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MOTIF ON $(x^2 - y^2)(Ax^2 + Ay^2 - (2A - 2)xy) = (2A + 3)(X^2 - Y^2)Z^5$

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Abstract

Astonishing values of x, y, X, Y, Z in two dissimilar techniques of the seventh degree diophantine equation $(x^2 - y^2)(Ax^2 + Ay^2 - (2A-2)xy) = (2A+3)(X^2 - Y^2)Z^5$ are perceived. Some engrossing connections between x, y, X, Y, Z and unusual numbers namely pronic, triangular, 4-dimensional figurate, and polygonal numbers are conferred.

Symbols worn:

$(Pr)_a$: Pronic number of rank $a = a(a+1)$

$(Tri)_a$: Triangular number of rank $a = \frac{a(a+1)}{2}$

(4DF) : 4D figurate number $= \frac{a^4 - a^2}{12}$

$(PenPy)_a$: Pentagonal Pyramidal number of rank $a = \frac{a^2(a+1)}{2}$

$T_{m,n}$: Polygonal numbers of rank n with sides

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$$m = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$$

(Ptope)$_a$: Pentatope number of rank $a = \frac{a(a+1)(a+2)(a+3)}{24}$.

1. Establishment

In the field of Mathematics, Number theory has an extraordinary and remarkable place in the era of research. Solving Diophantine equations of any degree is a challenging one [1-3] in number theory. There are dissimilar kinds of diophantine equations such as exponential, surd, logarithmic equations solved by mathematicians around the world [4-12]. One of the alluring Diophantine problem, to find the values of x, y, X, Y, Z in this paper is $(x^2 - y^2)(Ax^2 + Ay^2 - (2A-2)xy) = (2A+3)(X^2 - Y^2)Z^5$. Some engrossing connections between x, y, X, Y, Z and unusual numbers namely pronic, triangular, 4 dimensional figurate, and polygonal numbers are conferred.

2. Description of Method

Contemplate the seventh degree equation with five unknowns

$$(x^2 - y^2)(Ax^2 + Ay^2 - (2A-2)xy) = (2A+3)(X^2 - Y^2)Z^5. \quad (1)$$

Launching the linear modifications

$$x = u + v,\ y = u - v,\ X = 2u + v,\ Y = 2u - v \quad (2)$$

in (1), arrives

$$u^2 + (2A-1)v^2 = (2A+3)Z^5. \quad (3)$$

2.1. Motif I

Motif on $(x^2 - y^2)(Ax^2 + Ay^2 - (2A-2)xy) = (2A+3)(X^2 - Y^2)Z^5$.

Let us hold of

$$Z = a^2 + (2A-1)b^2 \quad (4)$$

where $a \neq b$.

Take $2A + 3$ as

$$2A + 3 = (2 + i\sqrt{2A-1})(2 - i\sqrt{2A-1}). \tag{5}$$

Utilize (4) and (5) in (3) applying the resolving activity, we have

$$(u + i\sqrt{2A-1}v) = (2 + i\sqrt{2A-1})(a + i\sqrt{2A-1}b)^5.$$

After specific algebraic computations and clarify, we obtain

$$u = (2a^5 - 5(2A-1)a^4b - 20(2A-1)a^3b^2 + 10(2A-1)^2a^2b^3$$

$$+ 10(2A-1)^2ab^4 - (2A-1)^3b^5)$$

$$v = (a^5 + 10a^4b - 10(2A-1)a^3b^2 - 20(2A-1)a^2b^3$$

$$+ 5(2A-1)^2ab^4 + 2(2A-1)^2b^5).$$

This replacement of u and v in (2) the solutions of equation (1) is, as follows

$$x(a, A, b) = (3a^5 - [10 - 5(2A-1)]a^4b - [30(2A-1)]a^3b^2 + [10(2A-1)^2$$

$$- 20(2A-1)]a^2b^3 + 15(2A-1)^2ab^4 + [2(2A-1)^2 - (2A-1)^3]b^5)$$

$$y(a, A, b) = (a^5 - [10 + 5(2A-1)]a^4b - 10(2A-1)a^3b^2 + [10(2A-1)^2$$

$$+ 20(2A-1)]a^2b^3 + 5(2A-1)^2ab^4 + [(2A-1)^3 + 2(2A-1)^2]b^5)$$

$$Z = a^2 + (2A-1)b^2.$$

3. Inspections

Case (i):

3.1. Choose $a = 0;\ A = 5;\ b = 1$

1. It is scrutiny that all $x(a, A, b), y(a, A, b), X(a, A, b), Y(a, A, b),$ are negative and $Z(a, A, b),$ is positive numbers.

2. $x(0, 5, 1) - y(0, 5, 1) - X(0, 5, 1) + Y(0, 5, 1) = 0$

3. $Z(0, 5, 1)$ is a perfect square.

4. $6Z(0, 5, 1)$ is a nasty number.

5. $x(0, 5, 1) + y(0, 5, 1) + X(0, 5, 1) + Y(0, 5, 1) \equiv 0 \pmod 9$.

Case (ii):

3.2. Choose $a = 1;\ A = 1;\ b = 1$

1. It is scrutiny that all $x(1, 1, 1), y(1, 1, 1), Y(1, 1, 1), Z(1, 1, 1)$ are positive and $X(1, 1, 1)$ is negative.

2. The succeeding forms a nasty number.

(i) $x(1, 1, 1) + X(1, 1, 1)$

(ii) $Y(1, 1, 1) + Z(1, 1, 1)$

3. $x(1, 1, 1) + X(1, 1, 1) + Z(1, 1, 1) = y(1, 1, 1)$

4. $y(1, 1, 1) = 2Y(1, 1, 1)$

5. $x(1, 1, 1) + y(1, 1, 1) + X(1, 1, 1) + Y(1, 1, 1) = 9Z(1, 1, 1)$

6. $x(1, 1, 1) + y(1, 1, 1) + Z(1, 1, 1)$ is a perfect square.

Case (iii).

3.3. Choose $a = a;\ A = 8;\ b = 1$

1. $x(a, 8, 1) + Y(a, 8, 1) = y(a, 8, 1) + X(a, 8, 1)$

2. $6Z(a, 8, 1) - 90$ is a nasty number.

3. $Z(a, 8, 1) - a^2 \equiv 0 \pmod{15}$

4. $Y(a, 8, 1) - x(a, 8, 1) + 4275 = 2755\,(pro)_a - 1140(4DF)_a - 2755a$

5. $2x(a, 8, 1) - X(a, 8, 1) = a(pro)_{a^2} + 24(ptope)_a + 108(4DF) - 314(PenPy)_a$ $- 58(Hep)_a + 1032a + 450.$

4. Motif II

From equation (3) we have $u^2 + (2A-1)v^2 = (2A+3)Z^5$, alternatively

$$u^2 + (2A-1)v^2 = (2A+3)Z^5 * 1. \tag{6}$$

Take 1 as $1 = \frac{[(A-1) + i\sqrt{2A-1}][(A-1) + i\sqrt{2A-1}]}{A^2}$ switch in (6) and apply the procedure, as in motif I we have

$$(u + i\sqrt{2A-1}v) = \frac{1}{A}(2 + i\sqrt{2A-1})(a + i\sqrt{2A-1}b)^5[(A-1) + (i\sqrt{2A-1})].$$

Equating real and imaginary parts yields, u and v as in the succeeding expressions

$$u = \frac{1}{A}[-a^5 - 5(2A-1)(A+1)a^4b + 10(2A-1)a^3b^2 + 10(2A-1)^2(A+1)a^2b^3$$

$$- 5(2A-1)^2ab^4 = (2A-1)]$$

$$v = \frac{1}{A}[(A+1)a^5 - 5a^4b - 10(A+1)(2A-1)a^3b^2 + 10(2A-1)a^2b^3$$

$$+ 5(A+1)(2A-1)^2ab^4 - (2A-1)^2b^5]$$

substitute u and v in equation (2), and the replacement of a by A_a, b by A_b, the integral solutions of equation (1) are as follows:

$$x(a, A, b) = A^4\{Aa^5 - 5[(2A-1)(A+1)+1]a^4b + 10[(2A-1) - (A+1)(2A-1)]$$

$$a^3b^2 + 10[(2A-1)^2(A+1) + (2A-1)]a^2b^3 - 5[(2A-1)^2 - (A+1)(2A-1)^2]$$

$$ab^4 - [(2A-1)^3 - (A+1) + (2A-1)^2]b^5\}$$

$$y(a, A, b) = A^4\{-(A+2)a^5 + 5[1 - (2A-1)(A+1)]a^4b + 10[(2A-1) + (A+1)(2A-1)]$$

$$a^3b^2 + 10[(2A-1)^2(A+1) - (2A-1)]a^2b^3 - 5[(2A-1)^2 - (A+1)(2A-1)^2]$$

$$ab^4 + [(2A-1)^2 - (2A-1)^3(A+1)]b^5\}$$

$$X(a, A, b) = A^4\{(A-1)a^5 - 5[2(2A-1)(A+1)+1]a^4 b + 10[2(2A-1) - (A+1)(2A-1)]$$

$$a^3 b^2 + 10[2(2A-1)^2(A+1) - (2A-1)]a^2 b^3 - 5[2(2A-1)^2 - (A+1)(2A-1)^2]$$

$$ab^4 - [(2A-1)^3(A+1) + (2A-1)^2]b^5\}$$

$$Y(a, A, b) = A^4\{-(A+3)a^5 - 5[2(2A-1)(A+1)-1]a^4 b + 10[2(2A-1) - (A+1)(2A-1)]$$

$$a^3 b^2 + 10[2(2A-1)^2(A+1) - (2A-1)]a^2 b^3 - 5[2(2A-1)^2 - (A+1)(2A-1)^2]$$

$$ab^4 - [2(2A-1)^3(A+1) - (2A-1)^2]b^5\}$$

$$Z(a, A, b) = A^2[a^2 + (2A-1)b^2].$$

5. Inspections

Case I:

5.1. Choose $a = a,\ A = 1,\ b = 0$

1. $x(a, 1, 0) + y(a, 1, 0) + 2aZ(a, 1, 0) + 24a(4DF) = 0$

2. $6Z(a, 1, 0)$ is a nasty number.

3. $y(a, 1, 0) - x(a, 1, 0) = Y(a, 1, 0)$

4. $x(a, 1, 0) + y(a, 1, 0) + Y(a, 1, 0) - 6aZ(a, 1, 0) = -6a(pro)_{a^2}$.

Case II.

5.2. Choose $a = 1,\ A = 1,\ b = 1$

1. $y(1, 1, 1) + Z(1, 1, 1) = Y(1, 1, 1)$

2. $y(1, 1, 1) - Z(1, 1, 1) = a$ nasty number.

3. $x(1, 1, 1)\ Z(1, 1, 1)$ is a perfect number.

4. $x(1, 1, 1) + y(1, 1, 1) + X(1, 1, 1) + Y(1, 1, 1) - 3$ is perfect square.

5. $Y(1, 1, 1) - X(1, 1, 1) = x(1, 1, 1)$

Case III:

5.3. Choose $a = 1$; $A = 1$; $b = b$

1. $5y(1,1,b)+X(1,1,b)+200b+15=600(ptope)_b-150(Sqpy)_b-25(Hex)_b$

2. $6\{Z(1,\ 1,\ b)-1\}$ is a nasty number.

3. $X(1,1,b)+Y(1,1,b)-2[x(1,1,b)+y(1,1,b)]-160b=120(4DF)_b$
$-(240)b(Tet)_b+320(PenPy)_b-320(Tri)_b$.

6. Termination

In this work, we have remarked various procedure of ascertaining limitless non-zero integer values to the non-homogeneous seventh degree Diophantine equation

$$(x^2-y^2)(Ax^2+Ay^2-(2A-2)xy)=(2A+3)(X^2-Y^2)Z^5.$$

One may make an effort to notice non-negative solutions of the above equations in diverse technique together with their similar inspections.

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