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d-LUCKY LABELING OF CERTAIN GRAPHICAL STRUCTURES

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Abstract

The concept of d-lucky labeling is such that the vertices of the graph G are labeled from the set of positive integers $\{1, 2, \ldots, r\}$ where r is the least positive integer meeting the condition $c(u) \neq c(v)$. $\eta_{dl}(G)$ denotes the d-lucky number of a graph G. We define $c(u) = d(u) + \sum_{v \in N(u)} l(v)$ where $d(u)$ signifies the degree of u, $N(u)$ denotes the open neighborhood of u and $l(v)$ is a label assigned to the vertex v. In this study, we have obtained the d-lucky number for some stellations of octahedron, dodecahedron and for some k-polytope and for some families of convex polytope.

1. Introduction

Graph theory has a beauty for its diverse features which makes the difficult networks or structures into simple graphs. Graph labeling is one of the main area of research which assigns labels to the edges or vertices or both of a graph. The method of assignment of labels define the type of labeling. In recent times the d-lucky labeling concept was studied by various authors for

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different graphs [1, 2]. We have obtained the $\eta_{dl}(G)$ for $HC(n)$ and $HCT(n)$ [3] and hexagonal networks. Further we have extended our concept to some polytopes and stellated polytopes.

Polytopes are geometric objects or figures which can be extended to n-dimensions. The n-dimensional polytope is also called as an n-polytope. The simplex represents the simplest form of polytope in a given space. We have examined 4-simplex and certain stellated polytopes. For extended study one can refer to Gallian survey.

2. Preliminaries

Definition [3]. Let $l : V(G) \to \{1, 2, r\}$ be a positive integer labeling of the vertices of a graph G. Define $c(u) = d(u) + \sum_{v \in N(u)} l(u)$ where $d(u)$ denotes u's degree, $N(u)$ denotes u's open neighborhood, and $l(v)$ denotes the label assigned to the vertex v. For any pair of adjacent vertices u and v in G, we define a labeling l as d-lucky labeling if $c(u) \neq c(v)$. The least positive integer r such that G has a d-lucky labeling with $\{1, 2, \ldots, r\}$ as the set of labels is the d-lucky number of a graph G, denoted by $\eta_{dl}(G)$.

3. Main Results

Theorem 1. *The n-simplex is d-lucky and the* $\eta_{dl}(G) = n + 1$.

Proof of Theorem 1. The n-simplex is a graph on $n + 1$ vertices and every vertex is adjacent to n number of vertices. Consider 4-simplex for illustration shown in Figure 1. There are 5 vertices each of degree 4.

We label the vertices from 1 to 5. We denote the pair of adjacent vertices as (u, v). Let us take all possible pairs of adjacent vertices namely (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5). We claim that $c(u) \neq c(v)$. For (1, 2): the degree of u with label 1 is 4 and sum of the labels of neighborhood of u is 14. Therefore $c(u)$ is 18. The degree of v with label 2 is 4 and sum of the labels of neighborhood v is 13. Therefore $c(v)$ is 17. For $(1, 3)$, $c(u) = 4 + 14 = 18$ and $c(v) = 4 + 12 = 16$. For $(1, 4) : c(u) = 4$

$+14 = 18$ and $c(v) = 4 + 12 = 16$. For $(1, 5) : c(u) = 4 + 14 = 18$ and $c(v) = 4 + 10 = 14$. For $(2, 3) : c(u) = 4 + 13 = 17$ and $c(v) = 4 + 12 = 16$.

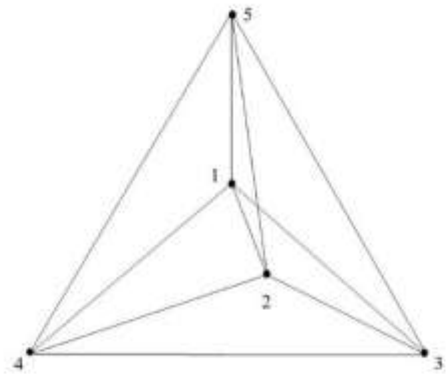

Figure 1. d-lucky labeling of 4-simplex.

For $(2, 4) : c(u) = 4 + 13 = 17$ and $c(v) = 4 + 11 = 15$. For $(2, 5) : c(u) = 4 + 13 = 17$ and $c(v) = 4 + 10 = 14$. For $(3, 4)$, $c(u) = 4 + 12 = 16$ and $c(v) = 4 + 11 = 15$. For $(4, 5) : c(u) = 4 + 11 = 15$ and $c(v) = 4 + 10 = 14$. For $(4, 5) : c(u) = 4 + 11 = 15$ and $c(v) = 4 + 10 = 14$. Therefore $c(u) \neq c(v)$ for all the pairs of vertices considered. This proves our claim. A similar argument holds for each pair of vertices that are contiguous. Thus the n-simplex satisfies d-lucky labeling and $\eta_{dl}(G) = n + 1$.

Theorem 2. *The* 3-*polytope admits d-lucky labeling and the* $\eta_{dl}(G) = 2$.

Proof of Theorem 2. Some of the 3-polytope are the stellation of octahedron and dodecahedron. We have considered the top view of this stellation as a graphical structure. The intersection of the planes are the vertices and we label these vertices as 1 and 2 such that it satisfies the condition $c(u) \neq c(v)$, for distinct vertices u and v.

For illustration of stellation of octahedron see Figure 2. We consider the centre vertex with label 1 as u and its adjacent vertex with label 1 as v and 2 in the inner tetrahedron.

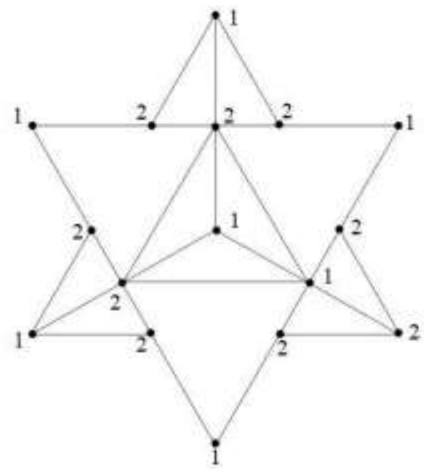

Figure 2. d-lucky labeling of octahedran.

We claim $c(u) \neq c(v)$. Consider the adjacent vertices with labels 1 and 1. The degree of u is 3 and sum of the labels of v is 5. So $c(u)$ is 8. The degree of v with label 1 is 6 and sum of the labels of u is 11. Therefore $c(v)$ is 17. Similarly the degree of v with label 2 is 6 and sum of the labels of u is 9 so $c(v)$ is 15. Therefore we can say that $c(u) = 8$, $c(v) = 17$ and $c(v) = 15$ which proves our claim $c(u) \neq c(v)$. Similar argument holds for each pair of vertices that are contiguous. Thus this stellation is d-lucky and $\eta_{dl}(G) = 2$.

For illustration of stellation of dodecahedron see Figure 3. We consider the centre vertex with label 1 as u and its adjacent vertex with label 1 and 2 as v on the outer decahedron. We claim $c(u) \neq c(v)$ for every pair of vertices u and v adjacent in G. Consider the adjacent vertices with labels 1 and 1. The degree of u is 10 and sum of the labels of v is 15 so $c(u)$ is 25. The degree of v with label 1 is 7 and sum of the labels of u is 10. Therefore $c(v)$ is 17.

Similarly the degree of v with label 2 is 3 and sum of the labels of u is 4 so $c(v)$ is 7.

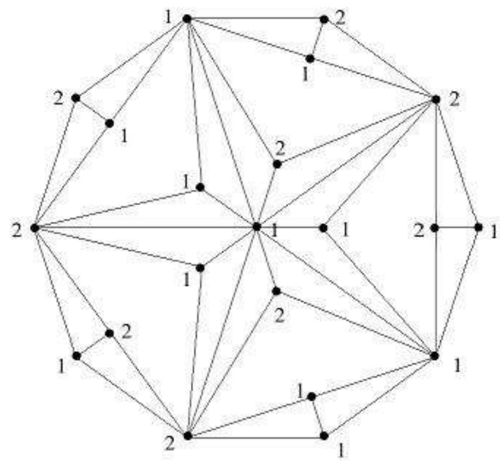

Figure 3. d-lucky labeling of dodecahedron.

Therefore we have $c(v) = 25, c(v) = 17$ and $c(v) = 7$ which proves our claim $c(u) \neq c(v)$. Similar argument holds for each pair of vertices that are contiguous. Thus the 3-polytope is d-lucky and $\eta_{dl}(G) = 2$.

Theorem 3. *The convex polytope* T_n *admits d-lucky labeling and the* $\eta_{dl}(G)$ *is* 2.

Proof of Theorem 3. The graph T_n consists of three sided faces, five sided faces and n-sided faces. Here we consider the graph T_8. It has three layers with eight vertices in each layer which we define as vertices in inner layer, middle layer and the outer layer. We label these vertices as 1 and 2 alternatively proceeding from the inner layer to outer layer satisfying the condition $c(u) \neq c(v)$ for every pair of distinct vertices.

The illustration is shown in Figure 4. We consider the vertex on the inner circle with label 1 as u and its adjacent vertex with label 2 as v. We claim $c(u) \neq c(v)$ for every pair of vertices adjacent to each other. The degree of u is 4 and sum of the labels of v is 7 so $c(u)$ is 11. The degree of v with label 1 is 4 and sum of the labels of u is 5.

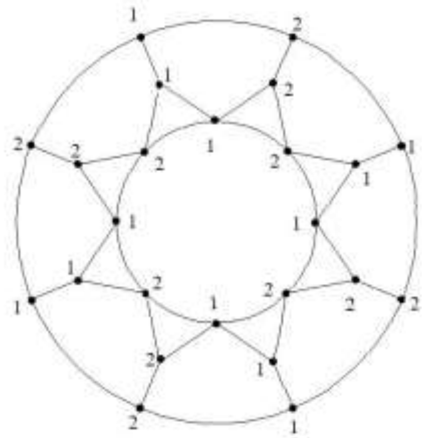

Figure 4. d-lucky labeling of T_8.

Therefore $c(u)$ is 9. Therefore $c(u) \neq c(v)$. For the same u we take a vertex v with label 1 from the middle layer. The degree of v with label 1 is 3 and sum of the labels of u is 4. Therefore $c(v)$ is 7. Therefore $c(u) \neq c(v)$ for every pair of vertices adjacent to each other in T_8. This proves our claim. Hence the graph T_n satisfies d-lucky labeling. The $\eta_{dl}(G)$ is 2.

Theorem 4. *The convex polytope* S_n *admits d-lucky labeling and* $\eta_{dl}(S_n)$ *is* 3.

Proof of Theorem 4. The graph S_n consists of three sided faces, four sided faces, five sided faces and n-sided faces. To prove that the graph S_n is d-lucky it should satisfy the condition $c(u) \neq c(v)$ for all the u and v pairs of vertices following. We begin the labeling from the interior circle as 1 and 2 alternatively, the second and the third layer vertices as 1 and the exterior circle as 2. See Figure 5.

We take $n = 8$ for illustration. We consider a pair of vertices (u, v) as (1, 2) on the inner circle. We claim $c(u) \neq c(v)$. The degree of u and v are 4 and sum of the labels of v and u are 7 and 5 respectively. So $c(u)$ is 11 and $c(v)$ is 9.

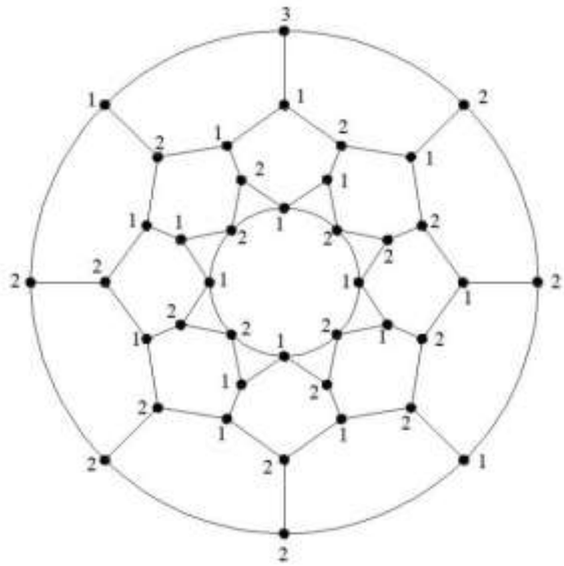

Figure 5. d-lucky labeling of S_8.

Therefore $c(u) \neq c(v)$. This proves our claim. Similar argument holds for every pair of vertices adjacent to each other. Hence the graph S_n satisfies d-lucky labeling and $\eta_{dl}(S_n) = 3$.

4. Conclusion

From the above results we can conclude that the n-simplex satisfies d-lucky labeling and $\eta_{dl}(G)$ is $n+1$. Also the first stellation of octahedron and the second stellation of dodecahedron which are the types of 3-polytope and some families of convex polytopes admit d-lucky labeling and the $\eta_{dl}(G)$ is 2. The d-lucky number for S_n is 3. Further we extend the concept to study the behaviour of other stellations of dodecahedron and polytopes for various values of n.

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