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ON FORECASTING TIME SERIES ANALYSIS UNDER FUZZY ENVIRONMENT

D. RAJAN and R. SUGUNTHAKUNTHALAMBIGAI

Associate Professor
T.B.M.L. College
Affiliated to Bharathidasan University
Porayar-609 307, Mayiladuthurai
District, Tamil Nadu, India
E-mail: dan_rajan@rediffmail.com

Assistant Professor
TNAU, Tamil Nadu, India
E-mail: sugunthа@tnau.ac.in

Abstract

Fuzzy time series is an effective tool for dealing with historical data. In fuzzy time series forecasting, various methods have been developed to build fuzzy relationships on time series data that have linguistic value for forecasting future value. However, the main problem in fuzzy time series forecasting is the accuracy of the estimated value. In this paper, to predict the average rainfall of a city in Trichy District using a fuzzy time series approach based on the average interval length.

1. Introduction

Fuzzy logic can work with any type of input whether it is imprecise, distorted or noisy input information. The fuzzy logic system construction given by Zadeh (1975) is easy and understandable. Fuzzy logic comes with the mathematical concepts of set theory and the reasoning is quite simple. It provides highly efficient solutions to complex problems in all areas of life as it resembles human reasoning and decision making. The concept of fuzzy logic is an analogous to the perception of emotions of human being and

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interpretation processes. Contrasting the classical controller approach, which is a point-to-point control system, fuzzy logic control system is a range-to-point or range to-range control system. Fuzzy logic system concept was introduced by Zadeh in the year 1965 [27]. Fuzzy logic is intended at a reinforcement of methods of reasoning which are estimated rather than meticulous. Mamdani et al., [14] applied the fuzzy logic in a practical application to control an automatic steam engine in the year 1974 which is almost after ten years the theory of fuzzy logic was recognized. Hsu and Chen in [1] and Chen in [2], presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series. It has the advantage of reducing the calculation, time and simplifying the calculation process. Rainfall is a stochastic procedure whose forthcoming event be contingent on some predecessors from other constraints such as the sea surface temperature for monthly to seasonal time scales, the surface pressure for weekly to daily time scale and other atmospheric constraints for daily to hourly time scale. Unpredictability of weather and climatic aspects, particularly those atmospheric constraints will be the major force for daily precipitation event. If unpredictability pattern could be documented and used for future path, feasibility of daily rainfall prediction is very much possible [3].

Based on the theory of fuzzy time series, Song et al. presented some forecasting methods [16], [19], [20], [21] to forecast the enrollments of the University of Alabama. One of the most critical issues in FTS models is the style of their universe of discourse partitioning, which affects their performance in forecasting [6]. Therefore, one of the main purposes of this study is to propose a new simulated annealing (SA) based model to find the right length of the intervals, thus improving forecasting results. Lee et al. [11, 12] introduced two methods based on fuzzy time series, the genetic algorithm, and simulated annealing heuristics to forecast temperature and the TAIFEX. Chen and Chung [4] developed first order and high order fuzzy time series models by using the genetic algorithm for enrolment forecasting. Park et al. [16] studied a two-factor high-order fuzzy time series using the PSO method, applying the model to TAIFEX and KOSPI 200 datasets.

2. Some Basic Concepts of Fuzzy Time Series

Definition 1. Let $Y(t)(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t)\ldots$ then $F(t)$ is called a fuzzy time-series defined on $y(t)$.

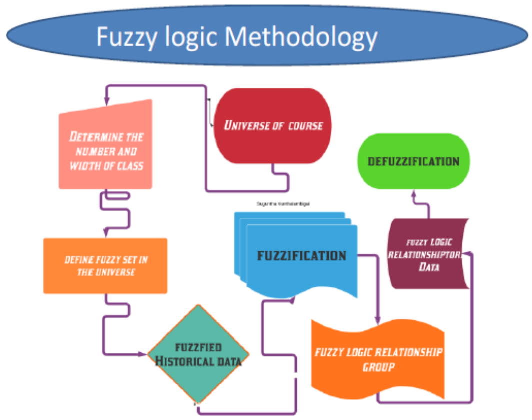
Definition 2. If there is a fuzzy relationship $R(t-1, t)$, such that $F(t) = F(t-1) \times R(t-1, t)$, where × is an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by $F(t-1) \to F(t)$.

Definition 3. Suppose $F(t-1) = Ai$ and $F(t) = Aj$ a fuzzy logical relationship is defined as $Ai \to Aj$ where Ai is named as the left-hand side of the fuzzy logical relationship and Aj the right-hand side.

Definition 4. Fuzzy logical relationships with the same fuzzy set on the left-hand side can be further grouped into a fuzzy logical relationship group. Suppose there are fuzzy logical relationships such that $Ai \to Aj1Ai \to Aj2\ldots$ then they can be grouped into a fuzzy logical relationship group $Ai \to Aj1, Aj2, \ldots$

Definition 5. Suppose $F(t)$ is caused by $F(t-1)$ only, and $F(t) = F(t-1) \times R(t-1, t)$. For any t, if $R(t-1, t)$ is independent of t, then $F(t)$ is named a time-invariant fuzzy time series; otherwise it is a time-variant fuzzy time series.

3. The Procedure of Forecasting with Fuzzy Time Series is Described as Follows



Step 1. Collect historical data.

Table 1. Historical data of enrollments of the year 2011 to 2017.

Blocks	Average Rainfall(mm)
Andanallur	811.14
Lalgudi	704.31
Manachanallur	631.81
Manapparai	518.41
Manikandam	560.18
Marungapuri	688.68
Musiri	518.34
Pullambadi	922.46
Thathaingarpet	617.08
Thiruverumbur	732.8
Thottium	445.83
Thurayur	736.22
Uppiliyapurm	591.39
Vaiyampaty	712.3

Step 2. Define the universe of discourse and intervals.

Let the $D_{\min}$ and $D_{\max}$ be the minimum and maximum value of historical data are 445.83 (D min) and 922.46 (D max). The universe of discourse can be defined by $U = [425, 925]$.

Then U is partitioned into ten intervals.

Table 2. Interval with their midpoints.

u_1	425−475	450
u_2	475−525	500
u_3	525−575	550
u_4	575−625	600
u_5	625−675	650
u_6	675−725	700
u_7	725−775	750
u_8	775−825	800
u_9	825−875	850
u_{10}	875−925	900

Step 3. Define fuzzy sets based on the intervals, and linguistic variables. Let U be the universe of discourse, where $U = \{u_1, u_2, u_3, \ldots, u_{10}\}$. The number of intervals will be in accordance with the number of linguistic variables (fuzzy sets) $A_1, A_2, \ldots, A_{10}$ to be considered.

Define ten fuzzy sets $A_1, A_2, \ldots, A_{10}$ as linguistic variables on the universe of discourse U. These fuzzy variables are being defined as:

Table 3. Label linguistic value of enrollments.

Fuzzified	Linguistic Value
A_1	very very very very few
A_2	very very very few

A_3	very very few
A_4	very few
A_5	Moderate
A_6	High
A_7	very High
A_8	very very High
A_9	very very very High
A_{10}	very very very very High

Defined fuzzy sets on U. The fuzzy sets A_i are expressed as follows:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0/u_9 + 0/u_{10}$$

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$$A_{10} = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_5 + 0/u_6 + 0/u_7 + 0/u_8 + 0.5/u_9 + 1/u_{10}$$

Step 4. Fuzzify historical data.

Table 4. Linguistic values for the enrolments of the year 2011 to 2017.

No.	Blocks	Average Rainfall(mm)	Linguistic value
1	Andanallur	811.14	A_9
2	Lalgudi	704.31	A_6
3	Manachanallur	631.81	A_5

4	Manapparai	518.41	A_2
5	Manikandam	560.18	A_3
6	Marungapuri	688.68	A_6
7	Musiri	518.34	A_2
8	Pullambadi	922.46	A_{10}
9	Thathaingarpet	617.08	A_5
10	Thiruverumbur	732.8	A_7
11	Thottium	445.83	A_1
12	Thurayur	736.22	A_7
13	Uppiliyapurm	591.39	A_4
14	Vaiyampaty	712.3	A_6

Step 5. Determine fuzzy logical relationships (FLRs) for all fuzzified data.

Table 5. The first-order fuzzy logical relationships on the enrolments.

No.	Relationships	No.	Relationships	No.	Relationships
1	A9 → A6	6	A6 → A2	11	A1 → A7
2	A6 → A5	7	A2 → A10	12	A7 → A4
3	A5 → A2	8	A10 → A5	13	A4 → A6
4	A2 → A3	9	A5 → A7		
5	A3 → A6	10	A7 → A1		

Step 6. Group fuzzy logical relationship as in step 6 having the same the left-hand sides and derive fuzzy logical relationships group (FLRG).

Table 6. Fuzzy logical relationship groups.

Groups	Fuzzy relation groups		Groups	Fuzzy relation groups	
G1	A9 → A6		G 6	A10 → A5	
G 2	A6 → A5	A6 → A2	G 7	A5 → A7	
G 3	A5 → A2		G 8	A7 → A1	A7→ A4
G 4	A2 → A3	A2 → A10	G 9	A1 → A7	
G 5	A3 → A6		G 10	A4 → A6	

Step 7. Calculate the forecasted enrolment.

Table 7. Historical, forecasted and error value.

Blocks	Actual	Predicted	Error	Blocks	Actual	Predicted	Error
Andanallur	811.14	700	13.7017	Pullambadi	922.46	900	2.434794
Lalgudi	704.31	650	7.711093	Thathaingarpet	617.08	525	14.92189
Manachanallur	631.81	500	20.86228	Thiruverumbur	732.8	750	2.347162
Manapparai	518.41	550	6.093632	Thottium	445.83	450	0.935334
Manikandam	560.18	575	2.645578	Thurayur	736.22	750	1.871723
Marungapuri	688.68	725	5.273857	Uppiliyapurm	591.39	700	18.36521
Musiri	518.34	500	3.538218	Vaiyampaty	712.3	700	1.726801

Table 8. Descriptive statistics of areal annual rainfall in Trichy District (from 2011 to 2017).

Minimum	= 445.83		
Maximum	= 922.46	Interquartile Range	= 172.62
Range	= 476.63	Sum of Squares	= 211479.47
Size	= 14	Mean Absolute Deviation	= 101.77
Sum	= 9190.95	Root Mean Square	= 667.90
Mean	= 656.50	Std Error of Mean	= 34.08

Median	= 660.2	Skewness	= 0.34
Mode	None	Kurtosis	= 3.91
Standard Deviation	= 127.54	Kurtosis Excess	= 0.07
Variance	= 16267.65	Coefficient of Variation	= 0.19
Mid-Range	= 684.14	Relative Standard Deviation	=19.42
Quarites	$Q_1 \to 560.18$ $Q_2 \to 660.245$ $Q_3 \to 732.8$		

4. Forecasting Performance Measures

The Mean Absolute Percentage Error (MAPE), value and applying Lewis's scale, provides some framework to judge the model. However, depending of the data set, as to whether there is a significant trend or seasonal component, the MAPE may under or overestimate the accuracy of the model. Average Forecasting Error (AFER) = 07.31%.

Table 9. A scale of judgment of forecast accuracy (Lewis (1982)).

MAPE	Judgment of Forecast Accuracy
Less than 10%	Highly accurate
11% to 20%	Good forecast
21% to 50%	Reasonable forecast
51% or more	Inaccurate forecast

Mean Absolute Deviation (MAD). One of the most popular forecasting performance measures of the size of the error is the mean absolute deviation (MAD). It is the size of overall forecasting error for a model. The MAD is calculated as the average of the absolute errors. The MAD value measures the amount of error. The smaller the MAD value, the better. Mean absolute deviation = 47.78.

Root Mean Square Error (RMSE): RMSE is a quadratic scoring rule that also measures the average magnitude of the error it's the square root of the average of square difference between prediction and actual observation. Root mean square error = 63.80.

5. Conclusion

In this paper we have proposed a simple computational method for fuzzy time series forecasting. We see that MAPE, MAD and RMSE of the forecasting result are given below:

Table 10. Method and Results.

	Method	Result
MAPE/AFER	$\frac{\mid Actual - predicted \mid}{Actual} \times 100$	07.31%
MAD/MAE	$\frac{\Sigma \mid Actual - predicted \mid}{n}$	47.78
RMSE	$\sqrt{\frac{(Actual - predicted)^2}{n}}$	63.80

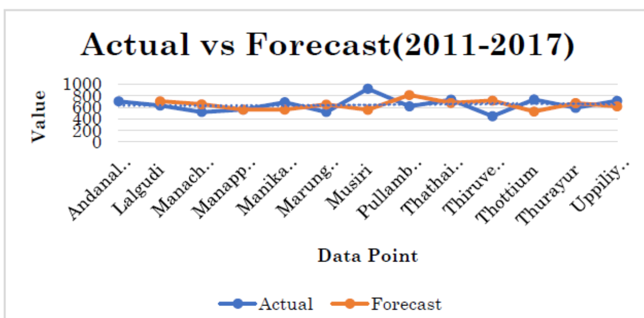

Figure 1. Graph of actual vs forecast (2011-2017).

The MAPE, value and applying Lewis's scale (1982), the MAPE value is 7.31 %(Less than 10%) then high accurate. The forecasted values obtained by the method show its suitability in fuzzy time series forecasting of crop production without any prior knowledge of the production governing parameters.

References

[1] S. M. Chen and C.-C. Hsu, A new method to forecasting enrollments using fuzzy time series, International Journal of Applied Science and Engineering 2(3) (2004), 234-244.

[2] S. M. Chen, Forecasting enrollments based on fuzzy time series, Fuzzy Sets and Systems 81 (1996), 311-319.

[3] S. M. Chen and N. Y. Chung, Forecasting enrollments of students by using fuzzy time series and genetic algorithms, International Journal of Information and Management Sciences 17 (2006), 1-17.

[4] S. M. Chen and N. Y. Chung, Forecasting enrollments using high-order fuzzy time series and genetic algorithms: research articles, International Journal of Information and Management Sciences 21 (2006), 485-501.

[5] Edvin Aldrian and Yudha Setiawan Djamil, Application of multivariate ANFIS for daily rainfall prediction: influences of training data size, Makara, Sains 12 (2008), 7-14.

[6] K. Huarng, Effective lengths of intervals to improve forecasting in fuzzy time series, Fuzzy Sets and Systems 123 (2001), 387-394.

[7] R. K. Klimberg and S. Ratick, A New Measure of Relative Forecast Error, INFORMS Fall 2000 Meeting, San Antonio, Nov. (2000).

[8] M. Lagrange, H. Andrieu, I. Emmanuel, G. Busquets and S. Loubrié, Classification of Rainfall Radar Images using the Scattering Transform, J. Hydrol. 556 (2018), 972-979. https://doi.org/10.1016/j.jhydrol.2016.06.063

[9] K. Lawrence, R. K. Klimberg and S. Lawrence, Fundamentals of Forecasting Using Excel, Industrial Press, Nov (2008).

[10] E. Leblois, Le Bassin Versant, Système Spatialement Structuré et Soumis au Climat, HDR, University of Grenoble, France, (2012).

[11] L. W. Lee, H. F. Wang and S. M. Chen, Temperature prediction and TAIFEX Forecasting based on high-order fuzzy logical relationships and genetic simulated annealing techniques, Expert Systems with Applications 34(1) (2008), 328-336.

[12] L. W. Lee, L. H. Wang and S. M. Chen, Temperature prediction and TAIFEX forecasting based on fuzzy logical relationships and genetic algorithms, Expert Systems with Applications 33(3) (2007), 539-550.

[13] C. D. Lewis, Industrial and business forecasting methods: A radical guide to exponential smoothing and curve fitting, London, Boston: Butterworth Scientific, (1982).

[14] E. H. Mamdani and S. Assilian, Int. J. Man Mach. Stud. 7 (1975), 1-13.

[15] T. Mentzer and C. C. Bienstock, Sales Forecasting Management, Sage Publications, (1998).

[16] J. I. Park, D. J. Lee, C. K. Song and M. G. Chun, TAIFEX and KOSPI 200 forecasting based on two-factors high-order fuzzy time series and particle swarm optimization, Expert Systems with Applications 37 (2010), 959-967.

[17] H. Rahmlow and R. Klimberg, Forecasting practices of MBA's, Advances in Business and Management Forecasting, Elsevier Science Ltd. 3 (2002), 113-123.

[18] I. V. Sideris, M. Gabella, R. Erdin and U. Germann, Real-time radar-rain-gauge merging using spatio-temporal Co-kriging with external drift in the alpine terrain of switzerland, Q. J. Roy. Meteor. Soc. 140 (2014), 1097-1111. https://doi.org/10.1002/ qj.2188

[19] Q. Song, A note on fuzzy time series model selection with sample autocorrelation functions, Cybernetics and Systems: An International Journal 34 (2003), 93-107.

[20] Q. Song and B. S. Chissom, Forecasting enrollments with fuzzy time series: Part II, Fuzzy Sets and Systems 62 (1994) 1-8.

[21] Q. Song and R. P. Leland, Adaptive learning defuzzification techniques and applications, Fuzzy Sets and Systems 81 (1996), 321-329.

[22] Q. Song and B. S. Chissom, Fuzzy time series and its models, Fuzzy Sets and Systems, 54(3) (1993), 269-277.

[23] Q. Song and B. S. Chissom, Forecasting enrollments with fuzzy time series part I, Fuzzy Sets and Systems 54(1) (1993), 1-9.

[24] S. Thorndahl, T. Einfalt, P. Willems, J. E. Nielsen, M.-C. ten Veldhuis, K. Arnbjerg-Nielsen, M. R. Rasmussen and P. Molnar, Weather radar rainfall data in urban hydrology, Hydrol. Earth Syst. Sci. 21(3) (2017), 1359-1380. https://doi.org/10.5194/hess-21-1359

[25] D. S. Wilks, Use of stochastic weather generators for precipitation downscaling, WIREs Clim. Change 1 (2010), 898-907. https://doi.org/10.1002/wcc.85

[26] S. E. Zick and C. J. Matyas, A shape metric methodology for studying the evolving geometries of synoptic-scale precipitation patterns in tropical cyclones, Ann. Am. Assoc. Geogr. 106(16) (2016), 1217-1235. https://doi.org/10.1080/24694452.2016.1206460

[27] L. A. Zadeh, J. Info. Control 8 (1965), 338-353.