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# RESOLVING SETS AND DIMENSION IN SPECIAL GRAPHS

**D. LITTLE FEMILIN JANA, R. JAYA, M. AROKIA RANJITHKUKAR, S. KRISHNAKUMAR and R. SENTHIL KUMAR**

Assistant Professor
Prince Shri Venkateshwara Padmavathy
Engineering College, India
E-mail: ponmar.feminadoss21@gmail.com

Assistant Professor
PSNA College of Engineering and Technology
India
E-mail: jayarajaram@psnacet.edu.in

Assistant Professor
M. Kumarasamy College of Engineering
Karur, India
E-mail: arockiaranjithkumar@gmail.com

Assistant Professor
Sethu Institute of Technology
Kariapatti, India
E-mail: krishmathew@gmail.com

Associate Professor of Mathematics
Auxillum College for Women
Ragunathapuram, Puthukottai – 62 India
E-mail: srisenthil2011@gmail.com

## Abstract

In this research paper, presented the idea resolving set using dominating set as a base set in special graphs like Bidiakis cube, Durer graph, Golomb graph and etc. In a graph

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$G = (V, E)$, The code of vertex $v$ with respect to the ordered set $W = \{w_1, w_2, w_3, \ldots, w_k\} \subseteq V(G)$ is defined by $C_w(v) = (d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$. The set $W$ is so-called a resolving set for $G$ if different nodes have different codes with respect to $W$. A resolving set having a minimum number of nodes is a minimum resolving set or a basis for $G$. The (metric) dimension Dim (G) is the quantity of nodes in a basis for $G = (V, E)$.

## 1. Introduction

Domination in graph theory is a natural model for many location problems in operations research. As an example, consider the following fire station problem. Suppose a county has decided to build some fire stations, to serve all of the towns in the county. They are to be placed in some towns so that every town either has a fire station or is a neighbor of a town which has a fire station. To save money, the county wants to build the minimum number of fire stations satisfying the above requirements.

A graph is an ordered pair $G = (V, E)$, where $V$ is a non-empty finite set, called the set of vertices of $G$, and $E$ is a set of unordered pairs (2- element subsets) of $V$, called the edges of $G$. If $xy \in E$, and $y$ are called adjacent and they are incident with the edge $xy$. For graph theoretic jargon we refer to Chartrand and Lesniak. The main focus of this paper is a generalization of the concept of domination in graphs. For an outstanding dealing of fundamentals of domination we refer to the book by Haynes et al. For survey of several advanced topics in domination we refer to the book edited by Haynes et al.

A graph $G' = (V', E')$ is a sub graph of $G = (V, E)$ if and only if $V' \subseteq V$ and $E' \subseteq E$. The order of a graph $G = (V, E)$ is $|V|$, the number of its vertices. The size of $G$ is $|E|$, the number of its edges. The degree of a vertex $x \in V$, denoted by $d(x)$, is the number of edges incident with it.

A set $S$ of vertices of $G$ is a dominating set of $G$ if every vertices of $G$ is dominated by at least one vertex of $S$. Equivalently: a set $S$ of vertices of $G$ is a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in $S$.

The minimum cardinality among the dominating sets of $G$ is called the domination number of $G$ and denoted by $\gamma(G)$. A dominating set of cardinality $\gamma(G)$ is referred to as a minimum dominating set.

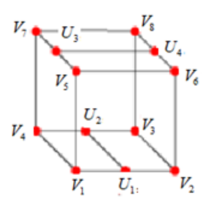
The resolving matrix of the finite graph $G(V, E)$ with respect to the ordered resolving set $R = \{v_1, v_2, v_3, \ldots, v_k\} \subseteq V(G)$ is defined by the $C_R(G) = [a_{ij}]$, $a_{ij}$ denotes the distance between the nodes $v_i$ and $v_j$. In $C_R(G) = [a_{ij}]$ rows contains the elements of $V(G) - R$ and column contains the elements in $R$. The resolving matrix is defined by

$$
\begin{array}{cc}
 & \begin{array}{ccccc} v_{r_1} & v_{r_2} & v_{r_3} & \cdots & v_{r_k} \end{array} \\
C_R(G) = \begin{array}{c} v_{c_1} \\ v_{c_2} \\ v_{c_3} \\ \vdots \\ v_{c_n} \end{array} & \begin{bmatrix} d(v_{c_1}, v_{r_1}) & d(v_{c_1}, v_{r_2}) & d(v_{c_1}, v_{r_3}) & \cdots & d(v_{c_1}, v_{r_k}) \\ d(v_{c_2}, v_{r_1}) & d(v_{c_2}, v_{r_2}) & d(v_{c_2}, v_{r_3}) & \cdots & d(v_{c_2}, v_{r_k}) \\ d(v_{c_3}, v_{r_1}) & d(v_{c_3}, v_{r_2}) & d(v_{c_2}, v_{r_3}) & \cdots & d(v_{c_3}, v_{r_k}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d(v_{c_n}, v_{r_1}) & d(v_{c_n}, v_{r_2}) & d(v_{c_n}, v_{r_3}) & \cdots & d(v_{c_n}, v_{r_k}) \end{bmatrix}
\end{array}
$$

$v_{r_i} \in R$ where $i = 1, 2, \ldots, k$ and $v_{c_j} \in V(G) - R$ and $j = 1, 2, \ldots, n$.

## 2. Main Results

**Bidiakis Cube:** Bidiakis cube is a 3-regular graph with 12 vertices and 18 edges.




$G(V, E)$: *Bidiakis cube*

The 12-node graph containing of a cube in which two opposite surfaces (say, top and bottom) be necessary edges drawn through them which join the midpoints of opposite sides of the faces in such a way that the orientation of the edges added on top and bottom are perpendicular to each other.

**Theorem 2.1.** *Let* $G(V, E)$ *is a Bidiakis Cube. Then the minimal* *resolving set* $R = \{U_i, U_j, U_k\}$ *where* $U_i, U_j, U_k \in V(G)$, *and the dimension of Bidiakis Cube* $G(V, E)$ *is* $Dim(G) = 3$.

**Proof.** Let $G(V, E)$ is a Bidiakis Cube. Let we assume the minimal dominating set $D = \{U_1, U_2, U_3, U_4\}$. Let $U_1, U_2, U_3, U_4$ contain the vertices in two opposite faces have edges drawn across them which connect the centers of opposite sides of the faces. Let $D = \{a, b, c, d\}$ it dominates the corner vertices of the Bidiakis Cube. Therefore $U_1, U_2, U_3, U_4$ is a dominating set of Bidiakis Cube. There solving matrix of $C_D(G)$ is

$$C_D(G) = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{array} \begin{array}{c} \begin{array}{cccc} U_1 & U_2 & U_3 & U_4 \end{array} \\ \begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 2 & 3 & 2 \\ 2 & 1 & 3 & 2 \\ 2 & 1 & 2 & 3 \\ 2 & 3 & 1 & 2 \\ 2 & 3 & 2 & 1 \\ 3 & 2 & 1 & 2 \\ 3 & 2 & 2 & 1 \end{bmatrix} \end{array}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$. Therefore $D = \{U_1, U_2, U_3, U_4\}$ is the resolve dominating set of Bidiakis Cube $G(V, E)$. Now we check $D = \{U_1, U_1, U_3, U_4\}$ is the minimal resolve dominating set of Bidiakis Cube.

Assume $D = \{U_1, U_2, U_3, U_4\}$ is a minimal resolve dominating set of Bidiakis Cube. There exist a set $R = (D - \{U_i\})$ where $U_i \in V(G)$ is the vertices in two opposite faces have edges drawn across them which connect the centers of opposite sides of the faces. Let $R = (D - \{U_1\})$ the resolving matrix of $C_R(G)$ is

$$C_R(G) = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ U_1 \end{array} \begin{array}{c} \begin{array}{ccc} U_2 & U_3 & U_4 \end{array} \\ \begin{bmatrix} 2 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 3 & 3 \end{bmatrix} \end{array}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$. Therefore $R = (D - \{U_i\})$ is the resolving set of Bidiakis Cube $G(V, E)$ but not dominating set. Since $D = \{U_1, U_2, U_3, U_4\}$ is a minimal dominating set of Bidiakis Cube. Assume $R = (D - \{U_i\})$ is not a minimal resolving set of Bidiakis Cube. There exist a set $R' = (D - \{U_i, U_j\})$ where $U_i, U_j \in V(G)$ is the vertices in two opposite faces have edges drawn across them which connect the centers of opposite sides of the faces. From the resolving matrix of $C_{R'}(G)$ we observe the code of $V_i, V_j \in V(G)$ are not distinct, i.e. $C_{R'}(v_i) = C_{R'}(v_j)$ for some $V_i, V_j \in (G)$, Therefore $R'$ is not a resolving set of Bidiakis Cube $G(V, E)$.

Hence $R = (D - \{U_i\})$ is a minimal resolving set of Bidiakis Cube. The set $R$ is a minimal resolving set or a basis of Bidiakis Cube $G(V, E)$. Therefore dimension of Bidiakis Cube $G(V, E)$ is cardinality of the basis $R$.

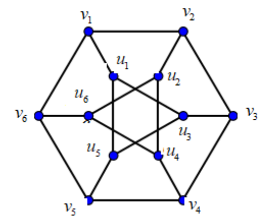
$$Dim(G) = |R| = |(D - \{U_i\})|$$

$$= |D| = |U_i|$$

$$= |\{U_1, U_2, U_3, U_4\}| - |U_i|$$

$$Dim(G) = 3$$

**Durer graph:** The Durer graph is an undirected cubic graph with 12 vertices and 18 edges.




$G(V, E)$: *Durer graph*

**Remark.**

1. Durer graph is a 3 regular graph.

2. $n$ the Durer graph 6 vertices forms a outer Hexagon and remaining 6

vertices form sinner 2 triangle region.

3. The 6 vertices of the inner triangles are adjacent to only one vertex on the Hexagon.

**Theorem 2.2.** *Let* $G(V, E)$ *is a Durer graph. Then the minimal resolving set* $R = (D - \{v_{(i+_6 3)}\})$, *where D is the minimal resolving set of Durer graph* $G(V, E)$. *The dimension of Durer graph* $G(V, E)$ *is* $Dim(G) = 3$.

**Proof.** Let $G(V, E)$ is a Durer graph. Let we assume $D = \{u_i, u_{(i+_6 1)}, v_i, v_{(i+_6 3)}\}$ is a minimal dominating set of Durer graph. The subset $\{u_i, u_{(i+_6 1)}\}$ dominates remaining inner vertices of Durer graph and the subset $\{v_i, v_{(i+_6 3)}\}$ remaining outer vertices of Durer graph. Note that $D - \{x\}$, is not a dominating set. If $x \in \{u_i, u_{i+1}\}$ there exist an inner vertex $u_j \in V(G)$ not dominated by $D$. Therefore $D = \{u_i, u_{(i+_6 1)}, v_i, v_{(i+_6 3)}\}$ is a minimal dominating set of Durer graph $G(V, E)$. The resolving matrix of $C_D(G)$ is

$$C_D(G) = \begin{array}{c c} & \begin{array}{cccc} u_1 & u_2 & v_1 & v_4 \end{array} \\ \begin{array}{c} u_3 \\ u_4 \\ u_5 \\ u_6 \\ v_2 \\ v_3 \\ v_5 \\ v_6 \end{array} & \begin{bmatrix} 1 & 2 & 2 & 2 \\ 3 & 1 & 3 & 1 \\ 1 & 3 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 1 \\ 2 & 2 & 1 & 3 \end{bmatrix} \end{array}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$. Therefore $D = \{u_i, u_{(i+_6 1)}, v_i, v_{(i+_6 3)}\}$ is resolving set of Durer graph $G(V, E)$. Assume $R - (D - \{v_{(i+_6 3)}\})$ is a resolving set of Durer graph. The code matrix of $C_D(G)$ is

$$C_R(G) = \begin{array}{c c} & \begin{array}{ccc} u_1 & u_2 & v_1 \end{array} \\ \begin{array}{c} u_3 \\ u_4 \\ u_5 \\ u_6 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \begin{bmatrix} 1 & 2 & 2 \\ 3 & 1 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \\ 2 & 2 & 1 \end{bmatrix} \end{array}$$

Note that code of the of every pair of vertices are distinct i.e. $C_R(v_i) \neq C_R(v_j)$ for every $v_i, v_j \in V(G)$. Therefore $R = (D - \{x\})$ is the resolving set but not a dominating of Durer graph $G(V, E)$ Assume $R = (D - \{x\})$ is not a minimal resolving set of Durer graph. There exist a set $R' = (D - \{x_i, x_j\})$ is the resolving set of Durer graph $G(V, E)$. From the resolving matrix of $C_R(G)$ we observe the code of $v_i, v_j \in D$, are same, i.e. $C_{R'}(v_i) = C_{R'}(v_j)$ for some $v_i, v_j \in D$. Therefore $R'$ is not a resolving of Durer graph $G(V, E)$. Hence $R = (D - \{v_{(i+_6 3)}\})$ is a minimal resolving set of Durer graph.

The set $R$ is a minimal resolving set or a basis of Durer graph $G(V, E)$. Therefore dimension of Durer graph $G(V, E)$ is cardinality of the basis $R$.

$$Dim(G) = |R| = |(D - \{v_{(i+_6 3)}\})|$$

$$= |D| - |\{v_{(i+_6 3)}\}|$$

$$= |\{u_i, u_{(i+_6 3)}, v_i, v_{(i+_6 3)}\}| - |v_{(i+_6 3)}|$$

$$Dim(G) = 3$$

**Note:** In a Durer graph $G(V, E)$, every minimal dominating set is a minimal resolve dominating set of $G(V, E)$.

## 4. Conclusion

Further we will work on different type of resolving set in graphs and also define the minimal resolving number of the different type of resolving set in graphs. In addition investigate the properties of that minimal resolving number of the different type of resolving set in graph.

## References

[1] R. C. Brigham, G. Chartrand, R. D. Dutton and P. Zhang, Resolving domination in graphs, Mathematical Bohemica 128(1) (2003), 25-36.

[2] G. Chartrand and P. Zhang, Introduction to Graph Theory, McGraw Hill International Edition, 2005.

[3] E. J. Cockayne, S. T. Hedetniemi and D. J. Miller Properties of hereditary hypergraphs and middle graphs Canada, Math. Bull. 21 (1978), 461-468.

[4] Gary Chartrand, Linda Eroh, Mark A. Johnson and Ortrud R. Oellermann, Resolvability in graphs and the metric dimension of a graph, Discrete Appl. Math. 105(1-3) (2000), 99-113.

[5] F. Harary, Graph Theory, Reading, MA, 1973.

[6] T. W. Haynes, Fundamentals of Domination in graphs', Marcel Dekker, New York, 1998.

[7] V. Saenpholphat and P. Zhang, Conditional resolvability in graphs: a survey, International Journal of Mathematics and Mathematical Sciences 38 (2004), 1997-2017.

[8] P. J. Slater, Dominating and reference sets in a graph, Journal of Mathematical and Physical Sciences 22 (1988), 445-455.

[9] I. G. Yero, D. Kuziak and J. A. Rodríguez-Velázquez, On the metric dimension of corona product graphs, Computers and Mathematics with Applications 61 (2011), 2793-2798.