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PARTIALLY BACKLOGGED TWO-WAREHOUSE ORDERING POLICY WITH SELLING PRICE DEPENDENT DEMAND UNDER INFINITE PLANNING HORIZON

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Abstract

In real life situation, the scope of any warehouse is limited. The space is required for stockpiling the surplus items for the inadequate ability W to the selfstores i.e. owned warehouse (O.W). The rented store (RW) is used for storing surplus units. In rented warehouse, the cost of storage is quite heavy in comparison of OW due to better facilities. From an economic perspective, stock of RW is to be consumed first. Moreover, this article is constructed with exponentially an action undergone and an instance of something happening depend on demand rate for decaying collection .The quantity of damage can be supposed to be constant in RW and time-depend on OW. Lacking of items permitted partially accumulated to fulfill the customer's demand. The main purpose of the task is to obtain the optimal stocked policy such as the total cost of its minimization under the given constraints. Numerical sample and sensitivity analyzes are carried out to check the stability of the proposed framework.

1. Introduction

Deterioration is a natural phenomenon in various products carried as inventory and many researchers has done plenty of work for determining the inventory system of deteriorating items under consideration of shortage or non-shortage. Recently many researchers focused on the study of deteriorating items reason might be the maximum physical commodities

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undergodeterioration over time. Fruits, vegetables, pharmaceuticals, milk, groceries items and other perishable items undergo from depletion by direct spoilage while stocked. Giri and Chaudhuri [11] the time dependent deterioration rate inventory model, had contributed in this area. Bhunia and Maiti [3] introduced optimal replenishment model with distinct warehouse to backlogged shortages and carried additional requirement of consumers. They also assumed demand is the function of time. Liang and Zhou [1] considered a two-stores inventory model become progressively worse for collections which is considered with trade credit with constant demand to minimize the total inventory cost. Kumar and Chanda [2] proposed a two-warehouse inventory model with deterioration for technology products with linearly launching a new product or service to tackle potential buyers where demand follows avant-grade diffusion to be judged. Recently, Panda et al. [4] explored nonlinear inventory model for worsen collection with price-assets dependent brusquely with advertisement factor and craft-commendation.

In present circumstances, it has been observed that a few clients might want to hang tight for accumulating during the deficiency period, but some of the customers would not wait for longer time. Hence, the opportunity cost because of lost deals ought to be occupied into account while demonstrating the model. Jalan et al. [11] opposed that the stockpile collection should be determined on the time duration of the given particulars. The pioneer who, described an application for the time-determination partially based on quantity collection. Singh et al. [6] developed an inventory framer for defective items with multi-variate demand and partial stockpiles. Sekar et al. [14] determined storage model for progressively worsen items with variable requirement under the increased amount on stocks due to unavailability which effects environment and its semi-filled items. The day, which holds the amount of content of a building on rent warehouse is much effective than the self warehouse, rent warehouse is that property where a complete list of items should be supplied as per the demand of the process of progressively worsen. Chakroborty et al. [5] investigated EOQ model with ramp-type demand rate to evaluate a turning point that follows negative exponential distribution and Weibull decination parameter for worsen items along with partially backlogged condition. However, some inventory system, like fancy groups, the duration of the waiting time for the upcoming restock becomes the important issue for analysation if in case the backlogging could be received or not. After a long duration of time it leads towards the small accumulators. Therefore, the accumulation rate is the quality of being subject to variation and is contingent on the waiting time for the upcoming restoration. In this paper, the quantity of accumulation is to be considered an exact ratio of the required quantity of for short span of the time.

Classical inventory models offer a variety of insights into optimal way to manage inventories of individual product where the, constant worth of requirement can be imagined is Nevertheless, nature of demand rate for physical goods might be depend on time as well as price due to the fluctuation in market's demand. Kumar et al. [6] concluded the goods in stock with weibull worsen goods which partially accumulated under discounted cash flow with inflation factor. Chaudhary and Sharma [13] evaluated EOQ model for physical goods with less availability due to inflation. Kumar [12] discovered profit function for inventory with fixed price and contingent demand of time variable where constant and varying deterioration rate. The demand for some perishable products has been seen as exponentially increased in nature when products are launched. Maragatham and Palani [7] explored single warehouse EOQ policy with price dependent demand and time dependent ordering cost as well as holding cost to minimize the total cost. Moreover they provide the less availability during lead time. Saha and Chakrabarti [8] analyzed total cost for production model with linear selling price and constant deterioration rate by using signed distance method of fuzzy theory.

In this framework, we developed a generalized deterministic two warehouse structure for constantly decaying articles is constant although rate of required quantity related to the handover cost of item. Moreover, it is also function of time which is exponential in nature.

Backorderes are authorized and partially accumulated. The remain part of given framework is arranged in that way i.e. required assumption and notations are given in section 2. In section 3, we captured the mathematical formulation. We provided numerical examples to illustrate the results graphically through sensitivity analysis. Finally, we describe the conclusions in section 5 and references in section 6.

2. Assumptions and Notations

We consider the following assumptions:

- A single-item inventory is considered over an infinite planning horizon.
- The demand rate is exponentially dependent on time and selling price.
- Deterioration is assumed constant in rented warehouse (R.W.) and variable in owned warehouse (O.W.)
- Lead-time is assumed zero.
- Shortages are partially backlogged

In addition, we use the following notations:

μ ordering cost per order.

θ deterioration rate

a demand parameter $a > 0$.

b demand parameter $b > 0$.

p selling price per unit

W_1 storage capacity of owned warehouse (O.W.).

W_2 storage capacity of rented warehouse (R.W.).

B backlogged unit

δ backlogging rate

T length of the cycle.

G owned warehouse cost

H rented warehouse cost

X shortage cost per unit

F Lost sale cost

R Coefficient of demand rate

Y deterioration cost R.W.

Z deterioration cost O.W.

$Io(t)$ inventory level at any time in owned warehouse.

$I_r(t)$ inventory level at any time in rented warehouse.

TC the total inventory cost per unit time of inventory system.

3. Model Formulation

In this model, initially the on-hand inventory level is W units. From W units, W_1 units are stored in owned warehouse and the remaining $(W - W_1) = W_2$ units are stored in rented ware house. The level of inventory in rented warehouse decreases due to demand of customers and also effect of deterioration of items in interval $(0, t_1)$. Also, in owned warehouse during interval $(0, t_1)$ level of inventory W_1 decreases due to deterioration and demand which is assumed to be constant. Furthermore, interval (t_1, t_2) in which the depletion of the inventory occurs due to effects of the demand and deterioration. At time t_2 the level of inventory in OW becomes zero. Therefore, shortages occur in the interval (t_2, T) which are partially backlogged.

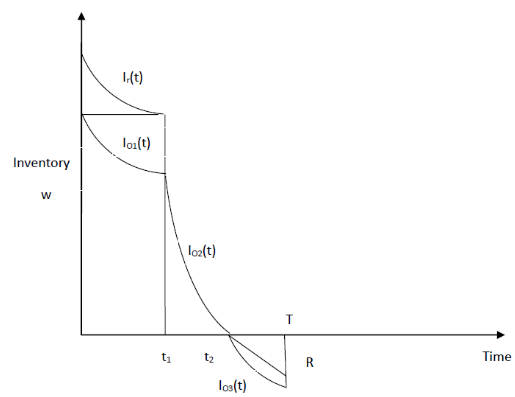

Figure 1. Inventory system.

The differential equation governing the system is given below

$$\frac{d}{dt} I_r(t) + \theta I_r(t) = -\text{Re}^{(at-bp)},\ 0 \leq t \leq t_1 \tag{1}$$

$$\frac{d}{dt} I_{o_1}(t) + \theta I_{o_1}(t) = R,\ 0 \le t \le t_1 \tag{2}$$

$$\frac{d}{dt} I_{o_2}(t) + \theta t I_{o_2}(t) = \mathrm{Re}^{(at-bp)},\ t_1 \le t \le t_2 \tag{3}$$

$$\frac{d}{dt} I_{o_3}(t) = -\delta e^{(at-bp)},\ t_2 \le t \le T. \tag{4}$$

With the boundary condition

$$I_r(t_1) = 0\ I_r(0) = W_2,\ I_{o_1}(0) = W_1,\ I_{o_2}(t_2) = 0,\ I_{o_3}(T) = -B. \tag{5}$$

On solving above equations, we get:

$$I_r(t) = \left(\frac{-\mathrm{Re}^{-pb}}{\theta - a}\right) e^{-at} + \frac{e^{(\theta-a)t_1}}{(\theta - a)} \mathrm{Re}^{-(pb+\theta t)} \tag{6}$$

$$I_{o_1}(t) = -R\left(t + \theta\frac{t^3}{6}\right)\left(1 - \frac{\theta t^2}{2}\right) + \left(1 - \frac{\theta t^2}{2}\right)\left(W_1\left(1 - \frac{\theta t^2}{2}\right) + R\left(1 + \frac{\theta t^2}{6}\right)\right) \tag{7}$$

$$I_{o_2}(t) = R(1 - bp)\left[(t_2 - t) - \frac{a}{2}(t_2^2 - t^2) - \frac{\theta}{3}(t_2^3 - t^3) + \frac{a\theta}{8}(t_2^4 - t^4)\right] \tag{8}$$

$$I_{o_3}(t) = -\theta\frac{\delta}{a} e^{-pb}(e^{-aT} - e^{-at}) - B. \tag{9}$$

By equation of continuity at $t = t_2$

$$B = \frac{\delta}{a} e^{-pb}(e^{-at_2} - e^{-aT}). \tag{10}$$

Now, the total inventory cost consists of the following cost components:

1. The ordering cost (OC) is μ. (11)

2. The inventory holding cost (HC) per cycle is given by

$$HC = H\int_0^{t_1} I_r(t)dt + G\int_0^{t_2} I_o(t)dt$$

$$HC = H\frac{R}{\theta - a}\left[\frac{3}{2}\theta t_1^2 - at_1^2 + bp_1^2(a - \theta) - \theta\frac{t_1^3}{2}(\theta - a)\right]$$

$$+ G\left[-R\frac{t_1^2}{2} - \frac{R}{8}\theta t_1^4 + \frac{Rt_1^6\theta^2}{72}\right]$$

$$G\left[Wt_1 + Rt_1 + \frac{R\theta}{24}t_1^4\right] + h_{ow}R(1-bp)\left[\left(t_2t_1 - \frac{t_1^2}{2}\right) - \frac{a}{2}\left(t_2^2t_1 - \frac{t_1^3}{3}\right)\right.$$

$$\left. - \frac{\theta}{3}\left(t_2^3t_1 - \frac{t_1^4}{4}\right) + \frac{a^8}{8}\left(t_2^4t_1 - \frac{t_1^5}{5}\right)\right]$$

$$+ GR(1-bp)\left[\left(\frac{t_2^2}{2}\right) - \frac{a}{2}\frac{t_2^3}{3} - \frac{\theta}{3}\frac{t_2^4}{4} + \frac{a\theta}{10}t_2^5\right]. \tag{12}$$

3. Shortage Cost

$$SC = -X\int_{t_2}^{T} I_{o_3}(t)dt$$

$$SC = -X\left[B(t_2 - T) - \frac{\delta}{a}e^{(bp-aT)}(T - t_2) - \frac{\delta}{a^2}e^{-bp}e^{-a(T-t_2)}\right] \tag{13}$$

4. Lost Sales Cost

$$LSC = F\int_{t_2}^{T}(1-\delta)\,\mathrm{Re}^{(at-bp)}\,dt$$

$$= F(1-\delta)R\left[\frac{e^{(aT-bp)} - e^{(at_2-bp)}}{a}\right]. \tag{14}$$

5. Deterioration Cost

$$DC = Y\left[W_2 - \int_0^{t_2}\mathrm{Re}^{(at-bp)}\,dt\right] + Z\left[W_1 - \int_0^{t_2}Rdt - \int_{t_1}^{t_2}\mathrm{Re}^{(at-bp)}\,dt\right]$$

$$= Y[W_2 - \mathrm{Re}^{-bp}(e^{at_2} - 1)] + Z[W_1 - Rt_1 - \mathrm{Re}^{-bp}(e^{at_2} - e^{at_1})]. \tag{15}$$

Therefore, the total average cost per unit time is $T.C = \frac{1}{T}$ [Ordering Cost + Holding Cost + Shortage Cost + Lost Sales Cost + Deterioration Cost] (16)

The necessary conditions for the total cost per unit time to be minimum are

$$\frac{\partial TC}{\partial t_2} = O,\ \frac{\partial TC}{\partial T} = O.$$

Provided

$$\left(\frac{\partial^2 TC}{\partial t_2^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_2 \partial T}\right)^2 > 0$$

and optimal value of pair (t_2, T) and TC can be calculated from above conditions. The below mentioned Figure 2 presents that model is convex.

4. Numerical Example

Consider an inventory system with the following parameter in proper units

$\mu = 500$, $K = 1200$, $L = 0.1$, $p = 2$, $a = 0.45$, $b = 0.003$, $\theta = 0.8$, $W_1 = 100$, $W_2 = 50$, $X = 0.5$, $L = 0.1$, $F = 22$, $G = 0.2$, $Y = 1.5$, $B = 32$, $\delta = 0.05$, $H = 13$.

Mathematica software calculated these optimum values $T^* = 11.859 - t_2^* = 5.2647$ and $TC^* = 135.972$.

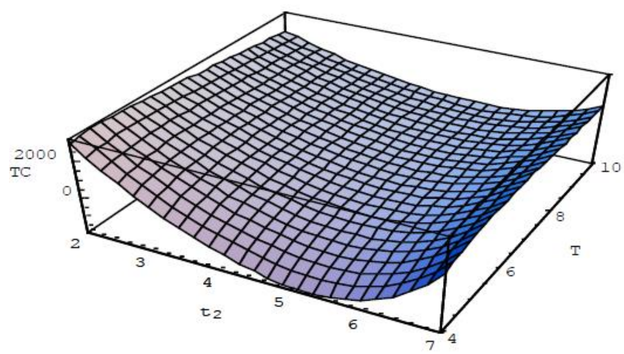

Figure 2. Convexity of the total cost function (T.C) with respect to t_2 and T.

5. Sensitivity Analysis

Sensitivity analysis w.r.t. assorted factors are as mentioned below:

% variation in p	p	t_2^*	T^*	T.C
– 15%	1.7	5.2647	11.8599	135.013
– 10%	1.8	5.2647	11.8596	135.034
10%	2.2	5.2648	11.8584	136.911
15%	2.3	5.2648	11.8582	136.931
% variation in θ	θ	t_2^*	T^*	T.C
– 15%	0.68	5.3435	11.6844	124.080
– 10%	0.72	5.3153	11.7361	128.229
10%	0.88	5.2206	12.0073	142.795
15%	0.92	5.2006	12.0910	145.782
% variation in A	A	t_2^*	T^*	T.C
– 15%	425	5.26479	11.9787	129.68
– 10%	450	5.26478	11.9383	131.77
10%	550	5.26477	11.7820	140.202
15%	575	5.26476	11.7443	142.328
% variation in δ	δ	t_2^*	T^*	T.C
– 15%	0.0425	5.2680	11.8167	139.247
– 10%	0.0450	5.2669	11.8306	138.155
10%	0.055	5.2625	11.8881	133.791
15%	0.0575	5.2614	11.9028	132.701
% variation in H	H	t_2^*	T^*	T.C
– 15%	11.05	5.2647	11.8628	135.769
– 10%	11.70	5.2647	11.8615	135.83
10%	14.30	5.2647	11.8565	136.108
15%	14.95	5.2647	11.8553	136.176

% variation in L	L	t_2^*	T^*	T.C
– 15%	0.085	5.3398	11.7641	128.624
– 10%	0.090	5.3130	11.7902	131.265
10%	0.110	5.2224	11.9480	139.747
15%	0.115	5.2031	11.9995	141.207
% variation in a	a	t_2^*	T^*	T.C
– 15%	0.3825	5.9512	15.8251	159.476
– 10%	0.4050	5.6965	14.0331	154.431
10%	0.4950	4.9115	10.4918	100.809
15%	0.5175	4.7774	10.0330	74.5998
% variation in X	X	t_2^*	T^*	T.C
– 15%	0.425	5. 2643	11.8394	134.639
– 10%	0.450	5.2645	11.8459	135.083
10%	0.550	5.2650	11.8722	136.862
15%	0.575	5.2652	11.8788	136.325

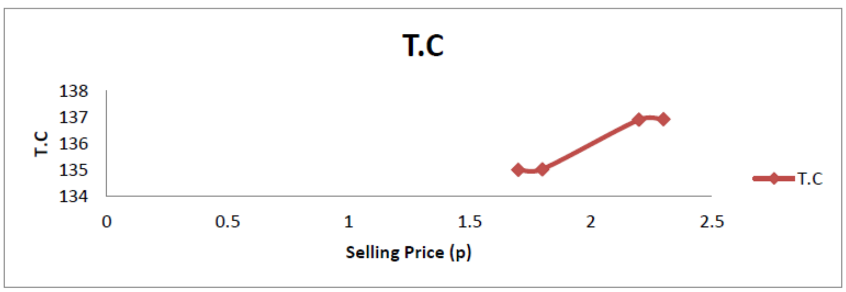

Figure 3. Variation in total cost with the variation in selling price.

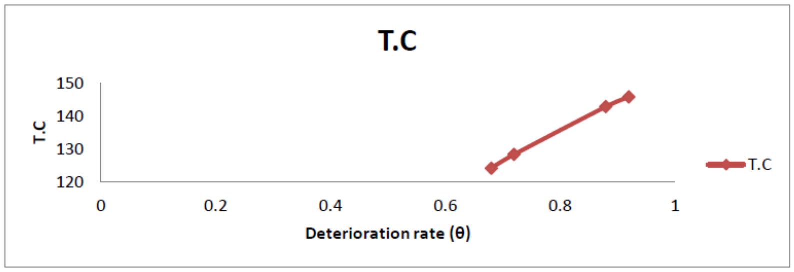

Figure 4. Variation in total cost with the variation in deterioration parameter.

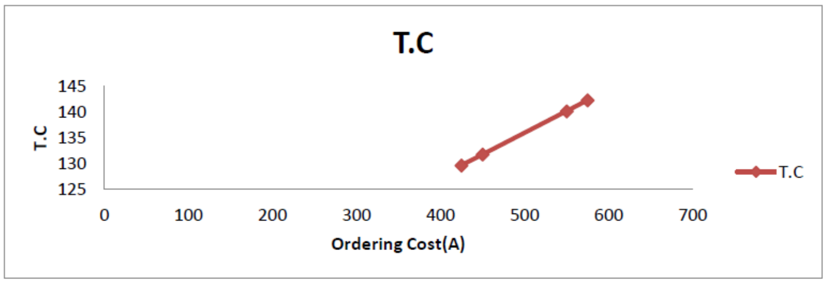

Figure 5. Variation in total cost with the variation in ordering cost.

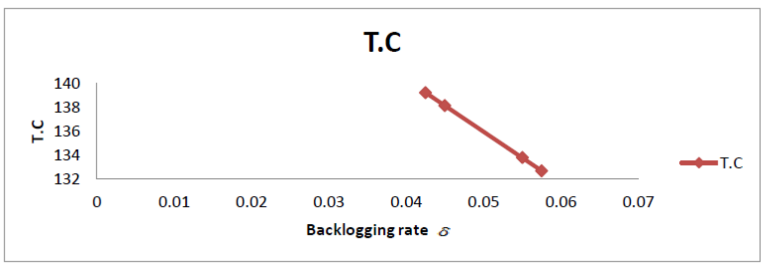

Figure 6. Variation in total cost with the variation in backlogging parameter.

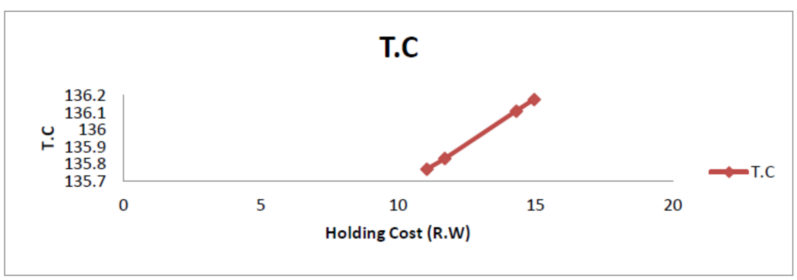

Figure 7. Variation in total cost with the variation in rented holding cost.

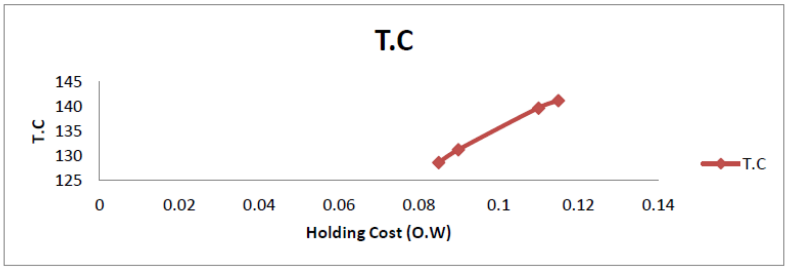

Figure 8. Variation in total cost with the variation in owned holding cost.

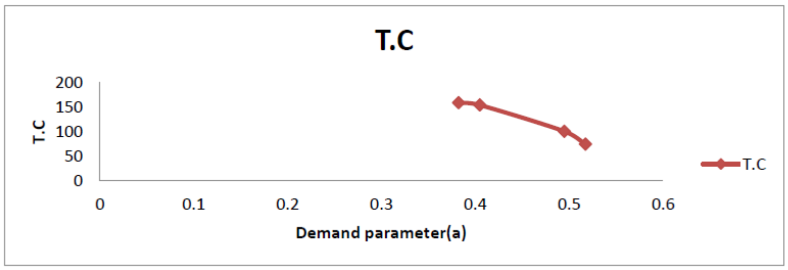

Figure 9. Variation in total cost with the variation in demand rate.

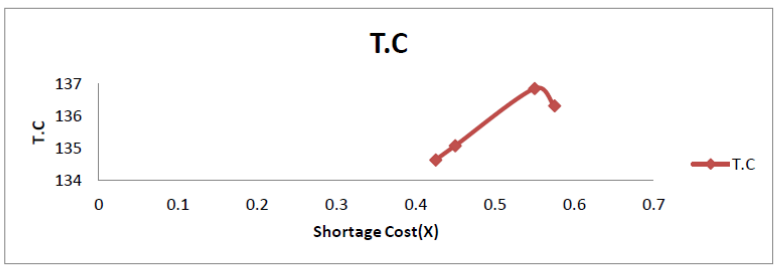

Figure 10. Variation in total cost with the variation in shortage cost.

The above table reveals the following:

(i) With increment in selling price parameter p^*, t_2^* increases very slightly, T^* total cycle time will decrease and total cost TC^* increases.

(ii) With increase in deterioration parameter θ, t_2^* increases, T^* total cycle time will increase and total TC^* cost increases.

(iii) With increase in demand parameter a^*, t_2^* increases, T^* total cycle time will increase and total cost TC^* decreases.

(iv) With variation in backlogging parameter 'δ' and from this, we have noticed that t_2^* increases, T^* total cycle time will increase and total cost TC^* decreases.

(v) With the variation in holding cost parameter of RW 'H' and we have observed that t_2^* unchanged, T^* total cycle time will decrease and total cost TC^* increases.

(vi) Variation in holding cost (G) of owned warehouse is discussed and in holding cost of the owned warehouse the T.C. of the system gradually increases.

(vii) Effect of ordering cost parameter (μ) we have noticed, t_2^* decreases, T^* total cycle time and total cost TC^* will increase rapidly.

(viii) With the variation in shortage cost per unit X, t_2^* increases, T^* total cycle time will increase and total cost TC^* increases.

5. Concluding Remarks

In the article, we developed inventory system considering time as well as selling price dependent exponential demand for the deteriorating items and noticed some useful results. According to our policies, facilities of RW are more better in every aspect in comparison of OW, therefore the cost of storage is quite high for RW. Furthermore, sensitivity analysis is carried out to check the robustness of the model and the result indicates the model is robust. Convexity has been proved by the graph. This proposed model is developed under some certain assumptions so, it is restricted to them. Shortages are endorsed in proposed system and the shortcoming items are partially backlogged. This paper can be generalised in several traditions namely fuzzy environment, power demand, permissible delay, inflation etc.

References

[1] Y. Liang and F. Zhou, A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment, Applied Mathematical Modelling 35(5) (2011), 2221-2231.

[2] A. Kumar and U. Chanda, Two-warehouse inventory model for deteriorating items with demand influenced by innovation criterion in growing technology market, Journal of Management Analytics 5(3) (2018), 198-212.

[3] A. K. Bhunia and M. Maiti, A two warehouse inventory model for deteriorating items

with a linear trend in demand and shortages, Journal of the Operational Research Society 49(3) (1998), 287-292.

[4] G. C. Panda, M. A. A. Khan and A. A. Shaikh, A credit policy approach in a two-warehouse inventory model for deteriorating items with price-and stock-dependent demand under partial backlogging. Journal of Industrial Engineering International 15(1) (2019), 147-170.

[5] D. Chakraborty, D. K. Jana and T. K. Roy, Two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribution deterioration under inflation and permissible delay in payments, Computers and Industrial Engineering 123 (2018), 157-179.

[6] N. Kumar, S. R. Singh and R. Kumari, An inventory model with time-dependent demand and limited storage facility under inflation, Advances in Operations Research, 2012.

[7] M. Maragatham and R. Palani, An Inventory Model for Deteriorating Items with Lead Time price Dependent Demand and Shortages. Advances in Computational Sciences and Technology 10(6) (2017), 1839-1847.

[8] S. Saha and T. Chakrabarti, A Fuzzy inventory model for deteriorating items with linear price dependent demand in a supply chain, International Journal of Fuzzy Mathematical Archive 13(1) (2017), 59-67.

[9] B. C. Giri, S. Pal, A. Goswami and K. S. Chaudhuri, An inventory model for deteriorating items with stock-dependent demand rate, European Journal of Operational Research 95(3) (1996), 604-610.

[10] S. R. Singh, S. Singhal and P. K. Gupta, A volume flexible inventory model for defective items with multi variate demand and partial backlogging, International Journal of Operations Research and Optimization 1(4), 54-68.

[11] A. K. Jalan, R. R. Giri and K. S. Chaudhuri, EOQ model for items with Weibull distribution deterioration, shortages and trended demand, International Journal of Systems Science 27(9) (1996), 851-855.

[12] S. Kumar, Optimization of deteriorating items inventory model with price and time dependent demand, Amity Journal of Operations Management 1(1) (2016), 64-76.

[13] R. R. Chaudhary and V. Sharma, A Model for Weibull Deteriorate Items with Price Dependent Demand Rate and Inflation, Indian Journal of Science and Technology 8(10) (2015), 975.

[14] T. Sekar, R. Uthayakumar and P. Mythuradevi, Limited capacity storehouse inventory model for deteriorating items with preservation technology and partial backlogging under inflation, Inventory Model 21(3) (2017), 377-405.