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ISOMORPHIC SINGLE VALUED NEUTROSOPHIC GRAPHS AND THEIR COMPLEMENTS

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Abstract

In this paper, basic definitions related to Single Valued Neutrosophic Graphs (SVNG) with examples are discussed. Some properties of isomorphism are introduced. Also isomorphism between single valued neutrosophic graphs is proved to be an equivalence relation. Also discussed about isomorphic neutrosophic graphs and their complements.

1. Introduction

The notion graph theory was first introduced by Euler in 1736. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the first theorem of graph

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theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. On the other hand, fuzzy graph theory as a generalization of Eulers graph theory was first introduced by Rosenfeld [7] in 1975. In 1965, Zadeh [9] proposed the theory of fuzzy set theory which is applied in many real applications to handle uncertainty. Atanassov [2] added a new component (which determines the degree of non-membership) in the definition of fuzzy set. Neutrosophy is a branch of philosophy, introduced by Florentin Smarandache in 1995, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The concept of Neutrosophic set was also introduced by F. Smarandache [3] which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In this research article we introduce the notion of isomorphism between neutrosophic graphs.

2. Single Valued Neutrosophic Graph

Defnition 2.1 [5]**.** A Single Valued Neutrosophic Graph (SVNG) is of the form $G = (A, B)$ where $A : V \to [0, 1]$ is a neutrosophic set in V and $B : V \times X \to [0, 1]$ is a neutrosophic relation on V such that

1. The functions $T_A : V \to [0, 1], I_A : V \to [0, 1]$ and $F_A : V \to [0, 1]$ denote the degree of membership, degree of indeterminacy and non-membership (Falsity) of the element $v_i \in V$; respectively, and $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for every $v_i \in V, (i = 1, 2, \ldots, n)$.

2. $E \subseteq V \times V$ where $T_B : V \times V \to [0, 1], I_B : V \times V \to [0, 1]$ and $F_B : V \times V \to [0, 1]$ are such that

$T_B(v_i, v_j) \leq \min\{T_A(v_i), T_A(v_j)\}$,

$I_B(v_i, v_j) \leq \min\{I_A(v_i), I_A(v_j)\}$, and $F_B(v_i, v_j) \leq \max\{F_A(v_i), F_A(v_j)\}$ and $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$ for every $(v_i, v_j) \in E (i, j = 1, 2, \ldots, n)$.

Example 2.2. Consider a graph $G = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{v_1v_2, v_2v_3, v_2v_5, v_3v_4, v_4v_5, v_5v_1\}$. Let A and B be the neutrosophic sets of V and E, respectively, as shown in following Tables. By simple calculations, it is easy to see that $G = (A, B)$ is a single valued neutrosophic graph as shown in Figure 2.1.

A	v_1	v_2	v_3	v_4
T	0.2	0.3	0.4	0.5
I	0.4	0.5	0.6	0.7
F	0.6	0.7	0.8	0.9

A	v_1v_2	v_2v_3	v_3v_4	v_4v_1
T	0.1	0.2	0.3	0.1
I	0.3	0.4	0.5	0.3
F	0.7	0.8	0.9	0.8

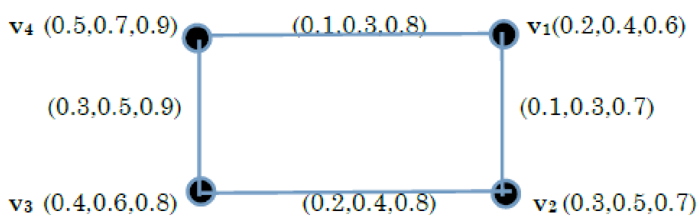

Figure 2.1. Single Valued Neutrosophic Graph.

Definition 2.3 [5]**.** A Single Valued Neutrosophic Graph $G = (A, B)$ is called complete if the following conditions are satisfied:

$$T_B(v_i, v_j) = \min\{T_A(v_i), T_A(v_j)\},$$

$$I_B(v_i, v_j) = \min\{I_A(v_i), I_A(v_j)\},$$

$$F_B(v_i, v_j) = \max\{F_A(v_i), F_A(v_j)\} \text{ for every } (v_i, v_j) \in E.$$

Example 2.4. Consider a Single Valued Neutrosophic Graph $G = (A, B)$

on the nonempty set $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$ as shown in Figure 2.2. By direct calculations, it is easy to see that G is complete.

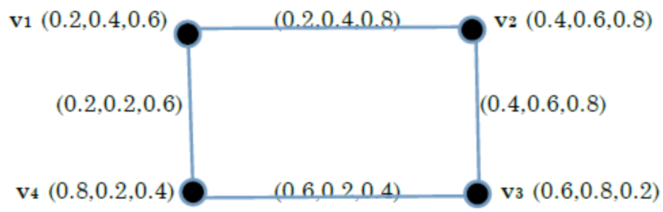

Figure 2.2. Complete Neutrosophic Graph.

Definition 2.5 [8]. Let $G = (A, B)$ be a single valued neutrosophic graph. The order of G, denoted $O(G)$ is defined as $O(G) = (O_T(G), O_I(G), O_F(G))$, where

$$O_T(G) = \sum_{V \in v} T_A(v) \text{ denotes the } T\text{-order of } G.$$

$$O_I(G) = \sum_{V \in v} I_A(v) \text{ denotes the } I\text{-order of } G.$$

$$O_F(G) = \sum_{V \in v} F_A(v) \text{ denotes the } F\text{-order of } G.$$

i.e. the order of G means also the number of vertices (or the cardinality of V).

Definition 2.6 [8]. Let $G = (A, B)$ be a single valued neutrosophic graph. The size of G, denoted $S(G)$ is defined as $S(G) = (S_T(G), S_I(G), S_F(G))$, where

$$S_T(G) = \sum_{v_i \neq v_j} T_B(v_i, v_j), \text{ denotes the } T\text{-size of } G.$$

$$S_I(G) = \sum_{v_i \neq v_j} I_B(v_i, v_j), \text{ denotes the } I\text{-size of } G$$

$$S_F(G) = \sum_{v_i \neq v_j} F_B(v_i, v_j), \text{ denotes the } F\text{-size of } G.$$

i.e. The size of G means also the number of edges (or the cardinality of E).

Example 2.7. Consider a complete NFG as in Figure 2.2 $O(G) = (2, 2, 2)$, $S(G) = (1.4, 1.4, 2.6)$.

3. Isomorphism on Neutrosophic Graphs

Definition 3.1. A homomorphism of neutrosophic graphs $h : G \to G'$ is a map $h : V \to V'$ which satisfies $A(v_i) \le A'(h(v_i))\,\forall v_i \in V$ i.e., $T_A(v_i) \le T_{A'}(h(v_i))$, $I_A(v_i) \le I_{A'}(h(v_i))$, $F_A(v_i) \le F_{A'}(h(v_i))\forall v_i \in V$ and $B(v_i, v_j) \le B'(h(v_i), h(v_j))\forall v_i, v_j \in V$ i.e., $T_B(v_i, v_j) \le T_{B'}(h(v_i), h(v_j))$, $I_B(v_i, v_j) \le I_{B'}(h(v_i), h(v_j))$, $F_B(v_i, v_j) \le F_{B'}(h(v_i), h(v_j))\forall v_i, v_j \in V$.

Definition 3.2. A weak isomorphism $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies $A(v_i) = A'(h(v_i))\forall v_i \in V$ i.e., $T_A(v_i) = T_{A'}(h(v_i))$, $I_A(v_i) = I_{A'}(h(v_i))$, $F_A(v_i) = F_{A'}(h(v_i))\forall v_i \in V$.

Example 3.3. Let $G = (A, B)$ and $G = (A', B')$ be neutrosophic graphs with underlying sets $V = \{v_1, v_2, v_3\}$ and $V' = \{v'_1, v'_2, v'_3\}$ where $A : V \to [0, 1]$, $B : V \times V \to [0, 1]$, $A' : V \to [0, 1]$ and $B' : V \times V \to [0, 1]$. Defining a map $h : V \to V'$ which satisfies $A(v_i) = A'(h(v_i))\forall v_i \in V$ i.e., $T_A(v_i) = T_{A'}(h(v_i))$, $I_A(v_i) = I_{A'}(h(v_i))$, $F_A(v_i) = F_{A'}(h(v_i))\forall v_i \in V$ and $B(v_i, v_j) \le B'(h(v_i), h(v_j))\forall v_i, v_j \in V$ i.e., $T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j))$, $I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j))$, $F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j))\forall v_i, v_j \in V$ Then $h : G \to G'$ is a weak isomorphism as shown in Figure 3.1.

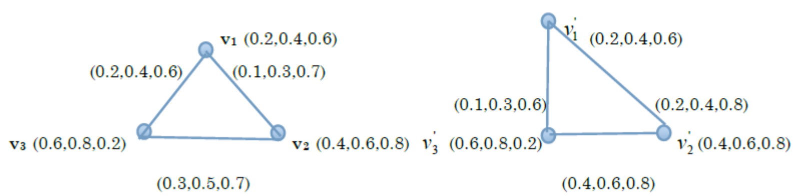

Figure 3.1. Weak Isomorphism.

Definition 3.4. A co-weak isomorphism $h : G \to G'$ is a map $h : V \to V'$ which is a bijective homomorphism that satisfies $B(v_i, v_j) = B'(h(v_i), h(v_j))\forall v_i, v_j \in V$.

i.e., $T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j)), I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j)),$ $F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j))\forall v_i, v_j \in V.$

Example 3.5. Let $G = (A, B)$ and $G' = (A', B')$ be neutrosophic graphs with underlying sets $V = \{v_1, v_2, v_3\}$ and $V' = \{v_1', v_2', v_3'\}$ where $A : V \to [0, 1]$, $B : V \times V \to [0, 1]$, $A' : V \to [0, 1]$ and $B' : V \times V \to [0, 1]$. Defining a map $h : V \to V'$ which satisfies $B(v_i, v_j) = B'(h(v_i), h(v_j))\forall v_i, v_j \in V$, i.e., $T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j)),$ $I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j)), F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j))\forall v_i, v_j \in V$

Then $h : G \to G'$ is a co-weak isomorphism as shown in Figure 3.2.

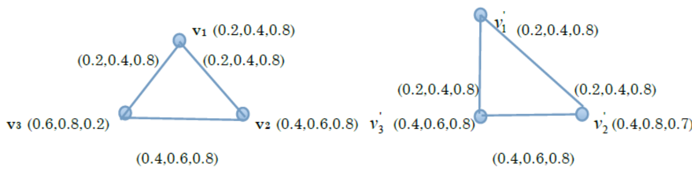

Figure 3.2. Co-weak isomorphism.

Definition 3.6. An isomorphism $h : G \to G'$ is a map $h : V \to V'$ which is bijective that satisfies $A(v_i) = A'(h(v_i))\forall v_i \in V$ i.e., $T_A(v_i) = T_{A'}(h(v_i)), I_A(h(v_i)) = I_{A'}(h(v_i)), F_A(v_i) = F_{A'}(h(v_i))\forall v_i \in V$ and $B(v_i, v_j) = B'(h(v_i), h(v_j))\forall v_i, v_j \in V$ i.e., $T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j)),$ $I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j)), F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j))\forall v_i, v_j \in V$. We denote it as $G \cong G'$.

Theorem 3.7. *For any two isomorphic neutrosophic graphs their order and size are same.*

Proof of Theorem 3.7. If $h : G \to G'$ is an isomorphism between the neutrosophic graphs G and G' with the underlying sets V and V' respectively then

$$T_A(v_i) = T_{A'}(h(v_i)), I_A(h(v_i)) = I_{A'}(h(v_i)), F_A(v_i) = F_{A'}(h(v_i))\forall v_i \in V \quad (1)$$

$$T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j)), I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j)),$$

$$F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j))\forall v_i, v_j \in V \quad (2)$$

(i) order of $G = (O_T(G), O_I(G), O_F(G))$

$$= \left(\sum_{v \in V} T_A(v), \sum_{v \in V} I_A(v), \sum_{v \in V} F_A(v) \right)$$

$$= \left(\sum_{v \in V} T_{A'}(h(v)), \sum_{v \in V} I_{A'}(h(v)), \sum_{v \in V} F_{A'}(h(v)) \right)$$

using equation (1)

$$= (O_T(G'), O_I(G'), O_F(G')) = \text{order if } G'$$

(ii) $S = (S_T(G), S_I(G), S_F(G))$

$$= \left(\sum_{v \in V} T_B(v_i, v_j), \sum_{v \in V} I_B(v_i, v_j), \sum_{v \in V} F_B(v_i, v_j) \right)$$

$$= \left(\sum_{v \in V} T_{B'}(h(v_i), h(v_j)), \sum_{v \in V} I_{B'}(h(v_i), h(v_j)), \sum_{v \in V} F_{B'}(h(v_i), h(v_j)) \right)$$

using equation (2)

$$= (S_T(G'), S_I(G'), S_F(G')) = S(G').$$

Hence the theorem.

Theorem 3.8. *Isomorphism between neutrosophic graphs is an equivalence relation.*

Proof of Theorem 3.8.

Let $G = (A, B)$, $G' = (A', B')$, $G'' = (A'', B'')$ be neutrosophic graphs with underlying sets V, V' and V'' respectively.

(i) Reflexive

Consider the identity map $h : V \to V$ such that $h(v) = v \forall v \in V$.

This h is a bijective map satisfying

$T_A(v_i) = T_A(h(v_i))$, $I_A(v_i) = I_A(h(v_i))$, $F_A(v_i) = F_A(h(v_i)) \forall v_i \in V$

$T_B(v_i, v_j) = T_B(h(v_i), (h(v_j)), I_B(v_i, v_j) = I_B(h(v_i), (h(v_j)),$ $F_B(v_i, v_j) = F_B(h(v_i), (h(v_j))\forall v_i, v_j \in V$. Hence h is an isomorphism of the neutrosophic graph to itself. Therefore it satisfies reflexive relation.

(ii) Symmetric

Let $h : V \to V'$ be an isomorphism of G onto G' then h is a bijective map such that

$$h(v) = v', v \in V. \tag{1}$$

Satisfying $T_A(v) = T_{A'}(h(v)), I_A(h(v)) = I_{A'}(h(v)), F_A(v) = F_{A'}(h(v)) \forall v \in V$

$$T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j)), I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j)),$$

$$F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j)) \forall v_i, v_j \in V. \tag{2}$$

As h is bijective, by equation (1) $h^{-1}(v') = v, \forall v' \in V'$. (3)

Using (3) in (2) we get

$T_A(h^{-1}(v')) = T_{A'}(v), I_A(h^{-1}(v')) = I_{A'}(v), F_A(h^{-1}(v')) = F_{A'}(v')$ for all $v' \in V'$

$$T_B(h^{-1}(v_i'), h^{-1}(v_j')) = T_{B'}(v_i', v_j'), I_B(h^{-1}(v_i'), h^{-1}(v_i')) = I_{B'}(v_i', v_j'),$$

$$F_B(h^{-1}(v_i'), h^{-1}(v_j')) = F_{B'}(v_i', v_j'), \text{ for all } v_i', v_j' \in V'. \tag{4}$$

Hence we get a 1-1, onto map $h^{-1} : V' \to V$, which is an isomorphism from G' to G. i.e., $G \cong G' \Rightarrow G' \cong G$. Therefore it satisfies symmetric property.

(iii) Transitive

Let $h : V \to V'$ and $g : V' \to V''$ be an isomorphisms of the neutrosophic graphs G onto G' and G' onto G'' respectively. Then goh is a 1-1 onto map from $V \to V''$ where

$$(goh)(v) = g(h(v)) \forall v \in V.$$

As $h : V \to V'$ is an isomorphism $h(v) = v', v \in V$ (5)

$$T_A(v) = T_{A'}(h(v)),\ I_A(h(v)) = I_{A'}(h(v)),\ F_A(v) = F_{A'}(h(v)) \forall v \in V$$

$$T_B(v_i, v_j) = T_{B'}(h(v_i), (h(v_j)),\ I_B(v_i, v_j) = I_{B'}(h(v_i), (h(v_j)),$$

$$F_B(v_i, v_j) = F_{B'}(h(v_i), (h(v_j)) \forall v_i, v_j \in V. \tag{6}$$

Using equation (5) in equation (6) we have

$$T_A(v) = T_{A'}(v'),\ I_A(v) = I_{A'}(v'),\ F_A(v) = F_{A'}(v') \forall v \in V \tag{7}$$

$$T_B(v_i, v_j) = T_{B'}(v_i', v_j'),\ I_B(v_i, v_j) = I_{B'}(v_i', v_j'),\ F_B(v_i, v_j) = F_{B'}(v_i', v_j')$$

$$\forall v_i, v_j \in V. \tag{8}$$

As $g : V' \to V''$ is an isomorphisms $g(v') = v''$, $v' \in V'$ (9)

$$T_{A'}(v') = T_{A''}(g(v')),\ I_{A'}(v') = I_{A''}(g(v')),\ F_{A'}(v') = F_{A''}(g(v')) \forall v' \in V' \tag{10}$$

$$T_{B'}(v_i', v_j') = T_{B''}(g(v_i'), (h(v_j')),\ I_{B'}(v_i', v_j') = I_{B''}(g(v_i'), (g(v_j')),$$

$$F_{B'}(v_i', v_j') = F_{B''}(g(v_i'), (g(v_j')) \forall v_i', v_j' \in V'. \tag{11}$$

Equations (5), (7) and (10) implies

$$T_A(v) = T_{A''}(g(v')) = T_{A''}(g(h(v))),\ I_A(v) = I_{A''}(g(v')) = I_{A''}(g(h(v))),$$

$$F_A(v) = F_{A''}(g(v')) = F_{A''}(g(h(v))). \tag{12}$$

Equations (5), (8) and (11) implies

$$T_B(v_i, v_j) = T_{B''}(g(v_i'), g(v_j')) = T_{B''}(g(h(v_i)), g(h(v_j))),$$

$$I_B(v_i, v_j) = I_{B''}(g(v_i'), g(v_j')) = I_{B''}(g(h(v_i)), g(h(v_j))),$$

$$F_B(v_i, v_j) = F_{B''}(g(v_i'), g(v_j')) = F_{B''}(g(h(v_i)), g(h(v_j))). \tag{13}$$

Equations (12) and (13) implies goh is an isomorphism between G and G''. i.e., $G \cong G''$.

(i), (ii) and (iii) imply isomorphism between neutrosophic graphs is an equivalence relation. Hence the theorem.

Theorem 3.9. *Weak isomorphism between neutrosophic graphs satisfies the partial order relation.*

Proof Theorem 3.9.

Let $G = (A, B)$, $G' = (A', B')$, $G'' = (A'', B'')$ be neutrosophic graphs with underlying sets V, V' and V'' respectively.

(i) Reflexive

Consider the identity map $h : V \to V$ such that $h(v) = v \forall v \in V$.

This h is a bijective map satisfying

$$T_A(v_i) = T_A(h(v_i)),\ I_A(h(v_i)) = I_A(h(v_i)),\ F_A(v_i) = F_A(h(v_i)) \forall v_i \in V$$

$$T_B(v_i, v_j) \leq T_B(h(v_i), h(v_j)),\ I_B(v_i, v_j) \leq I_B(h(v_i), h(v_j)),$$

$$F_B(v_i, v_j) \leq F_B(h(v_i), h(v_j)) \forall v_i, v_j \in V.$$

Hence h is a weak isomorphism of the neutrosophic graph to itself. Therefore it satisfies reflexive relation.

(ii) Anti Symmetric

Let h be a weak isomorphism between G and G' and g be a weak isomorphism between G' and G. i.e., $h : V \to V'$ is a bijective map such that $h(v) = v'$, $v \in V$ satisfying

$$T_A(v) = T_{A'}(h(v)),\ I_A(h(v)) = I_{A'}(h(v)),\ F_A(v) = F_{A'}(h(v)) \forall v \in V$$

$$T_B(v_i, v_j) \leq T_{B'}(h(v_i), h(v_j)),\ I_B(v_i, v_j) \leq I_{B'}(h(v_i), h(v_j)),$$

$$F_B(v_i, v_j) \leq F_{B'}(h(v_i), h(v_j)) \forall v_i, v_j \in V \tag{1}$$

and $g : V' \to V$ is a bijective map satisfying

$$T_{A'}(v') = T_A(g(v')),\ I_{A'}(v') = I_A(g(v')),\ F_{A'}(v') = F_A(g(v')) \text{ for all } \forall v' \in V'$$

$$T_{B'}(v_i', v_j') \leq T_B(g(v_i'), g(v_j')),\ I_{B'}(v_i', v_j') \leq I_B(g(v_i'), g(v_j')),$$

$$F_{B'}(v_i', v_j') \leq F_{B'}(g(v_i'), g(v_j')) \forall v_i', v_j' \in V'. \tag{2}$$

The inequalities (1) and (2) hold good on the finite sets V and V' only when G and G' have the same number of edges and the corresponding edges have same weight. Hence G and G' are identical.

(iii) Transitive

Let $h : V \to V'$ and $g : V' \to V''$ be weak isomorphism of the neutrosophic graphs G onto G' and G' onto G'' respectively. Then goh is a 1-1 onto map from $V \to V''$ where

$$(\text{goh})(v) = g(h(v)) \forall v \in V.$$

As $h : V \to V'$ is a weak isomorphism $h(v) = v'$, $v \in V$

$$T_A(v) = T_{A'}(h(v)),\ I_A(v) = I_{A'}(h(v)),\ F_A(v) = F_{A'}(h(v)) \forall v \in V$$

$$T_B(v_i, v_j) \le T_{B'}(h(v_i), h(v_j)),\ I_B(v_i, v_j) \le I_{B'}(h(v_i), h(v_j)),$$

$$F_B(v_i, v_j) \le F_{B'}(h(v_i), h(v_j)) \forall v_i, v_j \in V \tag{3}$$

As $g : V' \to V''$ is a weak isomorphism $g(v') = v''$, for all $v' \in V'$

$$T_{A'}(v') = T_{A''}(g(v')),\ I_{A'}(v') = I_{A''}(g(v')),\ F_{A'}(v') = F_{A''}(g(v)) \text{ for all } \forall v' \in V'$$

$$T_{B'}(v_i, v_j) \le T_{B''}(g(v_i'), g(v_j')),\ I_{B'}(v_i, v_j) \le I_{B''}(g(v_i'), g(v_j')),$$

$$F_{B'}(v_i, v_j) \le F_{B''}(g(v_i'), g(v_j')),\ \forall v_i', v_j' \in V'. \tag{4}$$

Equations (3) and (4) implies

$$T_A(v) = T_{A''}(g(v')) = T_{A''}(g(h(v))),\ I_A(v) = I_{A''}(g(v')) = I_{A''}(g(h(v))),$$

$$F_A(v) = F_{A''}(g(v')) = F_{A''}(g(h(v))) \tag{5}$$

$$T_B(v_i, v_j) \le T_{B''}(g(v_i'), g(v_j')) = T_{B''}(g(h(v_i)), g(h(v_j))),$$

$$I_B(v_i, v_j) \le I_{B''}(g(v_i'), g(v_j')) = I_{B''}(g(h(v_i)), g(h(v_j)))$$

$$F_B(v_i, v_j) \le F_{B''}(g(v_i'), g(v_j')) = F_{B''}(g(h(v_i)), g(h(v_j))). \tag{6}$$

Equations (5) and (6) implies goh is a weak isomorphism between G and G''. i.e., weak isomorphism satisfies transitivity.

(i), (ii) and (iii) implies weak isomorphism between neutrosophic graphs is a partial relation.

Hence the theorem.

4. Isomorphic Neutrosophic Graphs and Their Complements

Definition 4.1 [5]. The complement of a neutrosophic graph $G = (A, B)$ is a neutrosophic graph $\overline{G} = (\overline{A}, \overline{B})$, where

1. $\overline{V} = V$

2. $\overline{T}_A(v_i) = T_A(v_i)$, $\overline{I}_A(v_i) = I_A(v_i)$, $\overline{F}_A(v_i) = F_A(v_i)$, for all $v_i \in V$

3. $$\overline{T}_B(v_i, v_j) = \begin{cases} \min[T_A(v_i), T_A(v_j)] & \text{if } T_B(v_i, v_j) = 0, \\ \min[T_A(v_i), T_A(v_j)] - T_B(v_i, v_j) & \text{if } T_B(v_i, v_j) > 0, \end{cases}$$

$$\overline{I}_B(v_i, v_j) = \begin{cases} \min[I_A(v_i), I_A(v_j)] & \text{if } I_B(v_i, v_j) = 0, \\ \min[I_A(v_i), I_A(v_j)] - I_B(v_i, v_j) & \text{if } I_B(v_i, v_j) > 0, \end{cases}$$

$$\overline{F}_B(v_i, v_j) = \begin{cases} \max[F_A(v_i), F_A(v_j)] & \text{if } F_B(v_i, v_j) = 0, \\ \max[F_A(v_i), F_A(v_j)] - F_B(v_i, v_j) & \text{if } F_B(v_i, v_j) > 0, \end{cases}$$

for all $v_i, v_j \in V$.

Example 4.2. Consider a neutrosophic graph $G = (A, B)$ on the nonempty set $V = \{v_1, v_2, v_3, v_4\}$, $E = \{v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$. Neutrosophic graph $G = (A, B)$ and complement neutrosophic graph $\overline{G} = (\overline{A}, \overline{B})$ are shown in Figure 4.3.

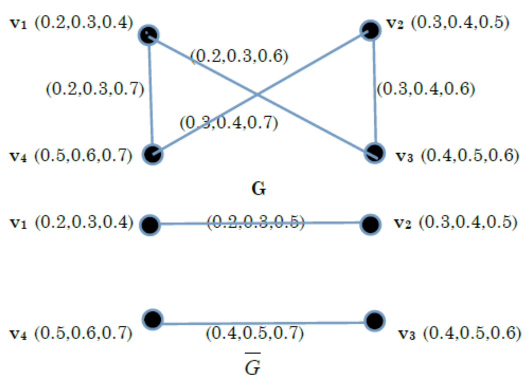

Figure 4.3. Neutrosophic graph G and its complement $\overline{G}$.

Theorem 4.3. *If two neutrosophic graphs are isomorphic then their complements are isomorphic.*

Proof of Theorem 4.3.

Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be the two neutrosophic graphs given. Assume $G_1 \cong G_2$.

There exists a bijective map $h : V_1 \to V_2$ satisfying

$$T_{A1}(v) = T_{A2}(h(v)),\ I_{A1}(v) = I_{A2}(h(v)),\ F_{A1}(v) = F_{A2}(h(v)) \text{ for all } v \in V$$

$$T_{B1}(v_i, v_j) = T_{B2}(h(v_i), (h(v_j)),\ I_{B1}(v_i, v_j) = I_{B2}(h(v_i), (h(v_j)),$$

$$F_{B1}(v_i, v_j) = F_{B2}(h(v_i), (h(v_j))\forall v_i, v_j \in V.$$

By definition

$$\overline{T}_{B_1}(v_i, v_j) = \min[T_{A_1}(v_i), T_{A_1}(v_j)] - T_{B_1}(v_i, v_j)$$

$$= \min[T_{A_2}(h(v_i)), T_{A_2}(h(v_j))] - T_{B_2}(h(v_i), h(v_j))$$

$$= \overline{T}_{B_2}(h(v_i), h(v_j)) \text{ for all } v_i, v_j \in V$$

Similarly $\overline{I}_{B_1}(v_i, v_j) = \overline{I}_{B_2}(h(v_i), h(v_j))$ for all $v_i, v_j \in V$

$\overline{F}_{B_1}(v_i, v_j) = \overline{F}_{B_2}(h(v_i), h(v_j))$ for all $v_i, v_j \in V$.

Hence $\overline{G_1} \cong \overline{G_2}$.

5. Conclusion

In this article isomorphism between neutrosophic graphs is proved to be an equivalence relation and weak isomorphism is proved to be a partial order relation. Also discussed about isomorphic neutrosophic graphs and their complements. Our future work will focus on neutrosophic forests and neutrosophic trees.

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