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MAX-MAX OPERATION ON α-UPPER LEVEL PARTITION OF FUZZY SQUARE MATRICES

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Abstract

In this paper, Max-max product of α-upper level partition of Fuzzy Square Matrix are defined and its some properties are established.

1. Introduction

Fuzzy Matrices assume an essential part in fuzzy set hypothesis. Fuzzy Matrices are effectively utilized when fuzzy uncertainty happens in an issue.

Zadeh [15] presented the hypothesis of fuzzy sets. The idea of segments of fuzzy matrix was presented by Kim and Roush [6]. Hashimoto [3] created sanctioned type of transitive fuzzy matrix.

Fuzzy sets by and large relies upon shaping the powers of a fuzzy matrix, where the product of two fuzzy matrices composed as an ordinary grid item yet with still up in the air by fuzzy logic operators. That is multiplication is supplanted by rationale MIN and summation is supplanted by logic MAX.

Max-min tasks are characterized to get the subsequent matrix, Kandasamy [5]. In 1977 G. Thomason [14] research the assembly of abilities of a square fuzzy matrix shaped by Max (min) items. Ragab et al. [9] introduced a few properties of the Min-max composition of fuzzy matrices.

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In this paper, Max-max activity for α-upper level partition of fuzzy square matrix was characterized. Max-max operation is more significant than Max-min activity. Properties of Max-max item on α-upper level partition of Fuzzy square matrices are created. Viz. Associative, Involution, complementation and distributive properties are inspected with counterexamples.

2. Preliminaries

Definition 2.1. A fuzzy matrix $A = (a_{ij})$ is a matrix of order $m \times n$ whose elements having values in the closed interval $[0, 1]$.

Definition 2.2. Let $A = (a_{ij})$ and $B = (b_{ij})$ be two fuzzy matrices of order $m \times n$. Some operators on fuzzy matrices whose elements are in the closed interval $[0, 1]$ are defined as,

$$(A + B) = Max\,(a_{ij}, b_{ij})$$

$$(A \cdot B) = Min\,(a_{ij}, b_{ij})$$

$\overline{A} = 1 - A = (1 - a_{ij})$, the complement of fuzzy matrix A.

3. α-Upper Level Partition of Fuzzy Square Matrix

Definition 3.1. The α-upper level partition of a fuzzy square matrix A is a Boolean matrix denoted by,

$$A^{(\alpha)} = (a_{ij})^{(\alpha)} \text{ such that}$$

$$(a_{ij})^{(\alpha)} = a_{ij} \text{ if } a_{ij} \geq \alpha$$

$$= 0 \text{ if } a_{ij} < \alpha \text{ where } \alpha \in [0, 1]$$

Definition 3.2. Let $A^{(\alpha)} = (a_{ij})^{(\alpha)}$ and $B^{(\alpha)} = (b_{ij})^{(\alpha)}$ be α-upper level partition of a fuzzy square matrix of order $n \times n$ then the following results are defined

(i) $a_{ij} \geq b_{ij} \Rightarrow (a_{ij})^{(\alpha)} \geq (b_{ij})^{(\alpha)}$

(ii) $(a_{ij}b_{ij})^{(\alpha)} = a_{ij}^{(\alpha)}b_{ij}^{(\alpha)}$

(iii) $(a_{ij} + b_{ij})^{(\alpha)} = a_{ij}^{(\alpha)} + b_{ij}^{(\alpha)}$

(iv) $(a_{ij} - b_{ij})^{(\alpha)} = a_{ij}^{(\alpha)} - b_{ij}^{(\alpha)}$

(v) $(a_{ij} \wedge b_{ij})^{(\alpha)} = a_{ij}^{(\alpha)} \wedge b_{ij}^{(\alpha)}$

(vi) $(a_{ij} \vee b_{ij})^{(\alpha)} = a_{ij}^{(\alpha)} \vee b_{ij}^{(\alpha)}$

Theorem 3.3. *A fuzzy matrix T is transitive if and only if all its upper level partitions are transitive.*

Proof. Let T be an $(n \times n)$ α-upper level partition of a fuzzy square matrix and it is called transitive if and only if

$$T^2 \leq T$$

$$\Leftrightarrow (T^2)^{(\alpha)} \leq T^{(\alpha)}$$

$$\Leftrightarrow (T^{(\alpha)})^2 \leq T^{(\alpha)}$$

$$\Leftrightarrow (T_{ij})^{(\alpha)}(T_{ij})^{(\alpha)} \leq (T_{ij})^{(\alpha)}$$

$$[(T_{ij})^{(\alpha)}]^2 \leq t_{ij} \text{ if } t_{ij} \geq \alpha$$

$$= 0 \text{ if } t_{ij} < \alpha$$

Thus T is transitive if and only if all of its upper level partitions are transitive.

Example 3.3.1.

Let $T = \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.6 & 0.2 & 0.5 \\ 0.4 & 0.1 & 0.3 \end{bmatrix}$

Take $\alpha = 0.2$

$$T^{(0.2)} = \begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.6 & 0.2 & 0.5 \\ 0.4 & 0 & 0.3 \end{bmatrix}$$

$$[T^{(0.2)}]^2 = \begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.6 & 0.2 & 0.5 \\ 0.4 & 0 & 0.3 \end{bmatrix}\begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.6 & 0.2 & 0.5 \\ 0.4 & 0 & 0.3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.6 & 0.2 & 0.5 \\ 0.4 & 0 & 0.3 \end{bmatrix} \leq \begin{bmatrix} 0.2 & 0 & 0.2 \\ 0.6 & 0.2 & 0.5 \\ 0.4 & 0 & 0.3 \end{bmatrix}$$

$$[T^{(0.2)}]^2 \leq T^{(0.2)}$$

Remark 3.4. A fuzzy matrix E is idempotent if and only if all of its upper level partitions are idempotent.

Definition 3.5. Let S be an $n \times n$ fuzzy matrix and S is symmetric if and only if all of its upper level partitions are symmetric.

Example 3.5.1.

$$S_{ij}^{(\alpha)} = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.3 & 0.2 & 0.7 \\ 0.5 & 0.7 & 0.1 \end{bmatrix}$$

Take $\alpha = 0.2$

$$S_{ij}^{(0.2)} = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.3 & 0.2 & 0.7 \\ 0.5 & 0.7 & 0 \end{bmatrix}$$

$$= S_{ji}^{(0.2)}$$

Definition 3.6. Let A be a fuzzy square matrix of order 'n'. The trace of α-upper level partition of a fuzzy matrix $A^{(\alpha)}$ denoted by $tr(A^{(\alpha)})$ and is defined as,

$$tr(A^{(\alpha)}) = \max(a_{ii}^{(\alpha)})$$

Example 3.6.1.

Let $A = \begin{bmatrix} 0.3 & 0 & 0.7 \\ 0.8 & 0.3 & 0.4 \\ 0.1 & 0.9 & 0.6 \end{bmatrix}$

Take $\alpha = 0.3$

$$A^{(0.3)} = \begin{bmatrix} 0.3 & 0 & 0.7 \\ 0.8 & 0.3 & 0.4 \\ 0 & 0.9 & 0.6 \end{bmatrix}$$

$$tr(A^{(\alpha)}) = \max(0.3, 0.6)$$

$$tr(A^{(\alpha)}) = 0.6$$

4. Max-max Product on α-Upper Level Partition of a Fuzzy Square Matrix

Definition 4.1. Let $A^{(\alpha)}$ and $B^{(\alpha)}$ be two α-upper level partition of a fuzzy square matrix of order n. Max-max product is defined as,

$$A^{(\alpha)} \circ B^{(\alpha)} = \underset{k=1}{\overset{n}{\vee}} (a_{ik}^{(\alpha)} \vee b_{kj}^{(\alpha)})$$

$$= Max\,(\max(a_{ik}^{(\alpha)}, b_{kj}^{(\alpha)})) \text{ if } a_{ij}^{(\alpha)}, b_{ij}^{(\alpha)} \geq \alpha$$

$$= 0 \text{ if } a_{ij}^{(\alpha)}, b_{ij}^{(\alpha)} < \alpha$$

where $\alpha \in [0, 1]$.

Example 4.1.1.

Let $A = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0.1 & 0.2 & 0.3 \\ 0.6 & 0.8 & 0.4 \end{bmatrix}$ and $B = \begin{bmatrix} 0.4 & 0.3 & 0.6 \\ 0.5 & 0.2 & 0.7 \\ 0.8 & 0.5 & 0.1 \end{bmatrix}$

Take $\alpha = 0.3$

$$A^{(0.3)} \circ B^{(0.3)} = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0 & 0 & 0.3 \\ 0.6 & 0.8 & 0.4 \end{bmatrix} \circ \begin{bmatrix} 0.4 & 0.3 & 0.6 \\ 0.5 & 0 & 0.7 \\ 0.8 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.7 & 0.7 \\ 0.8 & 0.5 & 0.7 \\ 0.8 & 0.8 & 0.8 \end{bmatrix}$$

Property 4.2. *Max-max product on* α*-upper level partition of a fuzzy square matrices are associative.*

$$A^{(\alpha)} \circ (B^{(\alpha)} \circ C^{(\alpha)}) = (A^{(\alpha)} \circ B^{(\alpha)}) \circ C^{(\alpha)}$$

Proof. Consider three α-upper level partition of a fuzzy square matrices of order $n \times n$ as,

$$A^{(\alpha)} = (a_{ij})^{\alpha}$$

$$B^{(\alpha)} = (b_{ij})^{\alpha}$$

$$C^{(\alpha)} = (c_{ij})^{\alpha} \text{ where } \alpha \in [0, 1]$$

$$(B^{(\alpha)} \circ C^{(\alpha)}) = \max(\max(b_{ij}^{(\alpha)} c_{ij}^{(\alpha)}))$$

$$A^{(\alpha)} \circ (B^{(\alpha)} \circ C^{(\alpha)}) = \max[\max(a_{ij}^{(\alpha)}, \max(\max b_{ij}^{(\alpha)} c_{ij}^{(\alpha)}))]$$

$$\max[\max[\max(a_{ij}^{(\alpha)}, b_{ij}^{(\alpha)}), c_{ij}^{(\alpha)}]]$$

$$= \max[\max[(A^{(\alpha)} \circ B^{(\alpha)}), c_{ij}^{(\alpha)}]]$$

$$= \max[\max(A^{(\alpha)} \circ B^{(\alpha)}), C^{(\alpha)}]$$

$$A^{(\alpha)} \circ (B^{(\alpha)} \circ C^{(\alpha)}) = (A^{(\alpha)} \circ B^{(\alpha)}) \circ C^{(\alpha)}$$

Hence Max-max product on α-upper level partitions are associative.

Example 4.2.1.

Let $A = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0.1 & 0.2 & 0.3 \\ 0.6 & 0.8 & 0.4 \end{bmatrix} B = \begin{bmatrix} 0.4 & 0.3 & 0.6 \\ 0.5 & 0.2 & 0.7 \\ 0.8 & 0.5 & 0.1 \end{bmatrix} C = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.2 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$

Take $\alpha = 0.3$

$$(B^{(0.3)} \circ C^{(0.3)}) = \begin{bmatrix} 0.4 & 0.3 & 0.6 \\ 0.5 & 0 & 0.7 \\ 0.8 & 0.5 & 0 \end{bmatrix} \circ \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.6 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.8 & 0.9 \end{bmatrix}$$

$$A^{(0.3)} \circ (B^{(0.3)} \circ C^{(0.3)}) = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0 & 0 & 0.3 \\ 0.6 & 0.8 & 0.4 \end{bmatrix} \circ \begin{bmatrix} 0.8 & 0.6 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.8 & 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.9 \end{bmatrix} \tag{1}$$

$$(A^{(0.3)} \circ B^{(0.3)}) = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0 & 0 & 0.3 \\ 0.6 & 0.8 & 0.4 \end{bmatrix} \begin{bmatrix} 0.4 & 0.3 & 0.6 \\ 0.5 & 0 & 0.7 \\ 0.8 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.7 & 0.7 \\ 0.8 & 0.5 & 0.7 \\ 0.8 & 0.8 & 0.8 \end{bmatrix}$$

$$(A^{(0.3)} \circ B^{(0.3)}) \circ C^{(0.3)} = \begin{bmatrix} 0.8 & 0.7 & 0.7 \\ 0.8 & 0.5 & 0.7 \\ 0.8 & 0.8 & 0.8 \end{bmatrix} \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.9 \end{bmatrix} \tag{2}$$

From (1) and (2), associative property is true.

Property 4.3. *Max-max product on* α*-upper level partition of a fuzzy square matrices are distributive.*

$$A^{(\alpha)} \circ (B^{(\alpha)} + C^{(\alpha)}) = (A^{(\alpha)} \circ B^{(\alpha)}) + (A^{(\alpha)} \circ B^{(\alpha)})$$

Proof.

$$B^{(\alpha)} + C^{(\alpha)} = \max(b_{ij}^{(\alpha)}, c_{ij}^{(\alpha)})$$

$$A^{(\alpha)} \circ (B^{(\alpha)} + C^{(\alpha)}) = \max[\max(a_{ij}^{(\alpha)}, \max(b_{ij}^{(\alpha)}, c_{ij}^{(\alpha)}))]$$

$$= \max[\max(a_{ij}^{(\alpha)}, b_{ij}^{(\alpha)}), \max(a_{ij}^{(\alpha)}, c_{ij}^{(\alpha)})]$$

$$= \max[(A^{(\alpha)} \circ B^{(\alpha)}), (A^{(\alpha)} \circ C^{(\alpha)})]$$

$$= A^{(\alpha)} \circ B^{(\alpha)} + A^{(\alpha)} \circ C^{(\alpha)}$$

$$(A^{\alpha} \circ B^{\alpha}) + (A^{\alpha} \circ C^{\alpha}) = \max[\max(\max(a_{ij}^{(\alpha)}, b_{ij}^{(\alpha)}), \max(a_{ij}^{(\alpha)}, c_{ij}^{(\alpha)}))]$$

$$= \max[\max[(a_{ij}^{(\alpha)}), \max(b_{ij}^{(\alpha)}, c_{ij}^{(\alpha)})]]$$

$$= \max[\max[A^{(\alpha)}, (B^{\alpha} \circ C^{\alpha})]]$$

$$= A^{(\alpha)}, (B^{\alpha} + C^{\alpha})$$

Example 4.3.1.

Consider $A = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0.2 & 0.7 \\ 0.6 & 0.5 & 0.1 \end{bmatrix} B = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0.4 \end{bmatrix}$

$$C = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.1 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

Take $\alpha = 0.3$

$$A^{(0.3)} = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0 & 0.7 \\ 0.6 & 0.5 & 0 \end{bmatrix} B^{(0.3)} = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0.4 \end{bmatrix}$$

$$C^{(0.3)} = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

$$B^{0.3} + C^{0.3} = \begin{bmatrix} 0.7 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.9 \\ 0.8 & 0.7 & 0.9 \end{bmatrix}$$

$$A^{(0.3)} \circ [B^{0.3} + C^{0.3}] = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0 & 0.7 \\ 0.6 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.7 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.9 \\ 0.8 & 0.7 & 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \end{bmatrix} \tag{3}$$

$$A^{(0.3)} \circ B^{(0.3)} = \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \end{bmatrix}$$

$$A^{(0.3)} \circ C^{(0.3)} = \begin{bmatrix} 0.8 & 0.5 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.6 & 0.9 \end{bmatrix}$$

$$(A^{(0.3)} \circ B^{(0.3)}) + (A^{(0.3)} \circ C^{(0.3)}) = \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \end{bmatrix} \tag{4}$$

From (3) and (4), distributive property is verified.

Remark 4.4. Similarly the distributive property

$(B^{\alpha} + C^{\alpha}) \circ A^{\alpha} = B^{(\alpha)} \circ A^{(\alpha)} + C^{(\alpha)} \circ A^{(\alpha)}$ are also true.

But $(A^C)^{\alpha} \circ [(A^C)^{\alpha} + (C^C)^{\alpha}] \neq [(A^C)^{\alpha} \circ (B^C)^{\alpha}] + [(A^C)^{\alpha} \circ (C^C)^{\alpha}]$.

Property 4.5. *For any* α*-upper level partition of a fuzzy matrix of order '*n*' such that*

$$[A^{(\alpha)} \circ B^{(\alpha)}]' = (B^{(\alpha)})' \circ (A^{(\alpha)})'.$$

Proof.

By definition, $A^{(\alpha)} \circ B^{(\alpha)} = \max(\max(a_{ij}^{\alpha}, b_{ij}^{\alpha}))$

$$(A^{(\alpha)} \circ B^{(\alpha)})' = \max(\max(a_{ij}^{(\alpha)})', (b_{ij}^{(\alpha)})')$$

$$= \max(\max((b_{ij}^{(\alpha)})', (a_{ij}^{(\alpha)})'))$$

$$= (B^{(\alpha)})' \circ (A^{(\alpha)})'$$

Example 4.5.1.

Consider $A = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0.2 & 0.7 \\ 0.6 & 0.5 & 0.1 \end{bmatrix} B = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0.4 \end{bmatrix}$

Take $\alpha = 0.3$

$$A^{(0.3)} = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0 & 0.7 \\ 0.6 & 0.5 & 0 \end{bmatrix} B^{(0.3)} = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0.4 \end{bmatrix}$$

$$(A^{(0.3)} \circ B^{(0.3)}) = \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.7 & 0.9 \end{bmatrix}$$

$$(A^{(0.3)} \circ B^{(0.3)}) = \begin{bmatrix} 0.8 & 0.8 & 0.8 \\ 0.7 & 0.7 & 0.7 \\ 0.9 & 0.9 & 0.9 \end{bmatrix} \tag{5}$$

$$(B^{(0.3)})' = \begin{bmatrix} 0.7 & 0 & 0.8 \\ 0.4 & 0.3 & 0.7 \\ 0.5 & 0.9 & 0.4 \end{bmatrix}$$

$$(A^{(0.3)})' = \begin{bmatrix} 0.4 & 0.5 & 0.6 \\ 0.3 & 0 & 0.5 \\ 0.5 & 0.7 & 0 \end{bmatrix}$$

$$(B^{(0.3)})' \circ (A^{(0.3)})' = \begin{bmatrix} 0.8 & 0.8 & 0.8 \\ 0.7 & 0.7 & 0.7 \\ 0.9 & 0.9 & 0.9 \end{bmatrix} \quad (6)$$

From (5) and (6)

$$[A^{(\alpha)} \circ B^{(\alpha)}]' = (B^{(\alpha)})' \circ (A^{(\alpha)})'.$$

Property 4.6. *The set of all* α*-upper level partition of a fuzzy square matrix* $A^{(\alpha)} = a_{ij}^{(\alpha)}$ *satisfies involution law.*

i.e., $\overline{\overline{A}} = A \quad (\overline{\overline{A^{\alpha}}}) = A^{(\alpha)}$.

Proof.

$$A^{(\alpha)} = a_{ij}^{(\alpha)}$$

$$\overline{A^{(\alpha)}} = 1 - a_{ij}^{(\alpha)}$$

$$\overline{\overline{A^{(\alpha)}}} = \overline{1 - a_{ij}^{(\alpha)}} = 1 - \overline{a_{ij}^{(\alpha)}}$$

$$= 1 - (1 - a_{ij}^{(\alpha)})$$

$$= a_{ij}^{(\alpha)}$$

$$(\overline{\overline{A^{\alpha}}}) = A^{(\alpha)}.$$

Example 4.6.1.

Consider $A = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.1 \\ 0.8 & 0.2 & 0.9 \end{bmatrix}$

Take $\alpha = 0.3$

$$A = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0 & 0.9 \end{bmatrix}$$

$$\overline{A^{(0.3)}} = \begin{bmatrix} 0.7 & 0.5 & 0.4 \\ 0.3 & 0.6 & 1 \\ 0.2 & 1 & 0.1 \end{bmatrix}$$

$$\overline{\overline{A^{0.3}}} = \begin{bmatrix} 0.3 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0 & 0.9 \end{bmatrix}$$

$$(\overline{\overline{A^{0.3}}}) = A^{(0.3)}.$$

Theorem 4.7. *Let* A, B *and* C *be* α*-upper level partition of a fuzzy square matrices of order n such that* $A^{\alpha} \leq B^{\alpha}$ *then* $A^{\alpha} \cdot C^{\alpha} \leq B^{\alpha} \cdot C^{\alpha}$.

Proof. Let $A^{(\alpha)} = (a_{ij})^{(\alpha)}$, $B^{(\alpha)} = (b_{ij})^{(\alpha)}$ and $C^{(\alpha)} = (c_{ij})^{(\alpha)}$ be α-upper level partition of a fuzzy square matrices of order 'n' such that,

$$A^{\alpha} \leq B^{\alpha}$$

$$(a_{ij})^{(\alpha)} \leq (b_{ij})^{(\alpha)}$$

$$(a_{ij})^{(\alpha)}(c_{ij})^{(\alpha)} \leq (b_{ij})^{(\alpha)}(c_{ij})^{(\alpha)}$$

$$\max(\max((a_{ij})^{(\alpha)}(c_{ij})^{(\alpha)})) \leq \max(\max((b_{ij})^{(\alpha)}(c_{ij})^{(\alpha)}))$$

$$A^{\alpha} \circ C^{\alpha} \leq B^{\alpha} \circ C^{\alpha}.$$

Example 4.7.1.

Consider $A = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0.2 & 0.7 \\ 0.6 & 0.5 & 0.1 \end{bmatrix} B = \begin{bmatrix} 0.7 & 0.2 & 0.5 \\ 0.6 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0.1 \end{bmatrix}$

$$C = \begin{bmatrix} 0.2 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0.1 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

Take $\alpha = 0.3$

$$A^{(0.3)} = \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0 & 0.7 \\ 0.6 & 0.5 & 0 \end{bmatrix} B^{(0.3)} = \begin{bmatrix} 0.7 & 0 & 0.5 \\ 0.6 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0 \end{bmatrix}$$

$$C^{(0.3)} = \begin{bmatrix} 0 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

Now we can show that,

$$A^{(0.3)} \circ C^{(0.3)} \leq B^{(0.3)} \circ C^{(0.3)}$$

$$\begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.5 & 0 & 0.7 \\ 0.6 & 0.5 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0.8 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0.3 & 0.9 \end{bmatrix} \leq \begin{bmatrix} 0.7 & 0 & 0.5 \\ 0.6 & 0.3 & 0.9 \\ 0.8 & 0.7 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0.5 & 0.6 \\ 0.7 & 0.4 & 0 \\ 0.8 & 0.3 & 0.9 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.5 & 0.9 \\ 0.8 & 0.7 & 0.9 \\ 0.8 & 0.6 & 0.9 \end{bmatrix} \leq \begin{bmatrix} 0.8 & 0.7 & 0.9 \\ 0.9 & 0.9 & 0.9 \\ 0.8 & 0.8 & 0.7 \end{bmatrix}.$$

Remark 4.8. Max-max product of α-upper level partition of a fuzzy square matrices are not commutative. Whenever $(A^C)^\alpha \circ (B^C)^\alpha$ and $(B^C)^\alpha \circ (A^C)^\alpha$ exists but are not commutative.

$$(A^C)^\alpha \circ (B^C)^\alpha \neq (B^C)^\alpha \circ (A^C)^\alpha.$$

5. Conclusion

In this paper Max-max operator on α-upper level partition of a fuzzy square matrices has been introduced. Some properties and results are examined with the help of numerical examples.

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