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DOMINATION PARAMETERS IN TOTAL GRAPH $T(G)$ OF A GRAPH G

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Abstract

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. The Total Graph $T(G, L(G), NIC)$ of G is a graph with vertex set $V(G) \cup E(G)$ where two points are adjacent if and only if they are adjacent points of G or they are adjacent lines of G or one is a point of G and another is a line of G incident with it. For simplicity, this graph is denote by $T(G)$. In this paper, we have studied some bounds for domination number of Total graph of a graph. Also, we have found the exact value the domination number of $T(G)$ for some graphs G.

Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. For two vertices u and v in a connected graph G, the distance $d(u, v)$ from u to v is the length of a shortest $u - v$ path in G. A vertex and an edge are said to cover each other if they are incident. A set of vertices which covers all the edges of a graph G is called a point cover for G, while a set of edges which

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covers all the vertices is a line cover. The smallest number of vertices in any point cover for G is called its point covering number or simply covering number and is denoted by $\alpha_0(G)$ or α_0. Similarly, α_1 is the smallest number of edges in any line cover of G and is called its line covering number. A set of vertices in G is independent if no two of them are adjacent. The largest number of vertices in such a set is called the point independence number of G and is denoted by $\beta_0(G)$ or β_0. Analogously, an independent set of edges of G has no two of its edges adjacent and the maximum cardinality of such a set is the line independence number $\beta_1(G)$ or β_1. Here G is a graph with p vertices and q edges. Number of vertices in $L(G)$ is q and number of edges in $L(G)$ is $1/2\,\Sigma di^2 - q$. For graph theoretic terminology refer [3].

The total graph $T(G)$ of a graph G is the graph with vertex set $V(G) \cup E(G)$ and two vertices of $T(G)$ are adjacent if and only if the corresponding elements of G are adjacent or incident. The concept of domination in graphs was introduced by Ore [6]. A set $D \subseteq V(G)$ is said to be a dominating set of G, if every vertex in $V(G) - D$ is adjacent to some vertex in D. D is said to be a minimal dominating set if $D - \{u\}$ is not a dominating set for any $u \in D$. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. We call a set of vertices a γ-set, if it is a dominating set with cardinality $\gamma(G)$. A dominating set D of a graph $G = (V, E)$ is a total dominating set, if the induced sub graph $\langle D \rangle$ has no isolated vertices. The total domination number $\gamma_t(G)$ of a graph G is the minimum cardinality of a total dominating set. In [1, 2], Bhanumathi and Mariselvi studied Eccentric domination in total graph of a graph. In [1], they have evaluated the domination parameters of total graphs of $K_{m,n}$, P_n^+, $K_{2n} - F$. In this paper, we study domination number of the total graph $T(G)$ of a graph G.

2. Domination Parameters in Total Graph $T(G)$ of a Graph G

Here, we find out some minimum dominating set of Total graph and the exact value of $\gamma(T(G))$ for some particular classes of graphs.

Theorem 2.1. (i) $\gamma(T(P_n)) = (2n-1)/5$ *if* $2n \equiv 1 (\text{mod } 5)$.

(ii) $\gamma(T(P_n)) = \lceil (2n-1)/5 \rceil$, *if* $2n \not\equiv 1 (\text{mod } 5)$.

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of path P_n and let $u_1, u_2, u_3, \ldots, u_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2, e_3, \ldots, e_{n-1}$ of path P_n to obtain $T(P_n)$ Thus $V(T(P_n)) = \{v_1, u_1, u_2, u_2, \ldots, u_{n-1}, v_n\}$ the graph $T(P_n)$ will have $(2n-1)$ vertices. The set $D = \{v_2\}$ is clearly minimum dominating set of $T(P_2)$ and $T(P_3)$ respectively with minimum cardinality. Hence $\gamma(T(P_2)) = 1$ and $\gamma(T(P_3)) = 1$, for $n \le 3$.

Case (i). Let $2n \equiv 1 (\text{mod } 5)$.

Let $D = \{v_2, u_4, v_7, u_9, \ldots, v_{n-1}\}$. The set D is a dominating set of $T(P_n)$. $|D| = \frac{2n-1}{5}$ Also for any vertex $v \in D$, the set $D - \{v\}$ is not a dominating set of $T(P_n)$. Also, $\left\lfloor \frac{2n-1}{1+\Delta(T(P_n))} \right\rfloor = \left\lfloor \frac{2n-1}{5} \right\rfloor \le \gamma(T(P_n))$. Therefore, $\gamma(T(P_n)) \le \frac{2n-1}{5}$, if $2n \equiv 1 (\text{mod } 5)$. Hence, $\gamma(T(P_n)) = \frac{2n-1}{5}$.

Case (ii). Let $2n \not\equiv 1 (\text{mod } 5)$.

The set $D \cup \{v_n\}$ or $D \cup \{u_{n-1}\}$ is a dominating set. Therefore, $\gamma(T(P_n)) \le \lceil (2n-1)/5 \rceil$. $\Delta(T(P_n)) = 4$. This implies that an vertex of $T(P_n)$ can dominate exactly 5 distinct vertices of $T(P_n)$ including itself. This implies that, $\left\lfloor \frac{2n-1}{1+\Delta(T(P_n))} \right\rfloor = \frac{2n-1}{5} \le \gamma(T(P_n))$. Hence, $\frac{2n-1}{5} \le \gamma(T(P_n)) \le \left\lfloor \frac{2n-1}{5} \right\rfloor$.

Theorem 2.2. (i) $\gamma(T(C_n)) = (2n)/5$, *if* $2n \equiv 0 (\text{mod } 5)$.

(ii) $\gamma(T(C_n)) = \lceil (2n)/5 \rceil$, *if* $2n \not\equiv 0 (\text{mod } 5)$.

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the cycle C_n and let

$u_1, u_2, u_3, \ldots, v_n$ be the added vertices corresponding to the edges $e_1, e_2, e_3, \ldots, e_n$ of the cycle C_n to obtain $T(C_n)$. Thus $V(T(C_n)) = \{v_1, u_1, v_2, u_2, \ldots, u_n, v_n\}$. The graph $T(C_n)$ will have $2n$ vertices.

Case (i). n is multiple of 5, $n \equiv 0(\text{mod } 5)$.

$D = \{v_1, u_3, v_6, u_8, \ldots, v_{n-2}\}$. The above set D is a dominating set of $T(C_n)$. Also for any vertex $v \in D$, the set $D - \{v\}$ is not dominating set of $T(C_n)$. Hence, it follows that $\gamma(T(C_n)) \leq (2n)/5$. Also $\left\lfloor \frac{2n-1}{1+\Delta(T(C_n))} \right\rfloor \leq \gamma(T(C_n))$. That is $(2n)/5 \leq \gamma(T(C_n))$. Therefore, $\gamma(T(C_n)) = (2n)/5$.

Case (ii). n is not multiple of 5, $n \not\equiv 0(\text{mod } 5)$.

The set $D = \{v_1, u_3\}$ is clearly minimum dominating set of $T(C_3)$. Hence $(T(C_3)) = 2$, if $n = 3$. The set $D \cup \{v_{n-1}\}$ or $D \cup \{u_{n-1}\}$ is a dominating set. Therefore $\gamma(T(C_n)) \leq \left\lceil \frac{2n}{5} \right\rceil$. Now $\deg(v_i) = 4 = \Delta(T(C_n))$, for $1 \leq i \leq 2n$ and each vertex of $T(C_n)$ can dominate exactly 5 distinct vertices of $T(C_n)$ including itself. This implies that, $\left\lfloor \frac{2n}{1+\Delta(T(C_n))} \right\rfloor = \left\lfloor \frac{2n}{5} \right\rfloor \leq \gamma(T(C_n))$. Hence, $\left\lfloor \frac{2n}{5} \right\rfloor \leq \gamma(T(C_n)) \leq \left\lceil \frac{2n}{5} \right\rceil$, and there is no minimum dominating set with $\left\lfloor \frac{2n}{5} \right\rfloor$ vertices. Therefore, $\gamma(T(C_n)) = \left\lceil \frac{2n}{5} \right\rceil$.

Theorem 2.3. $\gamma(T(G)) = 1$, *if and only if* $G = K_{1,n}$.

Proof. Radius of $T(G) = 1$, if and only if $G = K_{1,n}$.

Hence $\gamma(T(G)) = 1$, if and only if $G = K_{1,n}$.

Theorem 2.4. $\gamma(T(K_n)) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, & n \text{ is odd} \\ \frac{n}{2}, & n \text{ is even} \end{cases}$

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of K_n and let $u_{ij}, i < j, i, j = 1, 2, 3, \ldots, n$ be the added vertices corresponding to the edges $e_{ij} = v_i v_j$ of K_n to obtain $T(K_n)$. Thus $V(T(K_n)) = \{v_1, v_2, v_3, \ldots, v_n\} \bigcup_{i<j} \{u_{ij}\}$, $i, j = 1, 2, 3, \ldots, n$. The graph $T(K_n)$ has $n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$ vertices.

Case (i). n- odd

Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of K_n. $D = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n-2, n-1}, v_n\}$ is a minimum dominating set of $T(K_n)$. This implies that, $\gamma(T(K_n)) = \left\lceil \frac{n}{2} \right\rceil$.

Case (ii). n – even

Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of K_n. $D = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n-1, n}\}$ is a minimum dominating set of $T(K_n)$. Hence $\gamma(T(K_n)) = \frac{n}{2}$.

Theorem 2.5. $\gamma(T(W_n)) = \left\lceil \frac{n}{3} \right\rceil + 1$, *if* $n \geq 3$.

Proof. Let $v_1, v_2, v_3, \ldots, v_n, v$ (v is the central vertex of W_n) be the vertices of W_n and let $e_j = vv_j$, $j = 1, 2, \ldots, n$, $e_{12} = v_1 v_2$, $e_2 = v_2 v_3, \ldots, e_{n-1, n} = v_{n-1} v_n$, $e_{n1} = v_n v_1$ be the edges of W_n. Let $v_1, v_2, \ldots, v_n, v, u_1, u_2, \ldots, u_n, u_{12}, u_{23}, \ldots, u_{n-1, n}, u_{n1}$ be the corresponding vertices of $T(W_n)$. Thus $V(T(W_n))$ has $3n + 1$ vertices and $E(T(W_n))2n + 4n + \frac{n(n-1)}{2} + 3n = \frac{n^2 + 17n}{2}$ edges.

Case (i). $D = \{u_{12}, u_{45}, u_{78}, \ldots, u_{n-1, n}\}$. The above set D is a dominating set of $T(W_n)$. Also, for any vertex $w \in D$, the set $D - \{w\}$ is not a dominating set of $T(W_n)$. Hence it follows that the above set D is a minimal dominating set. Also there exists no dominating set with cardinality less than $|D|$.

Therefore, $\gamma(T(W_n)) = \left\lceil \frac{n}{3} \right\rceil + 1.$

Theorem 2.6. $\gamma(T(C_n^+)) = n,\ if\ n \geq 3.$

Proof. Let $G = C_n^+$ be a graph obtained from C_n by attaching exactly one pendant edge at each vertex of C_n. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, e_{34}, \ldots, e_{n-1}$ be the edges in C_n. Let u_i be the pendant vertex attached to v_i in $C_n^+, i = 1, 2, 3, \ldots, n.$ Then $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n, e_{11}, e_{22}, e_{33}, \ldots, e_{nn}, e_{12}, e_{23}, e_{34}, \ldots, e_{n-1}, n \in V(T(P_n^+)).$ Thus $V(T(P_n^+))$ has $4n$ vertices. C_n^+ has n in vertices of degree 1, and n vertices of degree 3. C_n^+ and $L(C_n^+)$ are subgraphs of $T(C_n^+)$. Let $D = \{v_1, v_2, v_3, \ldots, v_n\}$. D is a γ-set of C_n^+. Also D is a point cover for C_n^+. Therefore, $\gamma(T(C_n^+)) = \gamma(C_n^+) = n.$

Theorem 2.7. $\gamma(T(F_n)) = \lfloor (n\ /3) \rfloor + 1,\ if\ n \equiv 1 (\text{mod } 3)$

$$\gamma(T(F_n)) = \lfloor (n\ /3) \rfloor + 1,\ if\ n \not\equiv 1 (\text{mod } 3)$$

Proof. Let $v_1, v_2, v_3, \ldots, v_n, v$ (v is the central vertex of F_n) be the vertices of F_n and let $e_j = vv_j$, $j = 1, 2, \ldots, n,$ and $v_i v_j = e_{ij} (i < j = 1, 2, 3, \ldots, n)$ be the edges of F_n. Let $v_1, v_2, \ldots, v_n, v, u_1, u_2, \ldots, u_n, u_{12}, u_{23}, \ldots, u_{n-1, n}$ be the corresponding vertices of $T(F_n)$. Thus $V(T(F_n))$ has $3n$ vertices.

Case (i). $n \equiv 1 (\text{mod } 3)$

$S = \{v, u_{23}, u_{56}, u_{89}, \ldots, u_{n-2, n-1}\}$ is a minimum dominating set of $T(F_n)$ and $|S| = \lfloor (n / 3) \rfloor + 1.$ Hence, $\gamma(T(F_n)) = \lceil (n/3) \rceil + 1.$

Case (ii). $n \not\equiv 1 (\text{mod } 3)$

$S = \{v, u_{23}, u_{56}, u_{89}, \ldots, u_{n-1, n}\}$ is a minimum dominating set of $T(F_n)$ and $|S| = \lfloor (n / 3) \rfloor + 1.$ Hence, $\gamma(T(F_n)) = \lceil (n/3) \rceil + 1.$ When $G = T(F_2)$.

$S = \{v_1, v_2\}$ is a minimum dominating set of $T(F_2)$. Hence, $\gamma(T(F_2)) = 2$. In the following theorems, we fine some bounds for $\gamma(T(G))$.

Theorem 2.8. *If D is a point covering of G then* $\gamma(T(G))$.

Proof. Let $D \subseteq V(G)$ be the point covering of G. It is also a dominating set of G. Also D, is a dominating set of $T(G)$. Therefore, $\gamma(T(G)) \leq \alpha_0(G)$.

Theorem 2.9. *If* $\alpha_0(G) = \gamma(G)$, *then* $\gamma(T(G)) = \gamma(G)$.

Proof. Let D be a dominating set of G, which is also a point cover of G. Every edge of G is incident with some vertex in D. This implies that D is a dominating set of $T(G)$. Therefore, $\gamma(T(G)) = \gamma(G)$.

Remark 2.1. The converse of theorem 2.8 need not be true. That is $\gamma(T(G)) = \gamma(G)$ need not implies that $\gamma(G) = \alpha_0(G)$.

Theorem 2.10. *If* $\alpha_0(G) \geq \gamma(G) + 1$, *then* $\gamma(T(G)) \geq \gamma(G) + 1$.

Proof. Let D be a γ-set of G. $\alpha_0(G) \geq \gamma(G) + 1$, implies that D is not a point cover. Hence it cannot dominate $T(G)$. Therefore $\gamma(T(G)) \geq \gamma(G) + 1$.

Theorem 2.11. *For any graph* $G, \gamma(T(G)) = 1$, *if and only if G is* $K_{1,n}$, $n \geq 2$.

Proof. Assume $\gamma(T(G)) = 1$. Let D be a dominating set of $T(G)$ with $|D| = 1$.

Case (i). Let $D = \{v\}$, where $v \in V(G)$ is a dominating set of $T(G)$. The vertex v dominates all point vertices and line vertices in $T(G)$. Therefore v is adjacent with every vertices and incident with every edges in G. Hence, $G = K_{1,n}$.

Case (ii). Let $D = \{e\}, e \in E(G)$ is a dominating set of $T(G)$. If $e = uv$, $u, v \in V(G)$, the line vertex e dominate the point vertices u and v. It cannot dominate other point vertices. Hence the only possible is $G = K_2$.

Conversely. Assume $G = K_{1,n}$. Let v be the central vertex of $K_{1,n}$.

Each edges incident with v in G. Hence v dominate all point vertices and all line vertices in $T(G)$. Therefore, $\gamma(T(G)) = 1$.

Theorem 2.12. $\beta_0(T(G)) \geq \beta_1(G) + 1$ *or* $\beta_1(G)$.

Proof. Case (i). Let $D \subseteq E(G)$ be a perfect matching in G. Every point vertex in $T(G)$ is adjacent to some line vertex in D, and all other vertices are also dominated by D. Hence, D is the maximum independent set of vertices in $T(G)$. Therefore, $\beta_0(T(G)) = \beta_1(G)$.

Case (ii). Let there exists no perfect matching in G. Let D be a maximum matching in G. $D \subseteq E(G)$ with $|D| = \beta_1(G)$ and D is the edge independent set of G. Let v be a vertex in G, which is not incident with any edges in D, then $D \cup \{v\}$ is an independent set of $T(G)$. Therefore, $\beta_0(T(G)) \geq \beta_1(G) + 1$.

Remark 2.2. If G is disconnected then $T(G)$ is disconnected.

Remark 2.3 (i). G is an induced subgraph of G. Let $D \subseteq V(G)$ be a γ-set of G. D dominates all point vertices. Let $S \subseteq E(G)$ be a γ-set of $L(G)$. S dominates all line vertices. Hence $\gamma(T(G)) \leq \gamma(G) + \gamma(L(G))$.

(ii) Set of all point vertices form a dominating set of $T(G)$. Hence $\gamma(T(G)) \leq p$.

(iii) If G has no isolated vertices, set of all line vertices also form a dominating set of $T(G)$. Hence $\gamma(T(G)) \leq q$.

(iv) Let $S \subseteq E(G)$ be the maximum independent set of edges. Consider the sets of all vertices D in G, which are incident with edges in S. D contain at most $2|S|$ vertices. Also D dominates all the vertices in $T(G)$. Hence $\gamma(T(G)) \leq 2\beta_1(G)$.

(v) Let $S \subset V(G)$ be the maximum independent set of vertices of G. So all other vertices in $V - S$ are adjacent to any one vertex of S. Hence S is a dominating set of G. Hence $\gamma(T(G)) \leq \beta_0(G) + \gamma(L(G)) = \beta_0(G) + \beta_1(G)$.

(vi) The set of all point vertices is a minimum dominating set for $T(G)$ if and only if $G = \overline{K_n}$.

(vii) The set of all line vertices is a minimum dominating set for $T(G)$ if and only if $G = nK_2$.

Theorem 2.13. *Let* $p \geq 3$. *If* $G \neq K_{1,n}$ *is a Graph with radius* 1 *and diameter* 2, *the* $\gamma(T(G)) \leq \lceil (p+1)/2 \rceil$.

Proof. Let u be a central vertex. $e(u) = 1$ in G. In $T(G)$, u dominates all point vertices and line vertices incident with u in G. Remaining line vertices are dominated by a point cover S of $G - u$ and $|S| \leq \lceil (p-1)/2 \rceil$. $S \cup \{u\}$ is a dominating set of $T(G)$. Therefore, $\gamma(T(G)) \leq 1 + \lceil (p-1)/2 \rceil = \lceil (p-1)/2 \rceil$.

Theorem 2.14. *If* G *is a self-centered graph of diameter* 2, *then* $\gamma(T(G)) \leq \delta(G) + \alpha_0 \langle N_2(u) \rangle$, $u \in V(G)$.

Proof. Consider $u \in V(G)$ with degree $\delta(G)$. Consider $N_G(u) \subseteq V(G)$. In $T(G)$, elements in $N_G(u)$ dominates all points vertices and line vertices, which are not in $\langle N_2(u) \rangle$. Therefore, $D = N(u) \cup S \subseteq V(T(G))$, where S is a point cover of $\langle N_2(u) \rangle$ in G is a dominating set of $T(G)$. Hence, $\gamma(T(G)) \leq \delta(G) + \alpha_0 \langle N_2, (u) \rangle$.

Theorem 2.15. *If* $G = K_{1,n}$ *then* $\gamma t(T(G)) = 2$.

Proof. When $G = K_{1,n}$. Let $S = \{u, v\}$, where u is the central vertex. The central vertex dominates all point vertices and line vertices in $V(T(G)) - S$. The induced subgraph $\langle S \rangle$ has no isolated. Hence $\gamma t(T(G)) = 2$.

Theorem 2.16. *For any graph* G, $\gamma(T(G)) = 2$ *if and only if* (a) $\alpha_0(G) = 2$. (b) *G is one of the graphs* $K_{1,n} \cup K_2$, $n \geq 2$, C_4, K_3, P_4, (c) *G has a vertex u and an edge* $e = vw \in E(G)$ *such that* $V(G) = N[u] \cup \{v, w\}$ *and edge of G which are not incident with u are adjacent to e in a G.*

Proof. Assume $\gamma(T(G)) = 2$. Let D be a dominating set of $T(G)$ with $|D| = 2$.

Case (i). $D = \{u, v\}$. $u, v \in V(G)$. Since D dominates all point vertices and line vertices in $T(G)$. Each edge in G is incident with a vertex in D. That is D is a point cover of G.

Case (ii). $D = \{u, e\}$. $u \in V(G)$, $e \in E(G)$. Let $e = vw$, $v, w \in V(G)$. Then u dominates all the point vertices and e dominates v and w and line vertices which are adjacent to e in G.

Subcase (i). $\gamma(G) = 1$. u dominates all vertices of G and edges incident with u. Hence $r(G) = 1$ and $G \neq K_{1,n}$ and G has an edge $e \in E(G)$ such that edges of G which are not incident with u are adjacent to e in G.

Subcase (ii). $\gamma(G) = 2$. Hence, G is a graph with $r(G) = 1$ and $\alpha_0(G) = 2$ or $G = K_{1,n} \cup K_2$ or $u \in V(G)$ with $e(u) = 1$ and $e \in E(G)$ such that e is adjacent to all edges which are not incident with u.

Case (iii). $D = \{e_1, e_2\}$. Here D dominates 3 or 4 point vertices only. Hence $G = K_2 \cup K_1, K_{1,2}, C_3, C_4, P_4, 2K_2$. Converse is obvious.

Theorem 2.17. *$D \subseteq E(G)$ is a dominating set for $T(G)$ if and only if D is a line cover which dominates $L(G)$.*

Proof. $D \subseteq E(G)$ is a dominating set for $T(G)$ if and only if D is a dominating set for $L(G)$ and every point vertex in G is incident with some vertex in D. That is D covers all point vertices of G and D is a line cover for G and dominates $L(G)$.

Theorem 2.18. *If G has no isolated vertices, then $D = \{e_1, e_2\}$ is a γ-set of $T(G)$ if and only if $\alpha_1(G) = 2$.*

Proof. $D = \{e_1, e_2\} \subseteq E(G)$. D is a dominating set for $T(G)$ if and only if e_1, e_2 dominates all point vertices and $L(G)$.

Theorem 2.19. *For any graph G, $\gamma(T(G)) \leq \alpha_1(G) + \alpha_0\langle E(G) - D\rangle$. Where D is a minimum line cover of G.*

Proof. Let $D \subseteq E(G)$ be a minimum line cover of G. Therefore, $|D| = \alpha_1(G)$. Let D' be a subset of $V(T(G))$ corresponding to the edges in D. Then $D' \subseteq V(T(G))$ dominates all point vertices in $T(G)$. Let $D_1 = E(G) - D'$. The vertices of $L(G)$ in $V(T(G))$ are dominated by vertices of D_1. Hence, $\gamma(T(G)) \leq \alpha_1(G) + \alpha_0\langle E(G) - D\rangle$.

Theorem 2.20. *Let G be a graph with $diam(G) = 2$. If there exists $v \in V(G)$ such that $\langle N_2(v) \rangle$ is totally disconnected, then $\gamma(T(G)) \leq \Delta(G)$.*

Proof. Let $v \in V(G)$ be such that $\langle N_2(v) \rangle$ is totally disconnected. Let $D = N(v)$. Then $D \subseteq V(T(G))$, all the edges of G are incident with the vertices in D. Therefore, vertices of $V(T(G)) - D$ are adjacent to at least one vertex in D. Hence, $\gamma(T(G)) \leq |D| = |N(v)| \leq \Delta(G)$.

Theorem 2.21. *Let G be a graph with $u \in V(G)$ such that $N(u)$ independent, then $\gamma(T(G)) \leq p - \deg_G u$.*

Proof. Consider $D = V - N(u)$. D is a dominating set for G. Also, since there is no edge in $\langle N(u) \rangle$, D dominates all other point vertices and all line vertices in $T(G)$. Hence, $\gamma(T(G)) \leq p - \deg_G u$.

Conclusion

In this paper we computed the exact value of the domination number for Total graph of path, cycle, complete graph, Wheel graph and some special graphs. Also we found some upper bounds for domination number for Total graph $T(G)$ of a graph.

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