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VAGUE REGULAR ALPHA GENERALIZED IRRESOLUTE MAPPING IN TOPOLOGICAL SPACES

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Abstract

The purpose of this paper is to introduce the concept of vague regular α generalized irresolute mapping in vague topological spaces and also to establish some results with suitable examples. Further the notion of vague regular $\alpha T_{1/2}$ space, vague regular α generalized $T_{1/2}$ space are introduced and discussed.

1. Introduction

The concept of vague sets was introduced by Gau and Buehrer in 1993 [5] as an extension of fuzzy set theory. Many research were conducted on the notions of vague sets and vague topological spaces. The vague topological spaces and vague topological additive groups were introduced by Amarendra Babu et al. in 2017 [1]. On generalized continuous maps in topological spaces was introduced by K. Balachandran in 1991 [2]. The concept of irresolute functions was introduced by S. G. Crossley and S. K. Hildebrand in 1972 [4]. They proved that the irresolute functions are stronger than semi continuous but are independent of continuous functions. Also many researches worked and introduced various strong and weak forms of irresolute functions. By definition, a map $f : (X, \tau) \to (Y, \sigma)$ is called irresolute if $f^{-1}(V)$ is semi closed in (X, τ) for every semi closed set V in (Y, σ). A new class of vague

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generalized alpha mapping was introduced by L. Mariapresenti and I. Arockiarani in 2016 [7]. A vague regular alpha generalized continuous mapping was introduced by S. Bharathi, D. Poongodi in 2021 [3]. In this paper we introduce the concept of vague regular alpha generalized irresolute mapping and throughout the paper (X, τ), (Y, σ) and (X, μ) represent vague topological spaces. As an application we have also defined vague regular $\alpha T_{1/2}$ spaces and vague regular α generalized $T_{1/2}$ spaces.

2. Preliminaries

Definition 2.1[5]**.** Let X be the universe of discourse. A vague set is an object having the form $A = \{\langle X, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ is represented by a true membership function t_A and false membership function f_A, where $t_A(x)$ is the lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a sub interval $[t_A(x), 1 - f_A(x)]$ of $[0, 1]$. This expresses that if the actual grade of membership $\mu(x)$, then $t_A(x) \leq \mu(x) \leq f_A(x)$.

Definition 2.2[5]**.** Consider a two vague sets A and B of the form $A = \{\langle X, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle X, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$

Then the following properties are given below

(i) $t_A(x) \leqslant t_B(x)$ and $1 - f_A(x) \leqslant 1 - f_B(x)$ for all $x \in X \leftrightarrow A \subseteq B$

(ii) $A = B \leftrightarrow A \subseteq B$ and $B \subseteq A$

(iii) $A^C = \{\langle X, [f_A(x), 1 - t_A(x)]\rangle / x \in X\}$

(iv) $A \cap B = \{\langle x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$

(v) $A \cup B = \{\langle x, \max(t_A(x), t_B(x)), \max(1 - f_A(x), 1 - f_B(x))\rangle / x \in X\}$.

Definition 2.3[8]**.** A vague topology on X satisfies the following axioms

(i) $0, 1 \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is a vague topological space and any vague set A in τ is known as a VOS in X. A vague set A is a VCS iff its compliment is a VOS in X.

Definition 2.4[6]. A mapping $f : (A, \tau) \to (B, \sigma)$ is called

- Vague generalized continuous if $f^{-1}(F)$ is a vague generalized closed set in (X, τ) for every vague closed set F in (Y, σ).
- Vague generalized semi continuous if $f^{-1}(F)$ is a vague generalized semi closed set in (X, τ) for every vague closed set F in (Y, σ).
- Vague generalized pre continuous if $f^{-1}(F)$ is a vague generalized pre closed set in (X, τ) for every vague closed set F in (Y, σ).
- Vague generalized regular continuous if $f^{-1}(F)$ is a vague generalized regular closed set in (X, τ) for every vague closed set F in (Y, σ).
- Vague generalized α continuous if $f^{-1}(F)$ is a vague generalized α closed set in (X, τ) for every vague closed set F in (Y, σ).
- Vague generalized irresolute if $f^{-1}(F)$ is a vague generalized closed set in (X, τ) for every vague Generalized closed set F in (Y, σ).

Definition 2.5[2]. A mapping $f : (A, \tau) \to (B, \sigma)$ is called a vague regular α generalized continuous mapping if $f^{-1}(F)$ is a $Vr\alpha GCS$ in (X, τ) for every vague closed set F of (Y, σ).

3. Main Results

3.1. Vague regular α generalized irresolute mapping

Definition 3.1. A mapping $f : (A, \tau) \to (B, \sigma)$ is called a vague regular α generalized irresolute mapping if $f^{-1}(B)$ is a $Vr\alpha GCS$ in (X, τ) for every $Vr\alpha GCS$ B of (Y, σ).

Theorem 3.1. *A map* $f : (A, \tau) \to (B, \sigma)$ *is a* $Vr\alpha G$ *irresolute mapping iff the inverse image of every* $Vr\alpha GOS$ *in Y is a* $Vr\alpha GOS$ *in X.*

Proof. Assume that f is a $Vr\alpha G$ irresolute. Let B be any $Vr\alpha GOS$ in Y. Then B^C is a $Vr\alpha GCS$ in Y. Since f is a $Vr\alpha G$ irresolute mapping, $f^{-1}(B^C)$ is a $Vr\alpha GCS$ in X. Since $f^{-1}(B^C) = (f^{-1}(B))^C$, $f^{-1}(B)$ is a $Vr\alpha GOS$ in X. Hence the inverse image of every $Vr\alpha GOS$ in Y is $Vr\alpha GOS$ in X. The converse is obvious from the definition 3.1.

Theorem 3.2. *If a map* $f : (A, \tau) \to (B, \sigma)$ *is a* $Vr\alpha G$ *irresolute mapping then it is a* $Vr\alpha G$ *continuous but not conversely*.

Proof. Assume that f is a $Vr\alpha G$ irresolute. Let F be a vague closed set in Y. Since every vague closed set is a $Vr\alpha GCS$, F is a $Vr\alpha GCS$ in Y. Since f is a $Vr\alpha G$ irresolute, $f^{-1}(F)$ is a $Vr\alpha GCS$ in X. Therefore f is a $Vr\alpha G$ continuous.

Example 3.1. Let $X = p, q, Y = a, b$ with topologies $\tau = 0, G_1, X$ and $\sigma = 0, G_2, Y$. Then $G_1 = \{\langle x, [0.1, 0.3], [0.4, 0.6] \rangle\}$ and $G_2 = \{\langle x, [0.4, 0.6], [0.3, 0.2] \rangle\}$. Define a mapping $f : (A, \tau) \to (B, \sigma)$ by $f(p) = a$, $f(q) = b$. Then f is a $Vr\alpha G$ continuous mapping. Let $B = \{\langle x, [0.6, 0.7], [0.5, 0.8] \rangle\}$ is a $Vr\alpha GCS$ in Y. But $f^{-1}(B)$ is not a $Vr\alpha GCS$ in X. Therefore f is not a $Vr\alpha G$ irresolute mapping.

Theorem 3.3. *For any* $Vr\alpha G$ *irresolute map* $f : (A, \tau) \to (B, \sigma)$ *and any* $Vr\alpha G$ *continuous map* $g : (X, \tau) \to (Z, \mu)$. *The composition* $g \circ f : (X, \tau) \to (Z, \mu)$ *is a* $Vr\alpha G$ *continuous mapping*.

Proof. Let F be a vague closed set in Z. Since g is a $Vr\alpha G$ continuous, $g^{-1}(F)$ is a $Vr\alpha GCS$ in Y. Since f is a $Vr\alpha G$ irresolute, $f^{-1}(g^{-1}(F))$ is a $Vr\alpha GCS$ in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$. Therefore $g \circ f : (X, \tau)\beta(Z, \mu)$ is a $Vr\alpha G$ continuous mapping.

Theorem 3.4. *Let* $f : (X, \tau) \to (Y, \sigma)$ *and* $g : (X, \tau) \to (Z, \mu)$ *be two* $Vr\alpha G$ *irresolute mappings*, *then* $g \circ f : (X, \tau) \to (Z, \mu)$ *is a* $Vr\alpha G$ *irresolute*.

Proof. Let B be a $Vr\alpha GCS$ in Z. Then g is a $Vr\alpha G$ irresolute mapping. $g^{-1}(B)$ is a $Vr\alpha GCS$ in Y. Also f is a $Vr\alpha G$ irresolute mapping, $f^{-1}(g^{-1}(B))$ is a $Vr\alpha GCS$ in X. But $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(E)$.

Therefore but $g \circ f$ is a $Vr\alpha G$ irresolute mapping.

Remark. The irresolute maps and $Vr\alpha G$ irresolute mappings are independent of each other.

Definition 3.2. Let (X, τ) be a vague topological space. The vague regular α generalized closure for any vague set A is defined by, $Vr\alpha Gcl(A) = \bigcap\{K/K$ is a $Vr\alpha GCS$ in X and $A \subseteq K\}$. If A is a $Vr\alpha GCS$, then $Vr\alpha Gcl(A) = A$.

Remark 3.9. $A \subseteq Vr\alpha Gcl(A) \subseteq Vcl(A)$.

Theorem 3.5. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a mapping from a vague topological space X into a vague topological space Y. Then the following are equivalent*:

i. *f is a* $Vr\alpha G$ *irresolute*

ii. $f^{-1}(B)$ *is a* $Vr\alpha GOS$ *in X for every* $Vr\alpha GOS$ *B in Y*

iii. $Vr\alpha Gcl(f^{-1}(B) \subseteq f^{-1}(Vr\alpha Gcl(B)))$ *for every VS B in Y*

iv. $f^{-1}(Vr\alpha G\,\mathrm{int}(B)) \subseteq Vr\alpha G\,\mathrm{int}(f^{-1}(B))$ *for every VS B in Y.*

Proof. (i) $\to$ (ii) It is obvious from the definition 3.1.

(ii) $\to$ (iii) Let B be any vague set in y. Then $B \subseteq cl(B)$ also $f^{-1}(B) \subseteq f^{-1}(Vr\alpha Gcl(B))$. Since $Vr\alpha Gcl(B)$ is a $Vr\alpha GCS$ in Y. $f^{-1}(Vr\alpha Gcl(B))$ is a $Vr\alpha GCS$ in X. Therefore $Vr\alpha Gcl(f^{-1}(B)) \subseteq f^{-1}(Vr\alpha Gcl(B))$.

(iii) $\to$ (iv) It can be proved by taking the compliment of (iii).

(iv) $\to$ (i) Let B be any $Vr\alpha GOS$ in Y. Then $Vr\alpha G\,\mathrm{int}(B) = B$. By assumption we have $f^{-1} = f^{-1}(Vr\alpha G\,\mathrm{int}(B)) \subseteq Vr\alpha G\,\mathrm{int}(f^{-1}(B))$.

Therefore $f^{-1}(B)$ is a $Vr\alpha GOS$ in X. Hence f is a $Vr\alpha G$ irresolute mapping.

4. Applications

We introduce vague regular $\alpha T_{1/2}$ space, vague regular $\alpha T^{*}_{1/2}$ space, vague regular α generalized $T_{1/2}$ space and their characterizations also proved.

Definition 4.1. A vague topological space (X, τ) is said to be vague regular $\alpha T_{1/2}$ space, if every vague regular α generalized closed set is a vague α closed set in X.

Definition 4.2. A vague topological space (X, τ) is said to be vague regular $\alpha T^{*}_{1/2}$ space, if every vague regular α generalized closed set is a vague closed set in X.

Theorem 4.1. *A vague topological space* (X, τ) *is a vague regular* $\alpha T_{1/2}$ *space iff every vague* α *open set is a vague regular* α *generalized open set in* X.

Proof. Let A be a vague regular α generalized open set in X, then the compliment of A is a vague regular α generalized closed set in X. By hypothesis A^{C} is a vague α closed set in X. Therefore A is a vague α open set in X.

Conversely, let A be a vague regular α generalized closed set in X.

Then the compliment of A is a vague regular α generalized open set in X. By hypothesis A^{C} is a vague α open set in X. Therefore A is a vague α closed set in X. Hence (X, τ) is said to be vague regular $\alpha T_{1/2}$ space.

Theorem 4.2. *For a vague regular* $\alpha T_{1/2}$ *space in* (X, τ), *the following properties are equivalent*:

i. $A \in Vr\alpha GO(X)$

ii. $A \subseteq V\operatorname{int}(Vcl(V\operatorname{int}(A)))$

iii. *There exists a vague open set* G *such that* $G \subseteq A \subseteq V\operatorname{int}(Vcl(G))$.

Proof. (i) → (ii) is obvious. (ii) → (iii) Let $A \subseteq V\operatorname{int}(Vcl(V\operatorname{int}(A)))$. Then $V\operatorname{int}(A) \subseteq A \subseteq V\operatorname{int}(Vcl(V\operatorname{int}(A)))$. Therefore we have a vague open set $G = V\operatorname{int}(A)$ in X such that $G \subseteq A \subseteq V\operatorname{int}(Vcl(G))$.

(iii) → (i) Suppose there exists a vague open set $G = V\operatorname{int}(A)$ such that $G \subseteq A \subseteq V\operatorname{int}(Vcl(G))$ then $V\operatorname{int}(Vcl(G))^C \subseteq A^C$. That is $V\operatorname{int}(Vcl(V\operatorname{int}(A)))^C \subseteq A^C$, which implies $V\operatorname{int}(Vcl(V\operatorname{int}(A^C))) \subseteq A^C$. Therefore A^C is a vague α closed set in X. Then A is a vague α open set in X and A is a vague α generalized open set in X. Hence $A \in Vr\alpha GO(X)$.

Definition 4.3. A vague topological space (X, τ) is said to be a vague regular α generalized $T_{1/2}$ space, if every vague regular α generalized closed set in X is a vague α generalized closed set in X.

Theorem 4.3. *If a vague topological space (X, τ) is a vague regular α generalized $T_{1/2}$ space, then every vague regular α generalized open set is a vague α generalized open set in X.*

Proof. Let A be a vague regular α generalized open set in X. This implies A^C is a vague regular α generalized closed set in X. Since X is a vague regular α generalized $T_{1/2}$ space, then A^C is a vague α generalized closed set in X. Hence A is a vague α generalized open set in X.

Theorem 4.4. *Let $f : (X, \tau) \to (Y, \sigma)$ be a $Vr\alpha G$ irresolute mapping, then f is a vague irresolute mapping if X is a $Vr\alpha T^*_{1/2}$ space.*

Proof. Let F be a vague closed set in Y. Then F is a $Vr\alpha GCS$ in Y. Therefore $f^{-1}(F)$ is a $Vr\alpha GCS$ in X, by hypothesis. Since X is a $Vr\alpha T^*_{1/2}$ space, $f^{-1}(F)$ is a VCS in X. Hence f is a vague irresolute mapping.

Theorem 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a $Vr\alpha G$ irresolute mapping, then f is a vague α irresolute mapping if X is a $Vr\alpha T_{1/2}$ space.*

Proof. Let M be a vague α closed set in Y. Then M is a $Vr\alpha GCS$ in Y.

Therefore $f^{-1}(M)$ is a $Vr\alpha GCS$ in X, by hypothesis. Since X is a $Vr\alpha T_{1/2}$ space, $f^{-1}(M)$ is a $V\alpha CS$ in X. Hence f is a vague α irresolute mapping.

5. Conclusion

In this paper, we have introduced and studied some of the properties of the vague regular α generalized irresolute mapping in vague topological spaces. Also we studied the characterization of the vague regular $\alpha T_{1/2}$ space, vague regular $\alpha T_{1/2}^{*}$ space and vague regular α generalized $T_{1/2}$ space. These concepts will be helpful to raise our research work in vague topological spaces.

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