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APPLICATIONS OF GRAPH THEORY IN JOB SCHEDULING AND POSTMAN'S PROBLEM

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Abstract

The theme of graph theory is the modelling approach which needs accuracy in dealing with travelling problems. In this research paper the standard technique is used to show how graph theory and networks may be used to model travelling problems in a different view point with effective algorithms.

1. Introduction

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges. Graphs are one of the prime objects of study in discrete mathematics. certain discrete problems can be profitably analyzed using graph theoretic methods.

Graph theory is used in sociology, cloth mills, Indian railways, transport, airlines and defence organizations. There is a wide use of graphs in providing problem solving techniques. Various papers based on graph theory have been studied related to its uses and scheduling concepts and an overview has been presented here besides discussion of its uses in our day to day life.

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This paper introduces useful concepts from graph theory and it also shows how to apply network graph theory.

2. Applications of Graph Theory in Transport Network

Graph theory is a very natural and powerful tool in combinational operations research. A networks called transport network where a graph is used to model the transportation of commodity from one place to another the objective is to maximize the flow or minimize the cost within the prescribed flow.

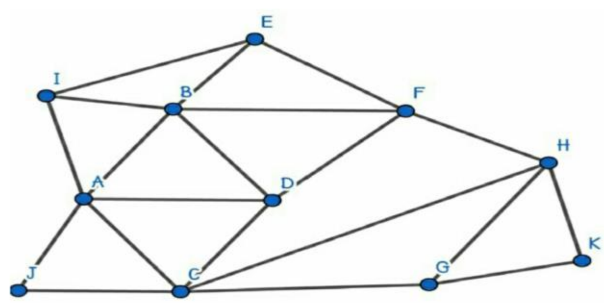
Highway systems can be thought of as transporting cars. A common feature of transportation system is the existence of a capacity associated with each edge. In transport geography most networks have an obvious spatial networks, which tend to be defined more by their links than by their nodes. A telecommunication system can also be represented as a network. Graph theory is considered as a fundamental tool in transport system networks.

Postman's Problem

In 1962, a mathematician called Kuan Mei-ko interested in a postman delivering the mails to a number of streets. Such that the total distance covered by the postman was considered as the shortest distance as possible. How could the postman ensure that the distance walked was maximum.

Example

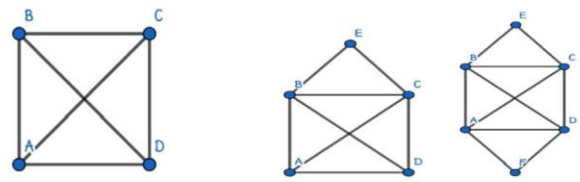
A postman has to start at A and he has to walk along 20 streets and return back to A. The length of the each street is given below. The problem is to find a train all the edges of a graph with minimum length.



Graph Figure 1

The lengths of the streets are $AB = 50$, $AJ = 50$, $AI = 70$, $JC = 50$, $IE = 80$, $BE70$, $BF = 50$, $DC = 70$, $DF60$, $FH60$, $CG = 70$, $GH = 70$, $GK = 60$, $HK = 60$, $EF = 70$, $CH = 120$, $IB = 60$.

We will return to solving this actual problem later, but initially we will look at drawing various graphs. The postman is transversable graphs given below.



Graph 1 Graph 2 Graph 3

From these graph we find:

• It is impossible to draw graph 1 without either taking the pen off the paper or re-tracing an edge.

• We can draw graph 2, but only by starting at either A or D in case the path will end at the other vertex of D or A.

• Graph 3 can be drawn regardless of the starting position and you will always return to the start vertex.

In order to establish the differences, we must consider the order of the vertices for each graph. The following

VERTEX	ORDER
A	3
B	3
C	3
D	3

Graph 1.

VERTEX	ORDER
A	3
B	4
C	4
D	3
E	2

Graph 2.

VERTEX	ORDER
A	4
B	4
C	4
D	4
E	2
F	2

Graph 3.

When the order of all the vertices is even the graph is transversable. When there are two odd vertices we can draw the graph but the start and end vertices are different. When there are four odd vertices the graph can't be drawn without repeating an edge.

Postman Algorithm:

An algorithm for finding an optimal postman route is

Step 1. List all odd vertices.

Step 2. List all possible pairing of odd vertices.

Step 3. For each pairing find the edges that connect the vertices with the minimum weight.

Step 4. Find the pairing such that the sum of the weight is minimized.

Step 5. On the original graph add the edges that have been found in step 4.

Step 6. The length of an optional postman route is the sum of all the edges added to the total found in step 4.

Step 7. A route corresponding to this minimum weight can then be easily found.

Now we apply the algorithm to the original problem.

Step 1. The odd vertices are A, B, C, E, G. I.

Step 2. There is five way of pairing these odd vertices namely A, B, C, E, G, I.

Step 3. The shortest way of joining the vertices is AE, CG, $B\Gamma$.

Step 4. These edges on to the original network in this diagram.

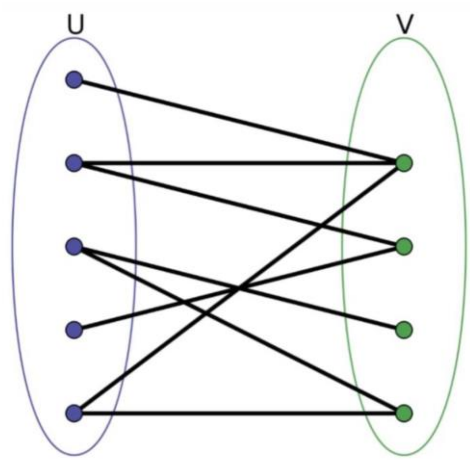
Step 5. The length of the optimal postman route is the sum of all the edges in the original network. Which is 1270mtr plus the answer found in the step 4, which is 250mtr. Hence the length of the optimal postman route is 1520mtr.

Step 6. One possible route corresponding to this length is can be found s but many other possible routes of the sum minimum length can be found.

Time Table Scheduling

In this problem timetabling of an institute is discussed in which there are four teachers $T1$, $T2$, $T3$, $T4$ and five subjects $S1$, $S2$, $S3$, $S4$, $S5$ to be taught. The requirement matrix is given as follow. To find the solution of timetabling problem, we use graph coloring technique.

P	S1	S2	S3	S4	S5
T1	2	0	1	1	0
T2	0	1	0	1	0
T3	0	1	1	1	0
T4	0	0	0	1	1



(Figure Bipartite graph with five subjects and four teachers)

In the above bipartite graph the edges of graph interpreted to four periods and finally the solution of time table is obtained.

–	1	2	3	4
T1	S1	S1	S3	S4
T2	S4	–	–	S2
T3	S2	S4	–	S3
T4	–	–	S4	S5

Digraph Methods in Networks

A digraph is a graph in which each line is directed. Those lines have arrows attached to them. A digraph stores nodes and edges with optional data, or attributes. These graphs hold directed edges. Here, self loops are allowed but the multiple edges are not. And the nodes can be arbitrary comprises a current node of a first type, where in at least a portion of one or more digraphs are created at the current node, wherein the one or more digraphs are defined in the multi-hop packet-switched communication network. A prior node of a second type, where in the current node of the first type receives the packet directly from the prior node of the second type or indirectly through a node of a third type.

3. Conclusion

It is now clear that graph theory and its offshoots, the theory of digraphs and networks, have blossomed not only as a branch of mathematics but also as systematic tools in operations research. This paper gives an overview of the applications of the networks in graph theory. Various papers based on graph theory and networks and overview has been presented here. And also postman's problem and time table scheduling have been worked out.

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