FUZZY CHROMATIC NUMBER OF FUZZY SOFT GRAPHS

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Abstract

In this paper, a new concept of fuzzy coloring of fuzzy soft graph is introduced. Fuzzy chromatic number of fuzzy soft graph, fuzzy soft sub graph, fuzzy soft bipartite graph and fuzzy soft tree has been discussed.

Introduction

Fuzzy Graph Theory was introduced by Azriel Rosenfied in 1975. In 1999, D. Molodtsov [7] introduced the concept of soft set theory. Molodstov’s soft sets give us new technique for dealing with uncertainty from the view point of parameters. Rajesh K. Thumbakara and Bobin George [12] introduced the notions of soft graph and investigate some of their properties.

The concept of fuzzy chromatic number was introduced by Munoz et al. [8]. In 2014, R. Jahir Hussain and K. S. Kanzul Fathima [4] introduced a new concept of fuzzy coloring and find fuzzy chromatic number of different fuzzy graphs.

This paper addresses the study of fuzzy chromatic number of fuzzy soft graphs. By using the concept of adjacency of vertices called strong adjacent and weak adjacent, the fuzzy chromatic number of fuzzy soft graphs, fuzzy soft sub graphs, fuzzy soft Bipartite graphs, fuzzy soft trees are studied.

1. Preliminaries

Definition 1.1. Let $S$ be an initial universe and $R$ be the set of parameters. Let $P(S)$ denotes the power set of $S$. A pair $(F, R)$ is called a soft set over $S$ where $F$ is a mapping given by $F : R \rightarrow P(S)$. 

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Definition 1.2. Let $S$ be an initial universe and $R$ be the set of parameters. Let $T \subseteq R$. A pair $(F, T)$ is called fuzzy soft set over $S$ where $F$ is mapping given by $F : T \rightarrow I^S$ where $I^S$ denotes the collection of fuzzy subsets of $S$.

Definition 1.3. Let $V$ be a non-empty finite set and $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all $(x, y) \in V \times V$. Then the pair $G = (\sigma, \mu)$ is called fuzzy graph over the set $V$.

Definition 1.4. Let $\{x_1, x_2, x_3, \ldots, x_n\}$ is a non empty set, $R$ is a parameter set and $T \subseteq R$. Also let

(i) $\alpha : T \rightarrow F(V)$ (collection of all fuzzy subsets in $V$)
   
   $e \mapsto \alpha(e) = \alpha_e$ (say )

   $\alpha_e : V \rightarrow [0, 1]$

   $(T, \alpha)$ : fuzzy soft vertex

(ii) $\beta : T \rightarrow F(V \times V)$ (collection of all fuzzy subsets in $V \times V$)

   $e \mapsto \beta(e) = \beta_e$ (say )

   $\beta_e : V \times V \rightarrow [0, 1]$

   $(T, \beta)$ : fuzzy soft edge

Then $((T, \alpha), (T, \beta))$ is called fuzzy soft graph if and only if $\beta_e(x, y) \leq \alpha_e(x) \wedge \alpha_e(y)$ for all $e \in T$ and this fuzzy soft graphs are denoted by $G^S(T, V)$.

Definition 1.5. The fuzzy soft graph $H^S(T, V) = ((T, \zeta), (T, \gamma))$ is called a fuzzy soft sub graph of $G^S(T, V) = ((T, \alpha), (T, \beta))$ if $\zeta_e(x) \leq \alpha_e(x)$ for all $x \in V, e \in T$ and $\gamma_e(x, y) \leq \beta_e(x, y)$ for all $x, y \in V, e \in T$.

Definition 1.6. A Path of length ‘$n$’ in a fuzzy soft graph is a sequence of distinct vertices $x_1, x_2, x_3, \ldots, x_n$ such that $\forall e \in T$ and $\beta_e(x_{i-1}, x_i) > 0 \forall i = 1, 2, 3, \ldots, n$. 

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Definition 1.7. The strength of connectedness between 2 nodes \( x, y \) in a fuzzy soft graph \( G^S(T, V) \) is 
\[
\beta^*_e(x, y) = \sup \{\beta^*_e(x_i, x_j) : k = 1, 2, 3, \ldots\}
\]
where \( \beta^*_e(x_i, x_j) = \sup \{\beta_e(x_1, x_{j+1}) \land \beta_e(x_2, x_{j+2}) \land \ldots \beta^*_e(x_{k+1}, x_j)\}\).

Definition 1.8. An arc \( (x, y) \) in fuzzy soft graph \( G^S(T, V) = ((T, \alpha), (T, \beta)) \) is said to be a strong arc if and only if \( \beta_e(x, y) = \beta^*_e(x, y) \) and \( x, y \) is said to be strong neighbours.

Definition 1.9. A fuzzy soft graph \( G^S(T, V) = ((T, \alpha), (T, \beta)) \) is fuzzy soft forest if it has a spanning fuzzy soft sub graph \( H^S(T, V) \), which is a forest, where all arcs \( (x, y) \) not in \( H^S(T, V) \), \( \beta_e(x, y) < \beta^*_e(x, y) \).

A connected fuzzy soft forest is called a fuzzy soft tree.

Definition 1.10. A fuzzy soft graph \( G^S(T, V) = ((T, \alpha), (T, \beta)) \) is fuzzy soft bipartite if it has a spanning fuzzy soft sub graph \( H^S(T, V) \) which is bipartite where for all edges \( (x, y) \) not in \( H^S(T, V) \). The weight of \( (x, y) \) in \( G^S(T, V) \) is strictly less than the strength of the pair \( (u, v) \) in \( H^S(T, V) \).

(i) \( \beta_e(u, v) < \gamma_e(u, v) \).

2. Fuzzy Chromatic Number of Fuzzy Soft Graphs

Definition 2.1. Two vertices \( x \) and \( y \) in a fuzzy soft graph \( G^S(T, V) \) are strong adjacent if \( (x, y) \) is a strong arc. Otherwise they are said to be weak adjacent.

Definition 2.2. The strong adjacent degree of a vertex ‘\( x \)’ is the number of vertices that are strong adjacent to \( x \). It is denoted by \( d_e^s(x) \), \( e \in T \).

The minimum strong adjacent degree of \( G^S(T, V) \) is 
\[
\delta_e^s(G^S(T, V)) = \min \{d_e^s(x) / x \in V, e \in T \}
\]

The maximum strong adjacent degree of \( G^S(T, V) \) is 
\[
\Delta_e^s(G^S(T, V)) = \max \{d_e^s(x) / x \in V, e \in T \}.
\]
Definition 2.3. A $k$-fuzzy soft vertex coloring of a fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is an assignment of $k$-colors, usually denoted by $1, 2, \ldots, k$ to the vertices of $G^S(T, V)$. Thus, a $k$-fuzzy vertex coloring of $G^S(T, V)$ is a mapping $\Omega_e : V(G^S(T, V)) \rightarrow \{1, 2, \ldots, k\}, e \in T$.

Definition 2.4. A fuzzy soft vertex coloring is proper if no two distinct strong adjacent vertices receive the same color. Thus a proper $k$-fuzzy soft vertex coloring $e \in T$ of $G^S(T, V)$ is a mapping $\Omega_e : V(G^S(T, V)) \rightarrow \{1, 2, \ldots, k\}$ such that $\Omega_e(v_1) \neq \Omega_e(v_2), e \in T$ whenever $v_1$ and $v_2$ are strong adjacent in $G^S(T, V)$.

Definition 2.5. A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ has a proper $k$-fuzzy soft vertex coloring then the vertex set has the partition $(v^1, v^2, \ldots, v^k)$ where $v^i$ denotes all the vertices of $V$ which are colored with the color $i$, each $v^i$ is a fuzzy independent set.

Definition 2.6. A fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is $k$-fuzzy soft colorable if $G^S(T, V)$ has a proper $k$-fuzzy soft vertex coloring.

Definition 2.7. The fuzzy chromatic number $\chi^S_f(G^S(T, V)), e \in T$ of a fuzzy soft graph $G^S(T, V) = ((T, \alpha), (T, \beta))$ is the minimum $k$ for which $G^S(T, V)$ is $k$-fuzzy soft colorable. $G^S(T, V)$ is said to be $k$-fuzzy chromatic if $\chi^S_f(G^S(T, V)) = k$.

Remark. It is customary to abbreviate a proper fuzzy soft vertex coloring as fuzzy soft coloring, a proper $k$-fuzzy soft vertex coloring as $k$-fuzzy soft coloring and $k$-fuzzy soft vertex colorable as $k$-fuzzy soft colorable.

3. Fuzzy Chromatic Number of Fuzzy Soft Sub Graphs

In a crisp graph, for any sub graph $H$ of $G$, $\chi(H) \leq \chi(G)$. But in fuzzy soft graph, it is not satisfied in general. Let $G^S(T, V)$ be fuzzy soft graph and $H^S(T, V)$ be a fuzzy soft sub graph of $G^S(T, V)$. 

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Case (i). If an edge \((x, y)\) is strong (weak) in \(G^S(T, V)\) and also the same in \(H^S(T, V)\) then the condition \(\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))\) is satisfied, since there is no change in minimum number of fuzzy independent sets.

Case (ii). If a strong arc in \(G^S(T, V)\) becomes weak arc in \(H^S(T, V)\) then the condition \(\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))\) is satisfied. Since the minimum number of fuzzy independent sets of \(H^S(T, V)\) is less than or equal to that of \(G^S(T, V)\).

Case (iii). If a weak arc in \(G^S(T, V)\) becomes strong arc in \(H^S(T, V)\) then the condition \(\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))\) may not be satisfied, since the minimum number of fuzzy independent sets of \(H^S(T, V)\) may not be less than or equal to that of \(G^S(T, V)\).

Thus we conclude that, the condition \(\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))\) is satisfied if a strong arc in \(G^S(T, V)\) becomes strong(weak) in \(H^S(T, V)\). But a weak arc in \(G^S(T, V)\) becomes strong arc in \(H^S(T, V)\), then the condition \(\chi_f^e(H^S(T, V)) \leq \chi_f^e(G^S(T, V))\) may not be satisfied.

4. Fuzzy Chromatic Number of Fuzzy Soft Bipartite Graphs

Theorem 4.1. A fuzzy soft graph \(G^S(T, V) = ((T, \alpha), (T, \beta))\) is 2-fuzzy soft colorable iff \(G^S(T, V)\) is fuzzy soft bipartite graph.

Proof. Suppose \(G^S(T, V)\) is 2-fuzzy soft colorable.

We have to show that \(G^S(T, V)\) is fuzzy soft bipartite.

Since \(G^S(T, V)\) is 2-fuzzy soft colorable,

Let \(V^*_1\) and \(V^*_2\) be the 2 partitions such that \(\forall e \in T, V^*_1\) is the set of all
vertices of \( G^S(T, V) \) which receive the color 1 and \( V^e_2 \) is the set of all vertices of \( G^S(T, V) \) which receive the color 2.

Then every strong arc in \( G^S(T, V) \) has one end in \( V^e_1 \) and other end in \( V^e_2 \).

Thus \( V \) can be partitioned into 2 fuzzy independent sets \( V^e_1 \) and \( V^e_2 \).

Thus \( G^S(T, V) \) is fuzzy soft bipartite graph.

Conversely, Suppose \( G^S(T, V) \) is fuzzy soft bipartite.

Then \( G^S(T, V) \) can be partitioned into 2 fuzzy independent sets \( V^e_1 \) and \( V^e_2 \), since every strong arc in \( G^S(T, V) \) has one end in \( V^e_1 \) and other end in \( V^e_2 \).

So, we can assign the color 1 to all the vertices in \( V^e_1 \) and color 2 to all vertices in \( V^e_2 \).

Hence \( G^S(T, V) \) is 2-fuzzy soft colorable.

The generalization of theorem can be stated as follows:

“A fuzzy soft graph \( G^S(T, V) = ((T, \alpha), (T, \beta)) \) is fuzzy soft \( k \)-partite iff \( G^S(T, V) \) is \( k \)-fuzzy soft colorable.”

Corollary 4.2. Let \( G^S(T, V) \) be a fuzzy soft graph such that \( G^* = (\mu^*, \rho^*) \) is \( P^e_p \), \( p \geq 2 \), where \( P^e_p \) path of length \( p \). Then \( \chi_e^f(G^S(T, V)) = 2 \).

Proof. Clearly every arc in \( G^S(T, V) \) is strong.

So vertex set can be partitioned into 2 sets \( V^e_1 \) and \( V^e_2 \), \( e \in T \) such that every strong arc has one end in \( V^e_1 \) and other end in \( V^e_2 \).
Thus $\chi^f_s(G^S(T, V)) = 2$.

**Corollary 4.3.** For a non-trivial fuzzy soft tree (forest), $\chi^f_s(G^S(T, V)) = 2$.

**Proof.** Since fuzzy soft tree (forest) is fuzzy soft bipartite. Therefore $\chi^f_s(G^S(T, V)) = 2$.

**Conclusion**

In this paper, the fuzzy chromatic number of fuzzy soft graphs, fuzzy soft sub graphs, fuzzy soft bipartite graphs and fuzzy trees are studied. We further extend this study on some more types of fuzzy soft graphs.

**References**


