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FUZZY $\mathcal{Z}$ CONTINUOUS MAPPINGS IN DOUBLE FUZZY TOPOLOGICAL SPACES

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Abstract

In this paper we introduce double fuzzy δ-continuous, double fuzzy δ-bicontinuous, double fuzzy $\mathcal{Z}$-continuous functions and study some of their properties in double fuzzy topological spaces.

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1. Introduction

Intuitionistic fuzzy sets were first introduced by Atanassov [1] in 1993, then Coker [2] introduced the notion of Intuitionistic fuzzy topological space in 1997. In 2005, Garcia and Rodabaugh [4] proved that the term intuitionistic is unsuitable in mathematics and applications and they introduced the name double for the term intuitionistic. In the past two decades many researchers [8, 9, 15] doing more applications on double fuzzy topological spaces. From 2011, El-Maghrabi and Mubarki [6] introduced and studied some properties of $\mathcal{Z}$-open sets and maps in topological spaces.

2. Preliminaries

Throughout this paper, X will be a non-empty set, I is the closed unit interval $[0, 1]$, $I_0 = (0, 1]$, $I_1 = [0, 1)$, $\iota \in I_0$, $\kappa \in I_1$ and always $\iota + \kappa \leq 1$. A fuzzy set μ is quasi-coincident with a fuzzy set υ denoted by $\mu q \upsilon$ iff there exists $x \in X$ such that $\mu(x) + \upsilon(x) > 1$ and otherwise they are not quasi-coincident which denoted by $\mu \bar{q} \upsilon$. The family of all fuzzy sets on X (resp. Y and Z) is denoted by I^X (resp. I^Y and I^Z). By $\underline{0}$ and $\underline{1}$, we denote the smallest and the largest fuzzy sets on X. For a fuzzy set $\mu(x) \in I^X$, $\underline{1} - \mu(x)$ denotes its complement. For $x \in X$, $\iota \in I_0$, a fuzzy point x_ι is defined by $x_r(y) = \iota$ if $x = y$ for all other y, $x_r(y) = 0$. All other notations are standard notations of fuzzy set theory.

Definition 2.1 [12]**.** A double fuzzy topology (τ, τ^*) on X is a pair of maps $\tau, \tau^* : I^X \to I$, which satisfies the following properties:

1. $\tau(\lambda) \leq \underline{1} - \tau^*(\lambda)$ for each $\lambda \in I^X$.

2. $\tau(\lambda_1 \wedge \lambda_1) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.

3. $\tau(\vee_{i \in \Gamma} \lambda_i) \leq \wedge_{i \in \Gamma} \tau(\lambda_i)$ and $\tau^*(\vee_{i \in \Gamma} \lambda_i) \leq \vee_{i \in \Gamma} \tau^*(\lambda_i)$ for each $\lambda_i \in I^X$, $i \in \Gamma$.

The triplet (X, τ, τ^*) is called a double fuzzy topological space (briefly, *dfts*). A fuzzy setλ is called an (ι, κ)- fuzzy open (briefly (ι, κ)- fo) set if $\tau(\lambda) \geq \iota$ and $\tau^*(\lambda) \leq \kappa$, λ is called an (ι, κ)- fuzzy closed (briefly (ι, κ)- *fc*) set iff $\underline{1} - \lambda$ is an (ι, κ)- *fo* set.

Definition 2.2 [5]**.** Let (X, τ, τ^*) be a *dfts*. Then double fuzzy interior and double fuzzy closure operators are defined from $I^X \times I_0 \times I_1 \to I^X$ as follows: $I_{\tau, \tau^*}(\lambda, i, \kappa) = \vee\{\mu \in I^X \mid \mu \leq \lambda, \tau(\mu) \geq i, \tau^*(\mu) \leq \kappa\}$,

$$C_{\tau, \tau^*}(\lambda, i, \kappa) = \wedge\{\mu \in I^X \mid \mu \geq \lambda, \tau(\underline{1} - \mu) \geq i, \tau^*(\underline{1} - \mu) \leq \kappa\},$$

where $i \in I_0$, and $\kappa \in I_1$ such that $i + \kappa \leq 1$.

Definition 2.3 [11]**.** Let (X, τ, τ^*) be a *dfts*. Then for each $i \in I_0$, $\kappa \in I_1$, a fuzzy set $\lambda \in I^X$, is said to be

1. (ι, κ)- fuzzy regular open (briefly (i, κ)-fro) set if $\lambda = I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$.

2. (ι, κ)- fuzzy regular closed (briefly (ι, κ)- frc) set iff $\underline{1} - \lambda$ is (ι, κ)- fro set.

Definition 2.4 [10]**.** Let (X, τ, τ^*) be a dfts. Then for each $i \in I_0$, $\kappa \in I_1$, and for fuzzy set $\lambda \in I^X$, we define the operators $\delta C_{\tau, \tau^*}$ and $\delta I_{\tau, \tau^*} : I^X \times I_0 \times I_1 \to I^X$ as follows

$$\delta I_{\tau, \tau^*}(\lambda, i, \kappa) = \vee\{\mu \in I^X \mid \mu \leq \lambda, \mu \text{ is an } (i, \kappa) - fro\}$$

$$\delta C_{\tau, \tau^*}(\lambda, i, \kappa) = \wedge\{\mu \in I^X \mid \mu \leq \lambda, \mu \text{ is an } (i, \kappa) - fro\}.$$

Definition 2.5 [3, 6, 10]**.** Let (X, τ, τ^*) be a *dfts*. Then for each $i \in I_0$, $\kappa \in I_1$, a fuzzy set $\lambda \in I^X$, is said to be

1. (ι, κ)- fuzzyδ open (briefly (ι, κ)- $f\delta$ set if $\lambda = \delta I_{\tau, \tau^*}(\lambda, i, \kappa)$.

2. (ι, κ)- fuzzy pre open (resp. (ι, κ)- fuzzy semi open) (briefly (i, κ)-fpo (resp. (i, κ)-fso)) set if $\lambda \leq I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$ (resp. $\lambda \leq C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa))$

3. (ι, κ)- fuzzy δ pre open (resp. (ι, κ)- fuzzy δ semi open, (ι, κ)- fuzzy b open, (ι, κ)- fuzzy Z open [14] and (ι, κ)- fuzzy e open) (briefly (i, κ)-$f\delta po$ (resp. (i, κ)-$f\delta so$, (i, κ)-fbo, (i, κ)-fZo and (i, κ)-feo)) set if $\lambda \leq I_{\tau, \tau^*}(\delta C_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$ (resp. $\lambda \leq C_{\tau, \tau^*}(\delta I_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$, $\lambda \leq C_{\tau, \tau^*}(I_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa) \vee I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$, $\lambda \leq C_{\tau, \tau^*}(\delta I_{\tau, \tau^*}(\lambda, i, \kappa)$, $i, \kappa) \vee I_{\tau, \tau^*}(C_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$ and $\lambda \leq C_{\tau, \tau^*}(\delta I_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa) \vee I_{\tau, \tau^*}$ $(\delta C_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa)$.

4. (ι, κ)- fuzzy δ pre closed (resp. (ι, κ)- fuzzy pre closed, (ι, κ)- fuzzy semi closed, (ι, κ)- fuzzy δ semi closed, (ι, κ)- fuzzy b closed, (ι, κ)- fuzzy Z closed and (ι, κ)- fuzzy e closed) (briefly (i, κ)-δpc (resp. (i, κ)-fpc, (i, κ)-fsc, (i, k)-$f\delta sc$, (i, κ)-fbc, (i, κ)-$f Zc$ and (i, κ)- fec)) set if $\underline{1} - \lambda$ is an (i, κ)-$f\delta po$ (resp. (i, κ)-fpo, (i, κ)-fso, (i, κ)-fbo, (i, κ)-fZo and (i, κ)-feo).

Definition 2.6 [10, 14]**.** Let (X, τ, τ^*) be a *dfts*. Then for each $i \in I_0$, $\kappa \in I_1$ and for fuzzy set $\lambda \in I^X$, we define the operators $\delta PC_{\tau, \tau^*}$ (resp. ZC_{τ, τ^*} and eC_{τ, τ^*}) and $\delta PI_{\tau, \tau^*}$ (resp. ZI_{τ, τ^*} and eI_{τ, τ^*}) : $I^X \times I_0 \times I_1 \to I^X$ as follows: $\delta PI_{\tau, \tau^*}$ (resp. ZI_{τ, τ^*} and $eI_{\tau, \tau^*})(\lambda, i, \kappa) = \vee\{\mu \in I^X \mid \mu \leq \lambda, \mu$ is an (i, κ)-$f\delta po$ (resp. (i, κ)-fZo and (i, κ)-feo)$\}$, $\delta PC_{\tau, \tau^*}$ (resp. ZC_{τ, τ^*} and $eC_{\tau, \tau^*})(\lambda, i, \kappa) = \wedge\{\mu \in I^X \mid \mu \geq \lambda, \mu$ is an (i, κ)-$f\delta pc$ (resp. (i, κ)-fZc and (i, κ)-fec)$\}$.

Definition 2.7 [14]. Let (X, τ, τ^*) be a $dfts$, $\lambda \in I^X$, $i \in I_0$ and $\kappa \in I_1$, λ is called an (i, κ)- fuzzy $Z - Q$- neighborhood of $x_t \in P_t(X)$ if there exists an (i, κ)-fZo set $\mu \in I^X$ such that $x_t q \mu$ and $\mu \leq \lambda$. The family of all (i, κ)- fuzzy $Z - Q$- neighborhood of x_t is denoted by ZQ- (x_t, i, κ).

Proposition 2.1 [14]. *Let* (X, τ, τ^*) *be a* $dfts$, $\lambda, \mu \in I^X$, *then*

1. $ZI_{\tau, \tau^*}(\underline{0}, i, \kappa) = \underline{0}$, $ZC_{\tau, \tau^*}(\underline{0}, i, \kappa) = \underline{0}$ *and* $ZI_{\tau, \tau^*}(\underline{1}, i, \kappa) = \underline{1}$, $ZC_{\tau, \tau^*}(\underline{1}, i, \kappa) = \underline{1}$.
2. $\underline{1} - ZI_{\tau, \tau^*}(\lambda, i, \kappa) = ZC_{\tau, \tau^*}(\underline{1} - \lambda, i, \kappa)$ *and* $\underline{1} - ZC_{\tau, \tau^*}(\lambda, i, \kappa) = ZI_{\tau, \tau^*}(\underline{1} - \lambda, i, \kappa)$.
3. *If* $\lambda < \mu$ *then* $ZI_{\tau, \tau^*}(\lambda, i, \kappa) < ZI_{\tau, \tau^*}(\mu, i, \kappa)$ *and* $ZC_{\tau, \tau^*}(\lambda, i, \kappa) < ZC_{\tau, \tau^*}(\mu, i, \kappa)$.
4. $ZC_{\tau, \tau^*}(\lambda \vee \mu, i, \kappa) > ZC_{\tau, \tau^*}(\lambda, i, \kappa) \vee ZC_{\tau, \tau^*}(\mu, i, \kappa)$.
5. $ZI_{\tau, \tau^*}(\lambda \vee \mu, i, \kappa) > ZI_{\tau, \tau^*}(\lambda, i, \kappa) \vee ZI_{\tau, \tau^*}(\mu, i, \kappa)$.
6. $ZC_{\tau, \tau^*}(\lambda \wedge \mu, i, \kappa) < ZC_{\tau, \tau^*}(\lambda, i, \kappa) \wedge ZC_{\tau, \tau^*}(\mu, i, \kappa)$.
7. $I_{\tau, \tau^*}(\lambda \wedge \mu, i, \kappa) < ZI_{\tau, \tau^*}(\lambda, i, \kappa) \wedge ZI_{\tau, \tau^*}(\mu, i, \kappa)$.

The operators $\delta I_{\tau, \tau^*}(\lambda, i, \kappa)$ and $\delta sI_{\tau, \tau^*}(\lambda, i, \kappa)$ satisfy the above properties.

Proposition 2.2 [14]. *Let* (X, τ, τ^*) *be a* $dfts$, $\lambda, \mu \in I^X$, *then*

1. $\lambda \leq ZC_{\tau, \tau^*}(\lambda, i, \kappa) < C_{\tau, \tau^*}(\lambda, i, \kappa) \leq \delta C_{\tau, \tau^*}(\lambda, i, \kappa)$.

2. $\delta I_{\tau, \tau^*}(\lambda, i, \kappa) = I_{\tau, \tau^*}(\lambda, i, \kappa) \leq ZI_{\tau, \tau^*}(\lambda, i, \kappa) \leq \lambda$.

Theorem 2.1 [14]. *Let* (X, τ, τ^*) *be a dfts, for each* $\lambda, \mu \in I^X$, *then the operator* (i, κ)-ZC_{τ, τ^*} *satisfies the following statements*

1. $ZC_{\tau, \tau^*}(ZC_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa) = ZC_{\tau, \tau^*}(\lambda, i, \kappa)$.

2. *If* λ *is* (i, κ)-fZc *set then* $ZC_{\tau, \tau^*}(\lambda, i, \kappa) = \lambda$.

3. *If* μ *is* (i, κ)-fZo *set then* $\mu q \lambda$ *iff* $\mu q ZC_{\tau, \tau^*}(\lambda, i, \kappa)$.

Theorem 2.2 [14]. *Let* (X, τ, τ^*) *be a dfts, for each* $\lambda, \mu \in I^X$, *then the operator* (i, κ)-ZI_{τ, τ^*} *satisfies the following statements*

1. $ZI_{\tau, \tau^*}(ZI_{\tau, \tau^*}(\lambda, i, \kappa), i, \kappa) = ZI_{\tau, \tau^*}(\lambda, i, \kappa)$.

2. *If* λ *is* (i, κ)-fZo *set then* $ZI_{\tau, \tau^*}(\lambda, i, \kappa) = \lambda$.

3. *If* $\lambda \leq \mu$ *then* $ZI_{\tau, \tau^*}(\lambda, i, \kappa) \leq ZI_{\tau, \tau^*}(\mu, i, \kappa)$.

4. $ZI_{\tau, \tau^*}(1 - \lambda, i, k) = \underline{1} - ZC_{\tau, \tau^*}(\lambda, i, \kappa)$ *and* $ZC_{\tau, \tau^*}(\underline{1} - \lambda, i, \kappa) = \underline{1} - ZC_{\tau, \tau^*}(\lambda, i, \kappa)$.

Definition 2.8 [13]. A function *f* from a $dfts$ (X, τ, τ^*) to a $dfts$ (Y, σ, σ^*) is called as double fuzzy continuous (resp. double fuzzy δ pre continuous, double fuzzy δ semi continuous, double fuzzy semi continuous [7], double fuzzy *Z* continuous and double fuzzy *e* continuous) (briefly $DFCts$, (resp. $DF\delta pCts$, $DF\delta sCts$, $DFsCts$, $DFMCts$ and $DFeCts$)) function if $f^{-1}(\mu)$ is an (i, κ)-fc (resp. (i, κ)-$f\delta pc$, (i, κ)-$f\delta sc$, (i, κ)-fsc, (i, κ)-fZc and (i, κ)-fec) set in I^X for every (i, κ)-fc set $\mu \in I^Y$ for all $i \in I_0$ and $\kappa \in I_1$.

Definition 2.9 [13]. A fuzzy set λ in a $dfts$ (X, τ, τ^*) is called an (i, κ)-fuzzy dense (resp. (i, κ)- fuzzy nowhere dense) if there exists no (i, κ)-fo (resp. non-zero (i, κ)-fo) set μ in (X, τ, τ^*) such that $\lambda < \mu < \underline{1}$ (resp. $\mu < C_{\tau, \tau^*}(\lambda, i, \kappa)$).

Lemma 2.1 [13]. *For a* $dfts$ (X, τ, τ^*) *every* (i, κ)- *fuzzy dense set is* (i, κ)-$f\delta po$.

3. A Double Fuzzy $\mathcal{Z}$ Continuous Functions

In this section we introduce the class of double fuzzy δ continuous and double fuzzy $\mathcal{Z}$ continuous functions and discuss about their properties.

Definition 3.1. A function f from a $dfts$ (X, τ, τ^*) to a $dfts$ (Y, σ, σ^*) is called as double fuzzy Z continuous (resp. double fuzzy δ continuous, double fuzzy pre continuous and double fuzzy b continuous) (briefly, $DFZCts$, (resp. $DF\delta Cts$, $DFpCts$ and $DFbCts$)) function if $f^{-1}(\mu)$ is an (i, κ)-fZc (resp. (i, k)-$f\delta c$, (i, k)-fpc and (i, k)-fbc) set in I^X for every (i, k)-fc set $\mu \in I^Y$ for all $i \in I_0$ and $\kappa \in I_1$.

Theorem 3.1. *Let* $f : (X, \tau, \tau^*) \to (X\ \eta, \eta^*)$ *be a mapping.* 1. *Every* $DF\ \delta Cts$ *function is* $DFCts$ *(resp.* $DF\ \delta pCts$ *and* $DF\ \delta sCts$ *) function.* 2. *Every* $DFCts$ *function is* $DFsCts$ *(resp.* $DFpCts$ *) function.* 3. *Every* $DF\ \delta pCts$ *function is* $DFeC\ ts$ *function.* 4. *Every* $DF\ \delta sCts$ *function is* $DFeCts$ *(resp.* $DFZCts$ *) function.* 5. *Every* $DFpCts$ *function is* $DFZCts$ *function.* 6. *Every* $DFsCts$ *function is* $DFbCts$ *function.* 7. *Every* $DFZCts$ *function is* $DFeCts$ *(resp.* $DFbCts$ *) function.*

Proof. We prove only (i), the others are similar. Let α be a (i, κ)-fo in I^Y.

By definition of $DF\ \delta Ctsf^{-1}(\alpha)$ is (i, α)-$f\delta o$ in I^X. By Theorem 3.4 in [14], $f^{-1}(\alpha)$ is (i, κ)-fo in I^X. Which implies f is $DFCts$.

Remark 3.1. The converse of the above theorem, in general, need not be true. It can be verified from the following examples.

Example 3.1. Let $X = Y = \{a, b, c\}$ and let the fuzzy sets $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ and α_7 are defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.5$, $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.9$, $\alpha_2(c) = 0.5$; $\alpha_3(a) = 0.2$, $\alpha_3(b) = 0.2$, $\alpha_3(c) = 0.2$, $\alpha_4(a) = 0.4$, $\alpha_4(b) = 0.4$, $\alpha_4(c) = 0.5$; $\alpha_5(a) = 0.5$, $\alpha_5(b) = 0.5$, $\alpha_5(c) = 0.5$, $\alpha_6(a) = 0.2$, $\alpha_6(b) = 0.4$, $\alpha_6(c) = 0.4$ and $\alpha_7(a) = 0.3$, $\alpha_7(b) = 0.0$, $\alpha_7(c) = 0.4$.

Consider the double fuzzy topologies (X, τ, τ^*), (Y, η_1, η_1^*), (Y, η_2, η_2^*), (Y, η_3, η_3^*), (Y, η_4, η_4^*), (Y, η_5, η_5^*) and (Y, η_6, η_6^*) with

$$\tau(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda\in\{\alpha_1,\alpha_2\}\\ 0, & \text{o.w.}\end{cases}\quad \tau^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda\in\{\alpha_1,\alpha_2\}\\ 1, & \text{o.w.},\end{cases}$$

$$\eta_1(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_2\\ 0, & \text{o.w.}\end{cases}\quad \eta_1^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_2\\ 1, & \text{o.w.},\end{cases}$$

$$\eta_2(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_3\\ 0, & \text{o.w.}\end{cases}\quad \eta_2^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_3\\ 1, & \text{o.w.},\end{cases}$$

$$\eta_3(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_4\\ 0, & \text{o.w.}\end{cases}\quad \eta_3^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_4\\ 1, & \text{o.w.},\end{cases}$$

$$\eta_4(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_5\\ 0, & \text{o.w.}\end{cases}\quad \eta_4^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_5\\ 1, & \text{o.w.},\end{cases}$$

$$\eta_5(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_6\\ 0, & \text{o.w.}\end{cases}\quad \eta_5^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_6\\ 1, & \text{o.w.},\end{cases}$$

$$\eta_6(\lambda)=\begin{cases}1 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_7\\ 0, & \text{o.w.}\end{cases}\quad \eta_6^*(\lambda)=\begin{cases}0 & \text{if } \lambda\in\{\underline{0},\underline{1}\}\\ \frac{1}{2}, & \text{if } \lambda=\alpha_7\\ 1, & \text{o.w.},\end{cases}$$

Then the identity function (i) $f:(X,\tau,\tau^*)\to(X\ \eta_1,\eta_1^*)$ is a (i) $DFCts$ (resp. $DF\delta pCts$) function but not a $DF\delta Cts$, (ii) $DFZCts$ but not a $DF\delta sCts$. Since the inverse image of the fuzzy set α_2 is an $\left(\frac{1}{2},\frac{1}{2}\right)$-$fo$, $\left(\frac{1}{2},\frac{1}{2}\right)$-$f\delta po$ and $\left(\frac{1}{2},\frac{1}{2}\right)$-$fZo$ set but not an $\left(\frac{1}{2},\frac{1}{2}\right)$-$f\delta o$ and $\left(\frac{1}{2},\frac{1}{2}\right)$-$f\delta o$, (ii) $f:(X,\tau,\tau^*)\to(X\ \eta_2,\eta_2^*)$ is a $DFpCts$ function but not a $DFCts$, since the inverse image of the fuzzy set α_3 is an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fpo$ set but not an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$go$ (iii) $f : (X, \tau, \tau^*) \to (X\ \eta_3, \eta_3^*)$ is a $DFsCts$ (resp. $DF\delta sCts$) function but not a $DFCts$, since the inverse image of the fuzzy set α_4 is an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fso$ (resp. $\left(\frac{1}{2}, \frac{1}{2}\right)$-$f\delta o$) set but not an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fo$ (iv) $f : (X, \tau, \tau^*) \to (Y\ \eta_4, \eta_4^*)$ is a $DFZCts$ function but not a $DFpCts$, since the inverse image of the fuzzy set α_5 is an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fZo$ set but not an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fpo$ (v) $f : (X, \tau, \tau^*) \to (Y\ \eta_5, \eta_5^*)$ is a $DFbCts$ function but not a $DFsCts$, since the inverse image of the fuzzy set α_6 is an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fbo$ set but not an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fso$ and (vi) $f : (X, \tau, \tau^*) \to (X\ \eta_6, \eta_6^*)$ is a $DFeCts$ function but not a $DFZCts$, since the inverse image of the fuzzy set α_7 is an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$feo$ set but not an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fZo$ set in (X, τ, τ^*).

Example 3.2. Let $X = Y = \{a, b, c\}$ and let the fuzzy sets α_8, α_9 and α_{10} are defined as $\alpha_8(a) = 0.7, \alpha_8(b) = 0.6, \alpha_8(c) = 0.5, \alpha_9(a) = 0.3, \alpha_9(b) = 0.3, \alpha_9(c) = 0.3$ and $\alpha_{10}(a) = 0.3, \alpha_{10}(b) = 0.4, \alpha_{10}(c) = 0.5$. Consider the double fuzzy topologies (X, τ, τ^*) and $(Y\ \eta, \eta^*)$ with

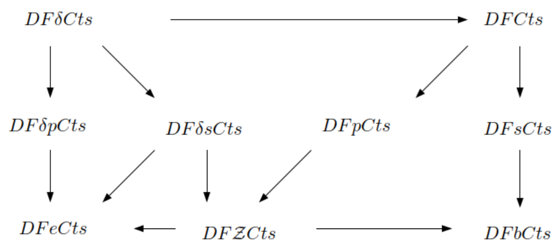
$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda \in \{\alpha_8, \alpha_9\} \\ 0, & \text{o.w.} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda \in \{\alpha_8, \alpha_9\} \\ 1, & \text{o.w.} \end{cases}$$

$$\eta(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \alpha_{10} \\ 0, & \text{o.w.} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{2}, & \text{if } \lambda = \alpha_{10} \\ 1, & \text{o.w.} \end{cases}$$

Then the identity function $f : (X, \tau, \tau^*) \to (Y\ \eta, \eta^*)$ is a $DFvCts$ function but not a $DFZCts$, since the inverse image of the fuzzy set α_{10} is an

$\left(\frac{1}{2}, \frac{1}{2}\right)$-$fbo$ set but not an $\left(\frac{1}{2}, \frac{1}{2}\right)$-$fZo$.

From the above theorem and examples, the following implications are hold.



Note: $A \to B$ denotes A implies B, but not conversely.

Definition 3.2. A mapping $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ is called $DFZCts$ at a fuzzy point x_r if the inverse image of each (i, κ)-Q neighbourhood of $f(x_r)$ is an (i, κ)-ZQ neighbourhood of $x_r \in I^X$.

Theorem 3.2. *A mapping* $f : (X, \tau, \tau^*) \to (X, \eta, \eta^*)$ *is* $DFZCts$ *iff it is* $DFZCts$ *at every fuzzy point* $x_r \in I^X$.

Theorem 3.3. *Let* (X, τ, τ^*) *and* (Y, η, η^*) *be dfts's and* $f : (X, \tau, \tau^*) \to (Y, \eta, \eta^*)$ *be a mapping. Then the following statements are equivalent*:

1. *f is* $DFZCts$ *function*. 2. $f^{-1}(\lambda)$ *is an* (i, κ)-fZo *set in* (X, τ, τ^*) *for each* (i, κ)-fo *set* λ *in* (Y, η, η^*) 3. $f^{-1}(\lambda)$ *is an* (i, κ)-fZc *set in* (X, τ, τ^*) *for each* (i, κ)-fc *set* λ *in* (Y, η, η^*). 4. $f(ZC_{\tau, \tau^*}(\lambda, i, \kappa)) \leq C_{\eta, \eta^*}(f(\lambda), i, \kappa)$, $\forall \lambda \in I^X$. 5. $ZC_{\tau, \tau^*}(f^{-1}(\lambda), i, \kappa) \leq f^{-1}(C_{\eta, \eta^*}(\lambda, i, \kappa))$, $\forall \lambda \in I^Y$. 6. $I_{\tau, \tau^*}(\delta C_{\tau, \tau^*}(f^{-1}(\lambda), i, \kappa), i, \kappa) \wedge C_{\tau, \tau^*}(I_{\tau, \tau^*}(f^{-1}(\lambda), i, \kappa), i, \kappa) \leq f^{-1}(C_{\eta, \eta^*}(\lambda, i, \kappa))$, $\forall \lambda \in I^Y$. 7. $f^{-1}(I_{\eta, \eta^*}(\lambda, i, k)) \leq ZI_{\tau, \tau^*}(f^{-1}(\lambda), i, \kappa)$, *for each* $\lambda \in I^Y$. 8.

$f^{-1}(I_{\eta,\eta^*}(\mu, i, \kappa)) \leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(f^{-1}(\mu), i, \kappa), i, \kappa) \vee I_{\tau,\tau^*}(C_{\tau,\tau^*}(f^{-1}(\mu), i, \kappa), i, \kappa)$

for each $\mu \in I^Y$.

Proof. (i) ⇒ (iii), (ii) ⇒ (i), (ii) ⇒ (viii), (iii) ⇒ (i), (v) ⇒ (vii) and (viii) ⇒ (ii) are direct to prove, other results are provided here.

(i) ⇒ (ii). Let λ be an (i, κ)-fo set in (Y, η, η^*), f is a DF $ZCts$ function, then we have $f^{-1}(\underline{1} - \lambda)$ is an (i, κ)-fZc set of (X, τ, τ^*). But $f^{-1}(\underline{1} - \lambda) = \underline{1} - f^{-1}(\lambda)$. Therefore $f^{-1}(\lambda)$ is an (i, κ)-fZo set of (X, τ, τ^*).

(iii) ⇒ (iv). Let $\lambda \in I^X$, since $\eta(I_{\eta,\eta^*}(f(\lambda), i, \kappa)) \geq i, \eta^*(I_{\eta,\eta^*}(f(\lambda), i, \kappa)) \leq \kappa$. Then by (iii), $f^{-1}(C_{\eta,\eta^*}(f(\lambda), i, \kappa))$ is an (i, κ)-fZc set of (X, τ, τ^*). Since $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(C_{\eta,\eta^*}(f(\lambda), i, \kappa))$, we have $ZC_{\tau,\tau^*}(\lambda, i, \kappa) \leq f^{-1}(C_{\eta,\eta^*}(f(\lambda), i, \kappa))$. Hence $f(ZC_{\tau,\tau^*}(\lambda, i, \kappa) \leq C_{\eta,\eta^*}(f(\lambda), i, \kappa)$.

(iv) ⇒ (v). For all $\lambda \in I^Y$ let $f^{-1}(\lambda)$ instead of λ in (iv), we have $f(ZC_{\tau,\tau^*}(f^{-1}(\lambda), i, \kappa)) \leq C_{\eta,\eta^*}(f(f^{-1}(\lambda)), i, \kappa) \leq C_{\eta,\eta^*}(\lambda, i, \kappa)$. It implies that $ZC_{\tau,\tau^*}(f^{-1}(\lambda), i, \kappa)) \leq f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa))$.

(vii) ⇒ (i). Let λ be an (i, κ)-fc set in (Y, η, η^*). Then $\underline{1} - \lambda = I_{\eta,\eta^*}(\underline{1} - \lambda, i, \kappa)$. By (vii), $f^{-1}(1 - \lambda) \leq ZI_{\tau,\tau^*}(f^{-1}(\underline{1} - \lambda), i, \kappa)$. But we know that $f^{-1}(\underline{1} - \lambda) \leq ZI_{\tau,\tau^*}(f^{-1}(\underline{1} - \lambda), i, \kappa)$. Thus, $f^{-1}(\underline{1} - \lambda) = ZI_{\tau,\tau^*}(f^{-1}(\underline{1} - \lambda), i, \kappa)$, that is, $f^{-1}(\underline{1} - \lambda)$ is (i, κ)-fZo set. Since $f^{-1}(\underline{1} - \lambda) = \underline{1} - f^{-1}(\lambda)$, $f^{-1}(\lambda)$ is (i, κ)-fZc set. Therefore *f* is $DFZc$ function.

(iii) ⇒ (vi): For all $\lambda \in I^Y$, since $C_{\eta,\eta^*}(\lambda, i, \kappa)$ is an (i, κ)-fc set in (Y, η, η^*), by (iii), we have that $f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa))$ is an (i, κ)-fZc set in (X, τ, τ^*). Hence $f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa)) \geq I_{\tau,\tau^*}(\delta C_{\eta,\eta^*}(f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa)), i, \kappa), i, \kappa)$

$\wedge C_{\eta,\eta^*}(I_{\tau,\tau^*}(f^{-1}(\lambda, i, \kappa)), i, \kappa) \geq I_{\tau,\tau^*}(\delta C_{\eta,\eta^*}(f^{-1}(\lambda), i, \kappa) \wedge C_{\eta,\eta^*}(I_{\tau,\tau^*}(f^{-1}(\lambda), i, \kappa), i, \kappa).$

(vi) $\Rightarrow$ (iii). For all $\lambda \in I^Y$, since $C_{\eta,\eta^*}(\lambda, i, \kappa)$ is an (i, κ)-fc set in (Y, η, η^*), and let $C_{\eta,\eta^*}(\lambda, i, \kappa)$ instead of λ in (vi), we have that $I_{\tau,\tau^*}(\delta C_{\eta,\eta^*}(f^{-1}(\delta C_{\eta,\eta^*}(\lambda, i, \kappa)), i, \kappa) \wedge C_{\tau,\tau^*}(I_{\tau,\tau^*}(f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa)), i, \kappa), i, \kappa)$ $\leq f^{-1}(C_{\eta,\eta^*}(C_{\eta,\eta^*}(\lambda, i, \kappa), i, \kappa) = f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa))$. Hence $f^{-1}(C_{\eta,\eta^*}(\lambda, i, \kappa))$ is an (i, κ)-fZc set in (X, τ, τ^*).

Proposition 3.1. *Let $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ DFZCts mapping and if for any fuzzy subset λ of X is (i, κ)- fuzzy nowhere dense then f is $DF\delta pCts$.*

Proof. Let $\tau_2 \geq i, \tau_2^*(\mu) \leq \kappa$. Since f is an $DFZCts$ mapping, then $f^{-1}(\mu)$ is an (i, κ)-fZo set in (X, τ_1, τ_1^*). Put $f^{-1}(\mu) = \lambda$ is an (i, κ)-fZo set in X. Hence $\lambda \leq C_{\tau,\tau^*}(\delta I_{\tau,\tau^*}(\lambda, i, \kappa), i, \kappa) \vee I_{\tau,\tau^*}(\lambda, i, \kappa), i, \kappa)$. But $\delta I_{\tau,\tau^*}(\lambda, i, \kappa) \leq I_{\tau,\tau^*}(\lambda, i, \kappa) \leq C_{\tau,\tau^*}(\lambda, i, \kappa)$, then $\delta I_{\tau,\tau^*}(\lambda, i, \kappa) \leq I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda, i, \kappa), i, \kappa)$. Since λ is (i, κ)- fuzzy nowhere dense and by Lemma 2.1, we have $\delta I_{\tau,\tau^*}(\lambda, i, \kappa) = \underline{0}$. Therefore f is $DF\delta pCts$.

Definition 3.3. A mapping $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is called double fuzzy δ-open map (briefly $DF\delta O$) if the image of every (i, κ)-fo set of (X, τ_1, τ_1^*) is (i, κ)-$f\delta o$ set in (Y, τ_2, τ_2^*).

Definition 3.4. A mapping $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ is called double fuzzy δ-bicontinuous (briefly, $DF\delta biCts$) if f is $DF\delta o$ map and $DF\delta Cts$ map.

Theorem 3.4. *If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be a $DF\delta biCts$ mapping then the inverse image of each (i, κ)-fZo set in (Y, τ_2, τ_2^*) under f is (i, κ)-fZo set in (X, τ_1, τ_1^*).*

Proof. Let f be a $DF\,\delta biCts$ and μ be a (i, κ)-fZo set in (Y, τ_2, τ_2^*). Then $\mu \leq C_{\tau_2, \tau_2^*}(\delta I_{\tau_2, \tau_2^*}(\mu, i, \kappa), i, \kappa) \vee I_{\tau_2, \tau_2^*}(C_{\tau_2, \tau_2^*}(\mu, i, \kappa), i, \kappa) \Rightarrow f^{-1}(\mu) \leq f^{-1}$ $(C_{\tau_2, \tau_2^*}(\delta I_{\tau_2, \tau_2^*}(\mu, i, \kappa), i, \kappa) \vee f^{-1}(I_{\tau_2, \tau_2^*}(C_{\tau_2, \tau_2^*}(\mu, i, \kappa), i, \kappa)) \leq C_{\tau_2, \tau_2^*}(f^{-1}$ $(\delta I_{\tau_2, \tau_2^*}(\mu, i, \kappa), i, \kappa) \vee f^{-1}(I_{\tau_2, \tau_2^*}(C_{\tau_2, \tau_2^*}(\mu, i, \kappa), i, \kappa))$.

Since f is a $DF\,\delta biCts$ mapping, then f is $DF\,\delta O$ map and $DF\,\delta Cts$ map. Then f is $DF\,\delta sCts$ map and $DF\,\delta pCts$ map. Hence $f^{-1}(\mu) \leq C_{\tau_2, \tau_2^*}(\delta I_{\tau_2, \tau_2^*}(f^{-1}(\mu), i, \kappa) \vee I_{\tau_2, \tau_2^*}(C_{\tau_2, \tau_2^*}(f^{-1}(\mu), i, \kappa)$. This shows that $f^{-1}(\mu)$ is (i, κ)-fZo set in (X, τ_1, τ_1^*).

Remark 3.2. If $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ be a $DF\,\delta biCts$ mapping. Then the inverse image of each (i, κ)-$f\delta po$ (resp. (i, κ)-$f\delta o$) set in Y under f is (i, κ)-fZo set in X.

Remark 3.3. Let (X, τ_1, τ_1^*) and (Y, τ_2, τ_2^*) be dfts's and $f : X \to Y$ be a mapping. The composition of two $DFZCts$ mappings need not be $DFZCts$ as shown by the following example.

Example 3.3. Let $X = Y = Z = \{a, b, c\}$ and let the fuzzy sets $\alpha_1, \alpha_2, \alpha_3$ and α_4 defined as $\alpha_1(a) = 0.3$, $\alpha_1(b) = 0.4$, $\alpha_1(c) = 0.5$; $\alpha_2(a) = 0.6$, $\alpha_2(b) = 0.9$, $\alpha_2(c) = 0.5$; $\alpha_3(a) = 0.6$, $\alpha_3(b) = 0.9$, $\alpha_3(c) = 0.5$ and $\alpha_4(a) = 0.4$, $\alpha_4(b) = 0.0$; $\alpha_4(c) = 0.5$. Consider the double fuzzy topologies (X, τ, τ^*) and (Y, η, η^*) with

$$\tau(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{6}, & \text{if } \lambda \in \{\alpha_1, \alpha_2\} \\ 0, & \text{o.w.} \end{cases} \quad \tau^*(\lambda) = \begin{cases} 0 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{5}{6}, & \text{if } \lambda \in \{\alpha_1, \alpha_2\} \\ 1, & \text{o.w}. \end{cases}$$

$$\sigma(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{6}, & \text{if } \lambda = \alpha_3 \\ 0, & \text{o.w.} \end{cases} \quad \sigma^*(\lambda) = \begin{cases} 0 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{5}{6}, & \text{if } \lambda = \alpha_3 \\ 1, & \text{o.w}. \end{cases}$$

$$\eta(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{1}{6}, & \text{if } \lambda = \alpha_4 \\ 0, & \text{o.w.} \end{cases} \quad \eta^*(\lambda) = \begin{cases} 0 & \text{if } \lambda \in \{\underline{0}, \underline{1}\} \\ \frac{5}{6}, & \text{if } \lambda = \alpha_4 \\ 1, & \text{o.w.} \end{cases}$$

Then the identity function $f : (X, \tau_1, \tau_1^*) \to (Y, \sigma, \sigma^*)$ and $g : (Y, \sigma, \sigma^*) \to (Z, \eta, \eta^*)$ are $DFZCts$ functions. But $g \circ f$ is not $DFZCts$ function, since the inverse image under $g \circ f$ of the fuzzy set α_4 is not an $\left(\frac{1}{6}, \frac{5}{6}\right)$-$fZo$ set in (X, τ, τ^*).

The next theorem gives the conditions under which the composition of two $DFZCts$ mappings is $DFZCts$.

Theorem 3.5. *Let* $(X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ *and* (Z, τ_3, τ_3^*) *be* $dfts$'s. *If* $f : (X, \tau_1, \tau_1^*) \to (Y, \tau_2, \tau_2^*)$ *and* $g : (Y, \tau s_2, \tau_1^*) \to (Z, \tau_3, \tau_3^*)$ *are mappings, then* $g \circ f$ *is* $DFZCts$ *mapping if*

1. f is $DFZCts$ *and g is* $DFCts$.

2. f is $DF\,\delta biCts$ *and g is* $DFZCts$ *mapping.*

Proof. (i) Let $\tau_3(\mu) \geq i$ and $\tau_3(\mu) \leq \kappa$. Since g is $DFCts$, then $\tau_3(g^{-1}(\mu)) \geq i$ and $\tau_3(g^{-1}(\mu)) \leq k$. Since f is $DFZCts$, then $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$ is (i, κ)-fZo set in (X, τ_1, τ_1^*). Hence $g \circ f$ is $DFZCts$.

(ii) Let $\tau_3(\mu) \geq i$ and $\tau_3(\mu) \geq \kappa$. Since g is $DFZCts$, then $g^{-1}(\mu)$ is an (i,κ)-fZo set in (Y, τ_2, τ_2^*). Since f is $DF\,\delta biCts$, by Theorem 3.4, $(g \circ f)^{-1}(\mu)$ is (i,κ)-fZo set in (X, τ_1, τ_1^*). Hence $g \circ f$ is $DFZCts$.

4. Conclusion

In this paper, we have introduced the double fuzzy δ-continuous, double fuzzy δ-bicontinuous, double fuzzy Z-continuous functions in double fuzzy topological spaces. Also some interesting properties and characterizations of the concepts are studied and we hope these investigations will further encourage other researchers to explore the interesting connections between this area of topology and fuzzy set.

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