method by the shooting method with the help of MATLAB software. These graphs represent velocity profile for Newtonian fluids $(n = 1)$ and the velocity profile for non-Newtonian fluids $(n \neq 1)$ respectively.

1. Introduction

This paper deals with the Deductive Group invariant analysis magneto hydrodynamics (MHD) flow for the said investigation problem. This physical phenomenon is represented by a system of PDEs. The one-parameter group

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theory of similarity analysis is applied for reducing the number of independent variables and hence the non-linear PDEs with boundary conditions are reduced to a non-linear ODE with corresponding boundary conditions. The Graphical solution of the reduced ODE is obtained in terms of velocity profile of Newtonian fluids $(n = 1)$ as well as the velocity profile of non-Newtonian fluids $(n \neq 1)$ with the help of MATLAB software. A considerable research work has been reported in the literature.

This physical phenomenon has been discussed by many researchers taking different aspects. Some of them are: Ishak et al. [1] have discussed the boundary layer flow numerically. The MHD flow of a non-Newtonian power law fluid was studied by constant transverse magnetic field over steady surface by Chaim [2]. Klemp and Aerivas [3] have studied the boundary layer flow to a moving stream. Cobble et al. [4] and Soundalgekar et al. [5] have discussed numerically the complex nature of the influence of a magnetic field for Newtonian fluid. Surati and Timol [6] have derived the similarity solutions for the said phenomenon under different physical conditions. Al. Salihi et al. [7] have applied the new systematic procedure in group theoretic methods for a class of boundary layer laws. Similarity analysis is used by Darji et al. [8] For a boundary layer flow of Sisko. Shukla et al. [9] have derived the similarity solution of forced convection flow of Powell-Eyring and Prandtl-Eyring fluids. In this paper we used Deductive group method of one parameter of similarity analysis proposed by Gaggioli et al. [10]. Bongar [11] has discussed the similarity solutions of MHD flow over a non-linear stretching surface. He has also examined the nature of solution and investigated the influence of the parameters via numerical solution.

2. Mathematical Formulation

Newtonian's law of viscosity state that the shear stress is proportional to velocity gradient. Fluids that obey this law are known as Newtonian fluids where as the others are known as non-Newtonian fluids which are defined according to the non-linear stress-strain relationship. The characteristic of non-Newtonian fluidis defined according to their stress-strain relationship. Some of them are Power-law fluids, Eyring fluids, Prandtls-Eyring fluids etc. We have considered the steady laminar flow of the later fluid electrically conducting incompressible fluid past a two-dimensional body.

This physical phenomenon is represented mathematically by the following equations:

Continuity equation:

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{k}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + u_{\infty}\frac{\partial u_{\infty}}{\partial y}\delta B^{2}(u - u_{\infty}) \tag{2.2}$$

where

- u and v are the velocity components in the coordinates x and y directions respectively,
- ρ denotes the density and δ denote the elective conductivity,
- $u_{\infty}(x)$ is the external velocity distribution and $B(x)$ is the imposed electrical transverse magnetic field with $B_0 > 0$, m are constants,
- $\tau_{xy} = k\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}$ is the non-linear model describing the non-Newtonian fluid with two parameters: the consisting coefficient k and the power law exponent n. The case $0 < n < 1$ corresponding to preudo plastic fluids, the $n > 1$ is known as dilatant or shear-thickening fluids. For $n = 1$, one recovers a Newtonian fluid.

The boundary conditions are:

(1) At the solid surface $y = 0$, neither slip nor mass transfer.

$$u(x, 0) = u_w(x), v(x, 0) = 0. \tag{2.3}$$

(2) Outside the viscous boundary layer the stream wise velocity component u should approaches u_{∞}.

$$\lim_{y \to \infty} u(x, y) = u_{\infty}(x) \tag{2.4}$$

We introduce the stream function $\psi(x, y)$ such that,

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{2.5}$$

Which satisfies the equation (2.1) identically and the above equation (2.2) reduced to

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 y}{\partial y^2} = \frac{k}{\rho}\left(\left|\frac{\partial^2 \psi}{\partial y^2}\right|^{n-1} \frac{\partial \psi}{\partial y^2}\right) + u_\infty \frac{\partial u_\infty}{\partial y} - \sigma B^2\left(\frac{\partial \psi}{\partial y} - u_\infty\right) \tag{2.6}$$

The conditions (2.3) and (2.4) are transformed into:

$$\frac{\partial \psi(x, 0)}{\partial y} = u_w(x), \frac{\partial \psi(x, 0)}{\partial x} = 0 \tag{2.7}$$

$$\lim_{y \to \infty} \frac{\partial \psi(x, y)}{\partial y} = u_\infty(x) \tag{2.8}$$

3. Deductive group Similarity Analysis

A one parameter deductive group analysis method is applied on PDEs (2.1) and (2.2). Using this method the two independent variables x and y are reduced into a single independent variable η. Hence, the above PDEs are transformed into an ODE with corresponding boundary conditions. The obtained ODE is called invariant similarity equation.

Invariant analysis:

The procedure is started with the group G, a class of transformations of one-parameter of the form:

$$G : \bar{s} = c^s(a)s + k^s(a) \tag{3.1}$$

where s stands for x, y, ψ, u, u_∞ and c's and k's are the real valued function at least differentiable in the real parameter a.

Equation (2.6) is said to be invariantly transformed for some function $H(a)$, if

$$\frac{\partial\overline{\psi}}{\partial\overline{y}}\frac{\partial^2\overline{\psi}}{\partial x\partial\overline{y}} - \frac{\partial\overline{\psi}}{\partial\overline{x}}\frac{\partial^2\overline{y}}{\partial\overline{y}^2} = \frac{k}{\rho}\frac{\partial}{\partial\overline{y}}\left(\left|\frac{\partial^2\psi}{\partial\overline{y}^2}\right|^{n-1}\frac{\partial^2\overline{\psi}}{\partial\overline{y}^2}\right) + \overline{u}_\infty\frac{\partial\overline{u}_\infty}{\partial y} - \delta B^2\left(\frac{\partial\overline{\psi}}{\partial\overline{y}} - u_\infty\right)$$

$$H(a) = \left[\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{k}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial^2\psi}{\partial y^2}\right|^{n-1}\frac{\partial^2\psi}{\partial y^2}\right) - u_\infty\frac{\partial u_\infty}{\partial y} - \delta B^2\left(\frac{\partial\psi}{\partial\overline{y}} - u_\infty\right)\right] \quad (3.2)$$

From equations (3.1) and (3.2)

We have,

$$\frac{c^\psi}{c^y}\frac{c^\psi}{c^y c^x}\frac{\partial^2\psi}{\partial y\partial x}\frac{\partial\psi}{\partial y} - \frac{c^\psi}{c^x}\frac{c^\psi}{(c^y)^2}\frac{\partial^2\psi}{\partial y^2}\frac{\partial\psi}{\partial x} - \frac{k}{\rho}n\left(\frac{c^\psi}{(c^y)^2}\right)^{n-1}\frac{\partial^2\psi}{\partial y^2}\frac{c^\psi}{(c^y)^3}\frac{\partial^3\psi}{\partial y^3}$$

$$-(c^{u_\infty}u_\infty + k^{u_\infty})\frac{c^{u_\infty}}{c^x}\frac{\partial u_\infty}{\partial x} + \delta B^2\left\{\frac{c^\psi}{c^y}\frac{\partial\psi}{\partial y} - (c^{u_\infty}u_\infty + k^{u_\infty})\right\} + R(a)$$

$$= H(a)\frac{\partial^2\psi}{\partial y\partial x}\frac{\partial\psi}{\partial y} - \frac{\partial^2\psi}{\partial y^2}\frac{\partial\psi}{\partial x} - \frac{k}{\rho}n\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial y^3} - (c^{u_\infty}u_\infty + k^{u_\infty})\frac{\partial u_\infty}{\partial x}$$

$$+ \delta B^2(c^{u_\infty}u_\infty + k^{u_\infty})\frac{\partial\psi}{\partial y} \quad (3.3)$$

The invariant of equation (3.3) implies,

$$\frac{(c^\psi)^2}{(c^y)^2 c^x} = \frac{(c^\psi)^n}{(c^y)^{2n+1}} = \frac{(c^{u_\infty})^2}{c^x} = c^{u_\infty} = H(a) \quad (3.4)$$

$$R(a) = k^{u_\infty}\frac{c^{u_\infty}}{c^x}\frac{\partial u_\infty}{\partial x} - \delta\beta^2 k^{u_\infty} \Rightarrow k^{u_\infty} = 0$$

From (3.4), we have

$$c^\psi = \frac{(c^y)^{\frac{2n-1}{n-2}}}{(c^y)^{\frac{1}{n-2}}} \quad (3.5)$$

$$c^\psi = c^{x+y} \quad (3.6)$$

$$c^{u_\infty} = c^x \quad (3.7)$$

$$c^{y} = (c^{x})^{\frac{n-1}{n+1}} \tag{3.8}$$

Also for the absolute invariance of the boundary conditions,

We have, $k^{u} = 0$ and $c^{u} = 1$ (3.9)

Now, $\lim_{y\to\infty}(c^{u}u(a) + k^{u}) = u_{\infty}(x)$ (3.10)

Where $u_{\infty}(x) = (x + \delta\beta_1)^{\delta}(x + \gamma_1)^{a}$, where a and δ are constants.

Thus the transformed group G is of the form,

$$G = \begin{cases} \overline{x} = c^{x}x + k^{x} \\ \overline{y} = c^{y}y + k^{y} \\ \overline{\psi} = c^{\psi}x + k^{\psi} \\ \overline{u} = u \\ \overline{u_{\infty}} = c^{u_{\infty}}u_{\infty} \end{cases} \tag{3.11}$$

The complete set of absolute invariants:

In this section we describe the set of absolute invariants so that the original problem (3.1) to (3.4) will transformed into similarity equations under the derived deductive group G given in (3.11).

Let $\eta = \eta(x, y)$ be the absolute invariant of the independent variable x and y, then the three absolute invariants for the dependent variables ψ, u and u_{∞} are given by,

$$\eta = y(x + \gamma_1)^{a}, \text{ in which } a = -\frac{\alpha_2}{\alpha_1}. \tag{3.12}$$

$$\psi(x, y) = f_1(\eta)(x + \beta_1)^{\delta} - \frac{\beta_3}{\alpha_3}, \text{ in which } \delta = \frac{2n}{n+1}. \tag{3.13}$$

$$u(x, y) = f_2(\eta)(x + \gamma_1). \tag{3.14}$$

$$u_{\infty}(x, y) = f_3(\eta)(x + \gamma_1). \tag{3.15}$$

Hence, the final boundary value problem for the phenomenon under investigation is:

$$\frac{nk}{\rho} c_1 (f_2'')^{n-1} f_1''' - c_2 a \eta f_1' f_1'' - (c_3 \delta + c_4 a)(f_1')^2 + c_5 (a \eta f_3' + f_3^2) = 0 \tag{3.16}$$

where

$$c_1 = (x + \delta\beta_1)^{n\delta} (x + \gamma_1)^{2na+1}$$

$$c_2 = (x + \delta\beta_1)^{2\delta} (x + \gamma_1)^{3a-1}$$

$$c_3 = (x + \delta\beta_1)^{2\delta} (x + \gamma_1)^{2a-1}$$

$$c_4 = (x + \delta\beta_1)^{2\delta} (x + \gamma_1)^{2a-1}$$

$$c_5 = (x + \gamma_1).$$

The corresponding boundary conditions are,

$$f_1(0) = 0,\ f_1'(0) = 1,\ \lim_{\eta \to \infty} f_1'(\eta) = 1. \tag{3.17}$$

This boundary value problem is solved by using fourth order Runge-Kutta method with the help of MATLAB software in terms of the following veloci by profile.

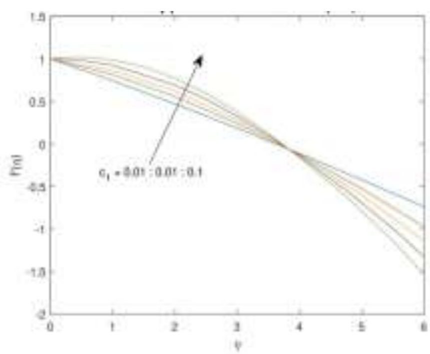

Figure 3.1. Velocity profile for non-Newtonian fluid $n \neq 1$.

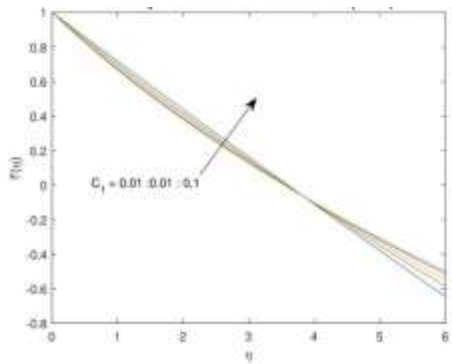

Figure 3.2. Velocity profile for non-Newtonian fluid $n = 1.8$.

4. Conclusion

This paper deals with the Deductive Group invariant analysis MHD flow in non-Newtonian power-law fluid over a stretching sheet. This phenomenon is discussed physically and it is represented by a system of PDEs. The application of the one-parameter group theory method reduces the number of independent variables by one and hence the governing system of non-linear PDEs with boundary conditions reduced to a non-linear ODE with corresponding boundary conditions. The numerical solutions in terms of graphs are obtained using fourth order Runge-Kutta method by the help of MATLAB software. These graphs given in figure (3.1) and (3.2) for different values of c_1 represent velocity profile for Newtonian fluids $(n = 1)$ and the velocity profile for non-Newtonian fluids $(n \neq 1)$ respectively.

References

[1] A. Ishak, R. Nazur and I. Pop Boundary Layer on a Moving Wall with Suction and Injection, Chinese Phys. Lett. 8 (2007), 2274-2276.

[2] T.C. Chiam, Solutions for the flow of a conducting power-law fluid in a transverse magnetic field and with a pressure gradient using Crocco variables. Acta. Mechanics 137, (1999), 225-235.

[3] J. B. Klemp, A. A. Acrivos, A moving wall boundary layer with reverse flow, J. fluid Mech. 76 (1976), 363-381.

[4] M. H. Cobble, Magneto fluid dynamics flow with a pressure gradient and fluid injection, J. Eng. Math. 11(1977), 249-256.

[5] V. M. Soundalgkar, H. S. Takher and M. Singh, Velocity and temperature field in MHD Falkner-Skan flow, J. Phys. Soc. Japan 50 (1981), 3139-3143.

[6] H. G. Surati and M. G. Timol, On the heat and transfer of the gas laminar forced convection flow with variable physical properties, International e-journal for Education and Mathematics 2(4) (2013), 37-43.

[7] A. K. Al-Salihi A. H. Hasmani and M. G. Timol, The new systematic procedure in group theoretic methods with applications of class of boundary layer flows, Int. J. of adv. In App. Math. and Mech. 1(1) (2013), 47-60.

[8] R. M. Darji and M. G. Timol, similarity analysis for unsteady natural convective boundary layer flow of Sisko fluid, Int. J. of Adv. Math. And Mech. 1(3) (2014), 22-36.

[9] H. Shukla, J. Patel, H. C. Surati, M. Patel and M. G. Timol, similarity solutions of forced convection flow of Powell-Eyring and Prandtl-Eyring fluids by Group-Theoretic method, Math. J. of Int. sci. 5 (2017), 151-165.

[10] R. A. Gaggioli and M. J. Moren, Similarity analysis via Group theory, AIAA Journal (1968), 10-14.

[11] G. Bonger, On similarity solutions of MHD flow over a nonlinear stretching surface in non-Newtonian power-law fluid, Electronic Journal of Qualitative Theory of Differential Equation 6 (2016), 1-12.