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SOME SIMPLE GRAPHS ARE LUCKY AND PROPER

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Abstract

In this paper we showed that bull, ladder rung, diamond, fork and cricket graphs are lucky and proper lucky. Also we computed the lucky number and proper lucky number.

1. Motivation

The graph which have finite number of nodes are called finite graph. Here we considered the few finite graphs and computed the following labeling. "The labeling is proper if nodes of graph are named by natural number with the condition that label of adjacent vertices is not same" [1]. "The lucky labeling graph has the sum of labels over neighboring nodes in the graph is not an identical" [1]. The graphs which admit the above condition are called lucky graph. The graphs which have above two conditions are called

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proper lucky graph. The lucky number and proper lucky number of G is indicated by $\eta(G)$ and $\eta_p(G)$ respectively [2]. The ladder rung graph is total vertex irregularity strength said by Nugroho Arif Sudibyo in 2019 [3]. Line graph of fork graph, bull graph, diamond graph and cricket graph are root square mean is derived by Sandhya in [3].

Bull Graph. The bull graph BG contains 5 vertices and edges are $V(G) = \{u_1, u_2, u_3, u_4, v_1\}$ and $E(G) = \{u_i, u_{i+1}; 1 \le i \le 3\} \cup \{v_1 u_1; 2 \le i \le 3\}$, refer figure 1(a) [3].

Ladder Rung Graph. The ladder rung graph LRG has $2n$ vertices are $V(G) = \{u_1, u_2, \ldots, u_n\} \cup \{v_1, v_2, \ldots, v_n\}$ and n edges are $E(G) = \{u_i u_j; 1 \le i \le n, 1 \le j \le n, i = j\}$, refer figure 1(b) [4].

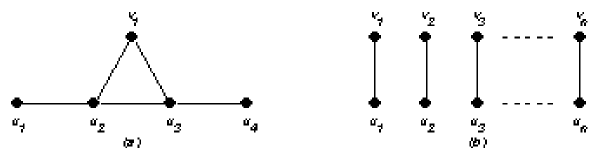

Figure 1. (a) Bull graph, (b) Ladder rung graph.

Fork Graph. The fork graph FG contains 5 vertices are $V(G) = \{u_1, u_2, u_3, v_1, v_2\}$ and 4 edges are $E(G) = \{u_i, u_{i+1}; 1 \le i \le 2\} \cup \{v_i, v_{i+1}; 1 \le i\} \cup \{u_i v_i; i = 2\}$, refer figure 2(a) [3].

Diamond Graph. The diamond DG graph contains 4 vertices $V(G) = \{u_1, u_2, u_3, v_1\}$ and 5 edges are $E(G) = \{u_i u_{i+1}; 1 \le i \le 2\} \cup \{v_i u_i; 1 \le i \le 3\}$, refer figure 2(b) [3].

Cricket Graph. The cricket graph CG contains 5 vertices are $V(G) = \{u_1, u_2, u_3, v_1, v_2\}$ and 5 edges are $E(G) = \{u_i u_{i+1}; 1 \le i \le 2\} \cup \{v_j u_{j+1}; j = 1\} \cup \{u_i v_j; i = 2, 1 \le j \ge 2\}$, refer figure 2(c) [3].

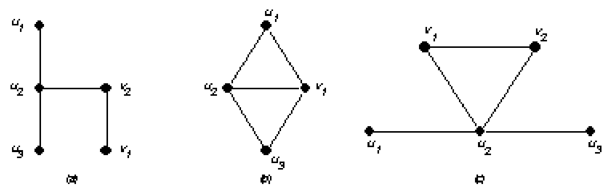

Figure 2. Cricket graph.

2. Main Results

Theorem 3.1. The *bull graph is lucky graph with* $\eta(BG) = \Delta(BG) - 1$.

Proof. Let $V(BG) \to \{1, 2\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i \neq 4 \\ 2 & i = 4 \end{cases} \qquad f(v_i) = 1 \qquad s(u_i) = \begin{cases} 1 & i = 1, i = 4 \\ 3 & i = 2 \\ 5 & i = 3 \end{cases} \qquad s(v_1) = 2.$$

Where f and s denote labeling and the sum over the neigbour vertices respectively. Thus bull graph is lucky graph and $\eta(BG) = \Delta(BG) - 1$.

Theorem 3.2. *The bull graph is proper lucky graph with* $\eta_p(BG) = \Delta(BG)$.

Proof. Let $V(BG) \to \{1, 2, 3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i \text{ is even} \\ 2 & i \text{ is odd} \end{cases} \qquad f(v_1) = 3 \qquad s(u_i) = \begin{cases} 1 & i = 1 \\ 2 & i = 4 \\ 5 & i = 3 \\ 7 & i = 2 \end{cases} \qquad s(v_1) = 3.$$

Therefore bull graph is proper lucky graph and $\eta_p(BG) = \Delta(BG)$.

Theorem 3.3. *The ladder rung graph is lucky and proper graph with* $\eta(BG) = \eta_p(LRG) = \delta(BG) + 1 = \Delta(BG) + 1$.

Proof. Let $V(LRG) \to \{1, 2\}$ be defined by

$$f(u_n) = 1 \; f(v_n) = 2 \qquad s(u_n) = 2 \qquad s(v_n) = 1.$$

Thus ladder rung graph is proper and lucky graph and $\eta(BG) = \eta_p(LRG) = \delta(BG) + 1 = \Delta(BG) + 1$.

Theorem 3.4. *The fork graph is lucky graph with* $\eta(FG) = \Delta(FG) - 2$.

Proof. Let $V(FG) \to \{1\}$ be defined by

$$f(u_i) = 1 \quad f(v_i) = 1 \quad s(u_i) = \begin{cases} 1 & i \neq 2 \\ 3 & i = 2 \end{cases} \quad s(v_n) = \begin{cases} 1 & i = 1 \\ 2 & i = 2 \end{cases}$$

Therefore fork graph is lucky graph and $\eta(FG) = \Delta(FG) - 2$.

Theorem 3.5. *The fork graph is proper lucky graph with* $\eta_p(FG) = \Delta(FG) - 1$.

Proof. Let $V(FG) \to \{1, 2\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i \neq 2 \\ 2 & i = 2 \end{cases} \quad f(v_i) = \begin{cases} 1 & i = 2 \\ 2 & i = 1 \end{cases} \quad s(u_i) = \begin{cases} 2 & i \neq 2 \\ 3 & i = 2 \end{cases} \quad s(v_n) = \begin{cases} 1 & i = 1 \\ 4 & i = 2 \end{cases}$$

Therefore for *k* graph is proper lucky graph and $\eta_p(FG) = \Delta(FG) - 1$.

Theorem 3.6. *The diamond graph is lucky graph with* $\eta(DG) = \Delta(DG) - 1$.

Proof. Let $V(DG) \to \{1, 2\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i = 1 \\ 2 & i \neq 1 \end{cases} \quad f(v_1) = 1 \quad s(u_i) = \begin{cases} 3 & i \neq 2 \\ 4 & i = 2 \end{cases} \quad s(v_1) = 5.$$

Therefore diamond graph is lucky graph and $\eta(DG) = \Delta(DG) - 1$.

Theorem 3.7. *The diamond graph is proper lucky graph with* $\eta_p(DG) = \Delta(DG)$.

Proof. Let $V(DG) \to \{1, 2, 3\}$ be defined by

$$f(u_i) = \begin{cases} 2 & i = 2 \\ 3 & i \neq 2 \end{cases} \quad f(v_1) = 1 \quad s(u_i) = \begin{cases} 3 & i \neq 2 \\ 7 & i = 2 \end{cases} \quad s(v_1) = 8.$$

Therefore diamond graph is proper lucky graph and $\eta_p(DG) = \Delta(DG)$.

Theorem 3.8. *The cricket graph is lucky graph with* $\eta(CG) = \Delta(CG) - 2$.

Proof. Let $V(CG) \to \{1, 2\}$ be defined by

$$f(u_i) = 1 \quad f(v_i) = \begin{cases} 1 & i = 1 \\ 2 & i = 2 \end{cases} \quad s(u_i) = \begin{cases} 1 & i \neq 2 \\ 5 & i = 2 \end{cases} \quad s(v_i) = \begin{cases} 2 & i = 2 \\ 3 & i = 1 \end{cases}.$$

Therefore cricket graph is lucky graph and $\eta(CG) = \Delta(CG) - 2$.

Theorem 3.9. *The cricket graph is proper lucky graph with* $\eta_p(CG) = \Delta(CG) - 1$.

Proof. Let $V(CG) \to \{1, 2, 3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i \neq 2 \\ 2 & i = 2 \end{cases} \quad f(v_i) = \begin{cases} 1 & i = 1 \\ 3 & i = 2 \end{cases} \quad s(u_i) = \begin{cases} 2 & i \neq 2 \\ 6 & i = 2 \end{cases} \quad s(v_i) = \begin{cases} 3 & i = 2 \\ 5 & i = 1 \end{cases}$$

Therefore cricket graph is proper lucky graph and $\eta_p(CG) = \Delta(CG) - 1$.

3. Conclusion

We proved the bull graph, ladder rung graph, fork graph, diamond graph and cricket graph are lucky and proper lucky graph and cardinality of it was mentioned.

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