



# SINGLE DEGREE OF FREEDOM FLEXIBILITY BASED MODEL ANALYSIS OF BRIDGE AND FRAME USING PUSHOVER TECHNIQUE

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## Abstract

This paper presents static nonlinear analysis of a concrete bridge and steel frame case study modeled as an equivalent single degree of freedom. The Principle is to evaluate the static nonlinear demand response using an elegant technique without any discretization required, so called flexibility-based method, which is implemented in a nonlinear program based on the direct stiffness method, offering significant advantages over existing stiffness-based approaches, since no discretization error occurs and all governing equations are satisfied exactly; after comparing the result with another program based displacement method.

## 1. Introduction

The nonlinear static procedure is intended to provide a simplified approach for directly determining the nonlinear response behavior of a structure at different levels of lateral displacements [1, 2]. There is five building performance levels. The performance level is for a combination of performance of structural and non-structural elements. In order to reach the overall building performance level, compatibility should be accomplished between the two; structural and non-structural performance. This difficulty arises when the flexibility-based finite element is implemented in a nonlinear analysis program based on the direct stiffness method, but gives several advantages the most important one is that the element offers

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Keywords: Flexibility based element, Static nonlinear, Response demand, Target displacement.

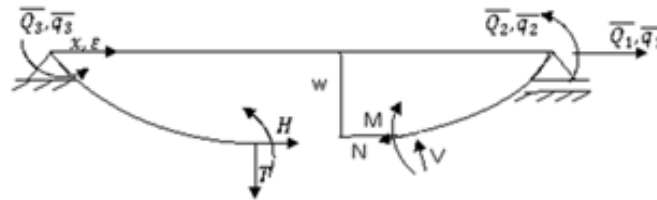
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significant advantages over existing stiffness-based approaches, since no discretization error occurs and all governing equations are satisfied exactly [3].

## 2. Flexibility Based Formulation

The use of flexibility instead of stiffness is motivated by the fact that the dynamically measured flexibility matrix is dominated by the lowest modes of a structure, which can be easily measured, while the dynamically measured stiffness matrix is dominated by the highest modes of the structure, which are hard if not impossible to measure [4].

The most important advantages of flexibility based formulation is that we are not obliged to discretize the structure to reach the accurate results. In this formulation the equilibrium is considered in the deformed element configuration in Figure.1



**Figure 1.** Equilibrium in Deformed Configuration.

The relation between nodal forces  $\bar{Q}$  in the system without rigid body modes and internal forces  $f(x)$  is:

$$f(x) = b[x, w(x)]\bar{Q} \quad (1)$$

$$\text{Where } b[x, w(x)] = \begin{bmatrix} 1 & 0 & 0 \\ -w(\xi) & \xi & \xi - 1 \end{bmatrix}, \xi = \frac{x}{L} \quad (2)$$

The compatibility condition for  $\bar{q}$  reduces to

$$\bar{q}_1 = \int_0^L \varepsilon(x) dx - \int_0^L \frac{1}{2} \chi(x) w(x) dx \quad (3)$$

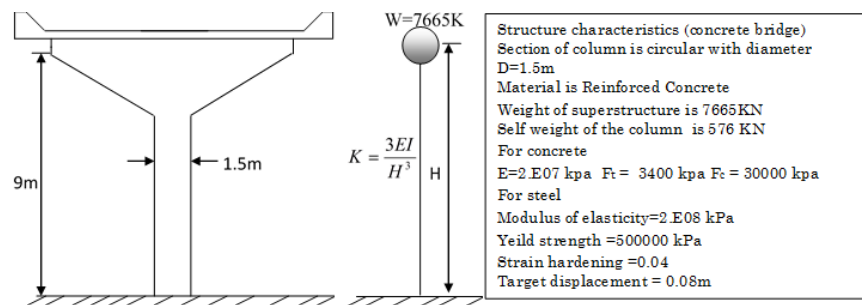
Thus the complete set of governing equations, the flexibility-based is

$$\bar{q}_1 = \int_0^L b^{*t}[x, w(x)]D(x)dx \quad (4)$$

$$\text{With } b^*[x, w(x)] = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{w(\xi)}{2} & \xi & \xi - 1 \end{bmatrix} \quad (5)$$

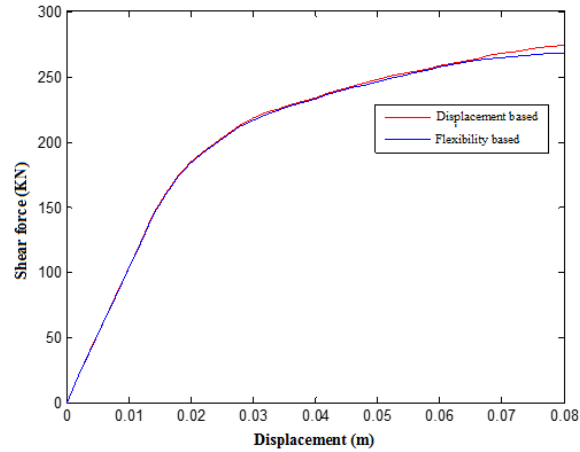
### 3. First example Case Study

The first example is a concrete bridge, which is modeled as single degree of freedom [5], figure bellow show the analytical model as well as structures characteristics. The goal of this first test example is the evaluation of response demand using flexibility formulation.

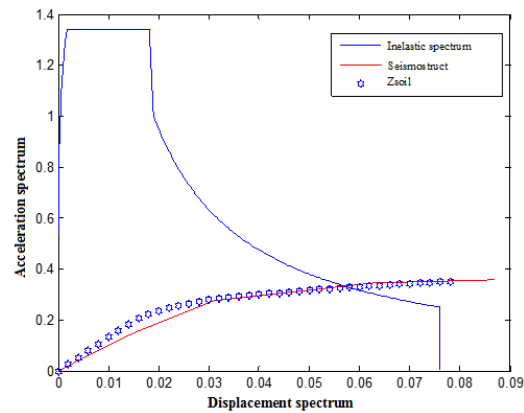


**Figure 2.** Analytical concrete bridge model and its characteristics.

The comparison of bridge capacity curve between flexibility and displacement based model as well as of the target displacement are presented in figures bellow.



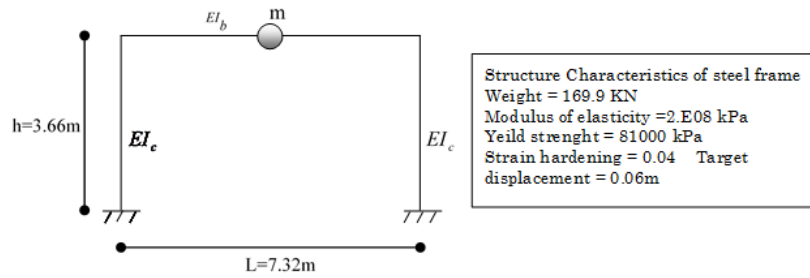
**Figure 3.** Comparison of bridge capacity curve between flexibility and displacement based model.



**Figure 4.** Target displacement of concrete bridge under inelastic spectrum.

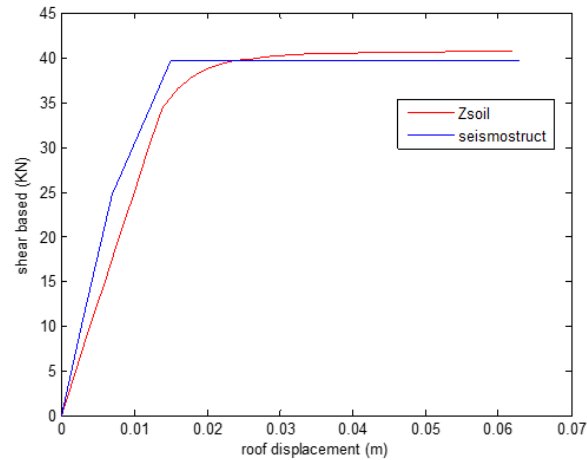
#### 4. Second Example Case Study

The second example is a steel frame, which is modeled as single degree of freedom [5] [6], figure below show the analytical model as well as structures characteristics. The goal of this first example is the evaluation of response demand using flexibility formulation.

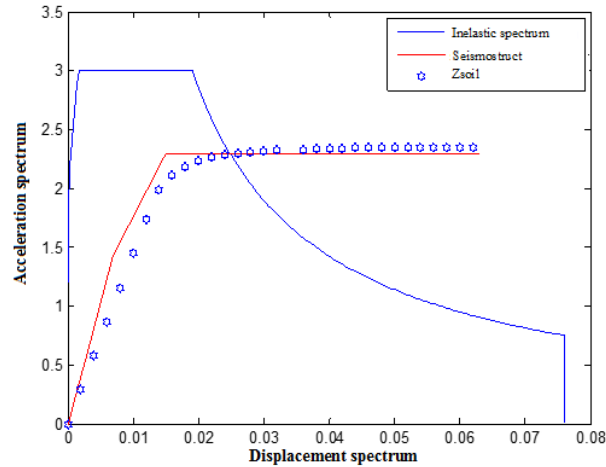


**Figure 5.** Analytical steel frame model and its characteristics.

The comparison of steel frame capacity curve between Zsoil software based on flexibility formulation and Seismostruct software based one displacement formulation model as well as of the target displacement are presented in figures bellow.



**Figure 5.** Comparison of steel frame capacity curve between flexibility and displacement based model.



**Figure 6.** Target displacement of steel frame under inelastic spectrum.

### 5. Results Discussion

Figure 3 allow as to have an idea about shear force base value of concrete bridge which is equal to 269.5 KN, we notice that it is obvious that the results between flexibility and displacement based model are very close each other. For the target, displacement is given by figure 4 which is equal to 0.06m.

The same observation for steel frame so figure 5 gives a shear force base value of steel frame which is about 40 KN, thus we can say that the results between flexibility and displacement based model are very close. For the target, displacement is given by figure 6, which is equal to 0.027m.

### 6. Conclusion

There is no doubt that the use of the static nonlinear analysis is more convenient and easy than the nonlinear dynamic method. Because this method has an undeniable advantage namely graphical interpretation and to allow the designer to follow more closely the nonlinear structure response. We must mention the great advantage of the elegant flexibility formulation method which allow also the designer to use fewer element because of the force interpolation function used in that formulation which is strictly satisfy element equilibrium at each point of the structure.

### References

- [1] Anil K. Chopra and Rakesh K. Goel, A modal pushover analysis procedure to estimate seismic demands for building, theory and preliminary evaluation, Prentice Hall, (2001).
- [2] P. Fajfar, Strucural analysis in earthquake engineering a breakthrough of simplified non-linear methods, Proceedings of 12th European Conference on Earthquake Engineering 843 (2002).
- [3] M. Belgasmia and S. Moussaoui, Comparison between static nonlinear and time history analysis using Flexi-Bility-Based model for an existing structure and effect of taking into account soil using domain reduction method for a single media, KSCE Journal of Civil Engineering 19(3) (2015), 651-663.
- [4] E. Reynders and G. Roeck, A local flexibility method for Vibration-Based damage localization and quantification, Journal of Sound and Vibration 329(12) (2010), 2367-2383.
- [5] S. Moussaoui and M. Belgasmia, Static nonlinear analysis case study european, Journal of Advances in Engineering and Technology 5(3) (2018), 156-161.
- [6] M. belgasmia, Definition of static nonlinear procedure and flexibility-based model with application on 2D model for an existing structure and comparing results with time history analysis, IGI Global USA, (2018), pp. 30.