Advances and Applications in Mathematical Sciences
Volume 18, Issue 1, November 2018, Pages 153-167

ON COMPLEX DYNAMICS OF SOME EIGHTH ORDER TECHNIQUES FOR NONLINEAR EQUATIONS

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Abstract

There are many techniques to solve nonlinear equations. These techniques are categorized by the order, informational efficiency and efficiency index. In this work we have taken the criteria, namely basins of attraction for checking the convergence domain of the techniques. This study is also called complex dynamics of iterative methods. We consider several techniques of order eighth and present the basin of attraction for respective examples. We measured that Kung-Traub and Sharma-Kumar techniques are consistently better than others.

1. Introduction

There are a number of different techniques for the numerical solution of nonlinear equations [13]. These techniques are categorized by their order of convergence (say, p), and the number of function and derivative evalutions (say, n) per step. To check the effectiveness of such techniques, there are two efficiency measures (see [13]) defined as $I = \frac{p}{n}$ (informational efficiency) and $E = p^{\frac{1}{n}}$ (efficiency index). Another measure, introduced recently, is the complex dynamics of iterative techniques. For example, see, (Amat et al. [1], Chicharro et al. [3], Chun et al. [4], Cordero et al. [5], Gutierrez et al. [7], Neta et al. [10], Scott et al. [11]). In 1974, Kung and Traub [8] introduced the concept of optimality. According to their hypothesis multipoint techniques

2010 Mathematics Subject Classification: 41A21, 65B99, 65H05.
Keywords: Rational Map, Basin of attraction, Extraneous fixed points.
Received January 5, 2018; Accepted April 26, 2018

without memory requiring $n+1$ function-evaluations have order of convergence at most 2^n. Such techniques are usually called optimal (see, for example, [8]). An optimal technique of order $p=2$ is the well known Newton's technique. Optimal techniques of order four were discussed in [1, 4, 10].

In this paper, we consider some eighth order optimal techniques and study their complex dynamics. Moreover, we will make the relation between conjugacy maps [5], extraneous fixed points [14] and the basins of attraction in our numerical trial. Rest of the paper is organised as follows. Section 2, the eighth order techniques that we have considered. In Section 3, we will check the conjugacy maps for each technique and find the extraneous fixed points [14]. Basins of attraction of the various optimal eighth order methods are shown in section 4. To study the complex dynamics, we choose the eighth order techniques proposed by Bi et al. [2], Cordero et al. [6], Kung-Traub [8], Liu-Wang [9] and Sharma-Kumar [12].

2. Techniques for the Relative Examination

In this section we tabulate the eighth-order techniques that we have taken here. To study the complex dynamics, we have taken eighth order techniques proposed by Bi et al. [2], Cordero et al. [6], Kung-Traub [8], Liu-Wang [9] and Sharma-Kumar [12].

Bi-Wu-Ren Technique (BWRT8):

$$w_k = l_k - \frac{g(l_k)}{g'(l_k)},$$

$$t_k = w_k - \frac{2g(l_k) - g(w_k)}{2g(l_k) - 5g(w_k)} \frac{g(w_k)}{g'(l_k)},$$

$$l_{k+1} = t_k - \frac{g(l_k) + (\gamma + 2)g(t_k)}{g(l_k) + \gamma g(t_k)} \frac{g(t_k)}{g[t_k, w_k] + g[t_k, l_k, l_k](t_k - w_k)}, \tag{1}$$

where $\gamma \in \mathbb{R}$ and $g[t_k, l_k, l_k] = \dfrac{g[t_k, l_k] - g'(l_k)}{t_k - l_k}$.

Cordero-Torregrosa-Vassileva Technique (CTVT8):

$$w_k = l_k - \frac{g(l_k)}{g'(l_k)},$$

$$t_k = l_k - \frac{g(l_k) - g(w_k)}{g(l_k) - 2g(w_k)} \frac{g(l_k)}{g'(l_k)},$$

$$l_{k+1} = \eta_k - \frac{3(\beta_2 + \beta_3)(\eta_k - t_k)}{\beta_1(\eta_k - t_k) + \beta_2(w_k - l_k) + \beta_3(t_k - l_k)} \frac{g(t_k)}{g'(l_k)}, \tag{2}$$

where $\beta_i \in \mathbb{R}(i = 1, 2, 3)$, $\beta_2 + \beta_3 \neq 0$ and

$$\eta_k = t_k - \frac{g(t_k)}{g'(l_k)}\left(\frac{g(l_k) - g(w_k)}{g(l_k) - 2g(w_k)} + \frac{1}{2}\frac{g(t_k)}{g(w_k) - 2g(t_k)}\right)^2.$$

Kung-Traub Technique (KTT8):

$$w_k = l_k - \frac{g(l_k)}{g'(l_k)},$$

$$t_k = w_k - \frac{g(l_k)}{g'(l_k)} \frac{g(w_k)g(l_k)}{(g(l_k) - g'(w_k))^2},$$

$$l_{k+1} = t_k - \frac{g(l_k)}{g'(l_k)} \frac{g(l_k)g(w_k)g(t_k)}{(g(l_k) - g(w_k))^2} \frac{g(l_k)^2 + g(w_k)(g(w_k) - g(t_k))}{(g(l_k) - g(t_k))^2(g(w_k) - g(t_k))}. \tag{3}$$

Liu-Wang Technique (LWT8):

$$w_k = l_k - \frac{g(l_k)}{g'(l_k)},$$

$$t_k = w_k - \frac{g(l_k)}{g'(l_k)} \frac{g(l_k)}{g(l_k) - 2g(w_k)},$$

$$l_{k+1} = t_k - \frac{g(t_k)}{g'(l_k)}\left[\left(\frac{g(l_k) - g(w_k)}{g(l_k) - 2g(w_k)}\right)^2 + \frac{g(t_k)}{g(w_k) - \beta_1 g(t_k)} + \frac{4g(t_k)}{g(l_k) + \beta_2 g(t_k)}\right], \tag{4}$$

where β_1, $\beta \in \mathbb{R}$.

Sharma-Kumar Technique (SKT8):

$$w_k = l_k - \frac{g(l_k)}{g'(l_k)},$$

$$t_k = w_k - \frac{g(l_k)g(w_k)}{g[w_k, l_k]^2},$$

$$l_{k+1} = t_k - \frac{1}{h_1 - h_2 + h_3} \frac{g'(l_k)g(t_k)}{g[t_k, l_k]^2}, \tag{5}$$

where h_1, h_2 and h_3 are defined as:

$$h_1 = \frac{w_k - t_k}{w_k - l_k},$$

$$h_2 = \frac{(t_k - l_k)^2 g'(l_k)}{(w_k - t_k)(w_k - l_k)g[w_k, l_k]},$$

$$h_3 = \frac{(t_k - l_k) g'(l_k)}{(w_k - t_k) g(t_k, l_k]}.$$

3. Compatible Conjugacy Maps For Quadratic Polynomials

Theorem 3.1 (Bi-Wu-Ren Technique, BWRT8)**.** *Rational map* $R_p(z)$ *emerging from the technique* (1) *implemented on* $p(z) = (z - a_1)(z - a_2)$, $a_1 \neq a_2$, $R_p(z)$ *is conjugate via the Mobius transformation given by* $M(z) = \frac{z - a_1}{z - a_2}$ *to*

$$S(z) = z^9 \frac{N_z}{D_z},$$

where

$$N_z = (3 + 2z)^2(-8 - 20z + 2z^2 + 45z^3 + 70z^4 + 70z^8 + 45z^9 + 20z^{10}$$

$$+ 4z^1 1 + z^5(95 + 6\gamma) + z^7(95 + 6\gamma) + z^6(108 + 13\gamma)), \tag{6}$$

$$D_z = (2 + 3z)^2(4 + 20z + 45z^2 + 70z^3 + 70z^7 + 45z^8 + 2z^9 - 20z^{10}$$

$$- 8z^{11} + z^4(95 + 6\gamma) + z^6(95 + 6\gamma) + z^5(108 + 13\gamma)) \tag{7}$$

and for $\gamma = 1$

$$N_z = (3+2z)^2(-8-20z+2z^2+45z^3+70z^4+101z^5+121z^6$$
$$+101z^7+70z^8+45z^9+20z^{10}+4z^{11}), \quad (8)$$

$$D_z = (2+3z)^2(4+20z+45z^2+70z^3+101z^4+121z^4$$
$$+101z^6+70z^7+45z^8+2z^9-20z^{10}-8z^{11}). \quad (9)$$

Theorem 3.2 (Cordero-Torregrosa-Vassileva Technique, CTVT8). *Rational map* $R_p(z)$ *emerging from the technique* (2) *implemented on* $p(z) = (z-a_1)(z-a_2)$, $a_1 \neq a_2$, $R_p(z)$ *is conjugate via the Mobius transformation given by* $M(z) = \frac{z-a_1}{z-a_2}$ *to*

$$S(z) = z^8\,\frac{N_z}{D_z},$$

where

$$N_z = (44\beta_2+92\beta_3+16z^{30}(\beta_2+\beta_3)+16z^{29}(4\beta_2+5\beta_3)+20z(6\beta_2+13\beta_3)$$
$$+16z^{28}(13\beta_2+17\beta_3)+16z^{27}(\beta_1+32\beta_2+44\beta_3)+16z^{26}(4\beta_1+67\beta_2+95\beta_3)$$
$$16z^{25}(12\beta_1+124\beta_2+179\beta_3)+16z^{24}(25\beta_1+207\beta_2+303\beta_3)+4z^3(35\beta_1$$
$$+201\beta_2+371\beta_3)+4z^4(90\beta_1+457\beta_2+763\beta_3)+z^2(48\beta_1+484\beta_2+844\beta_3)$$
$$+4z^5(134\beta_1+642\beta_2+1133\beta_3)+8z^8(281\beta_1+1215\beta_2+1951\beta_3)+4z^{22}(294\beta_1$$
$$+1851\beta_2+2769\beta_3)+8z^{10}(462\beta_1+1969\beta_2+3123\beta_3)+4z^{21}(438\beta_1+2526\beta_2$$
$$+3793\beta_3)+8z^{12}(636\beta_1+2669\beta_2+4191\beta_3)+8z^{14}(721\beta_1+3051\beta_2+4747\beta_3)$$
$$+4z^9(695\beta_1+2880\beta_2+4774\beta_3)+4z^{20}(614\beta_1+3305\beta_2+4995\beta_3)$$
$$+4z^{19}(811\beta_1+4041\beta_2+6163\beta_3)+4z^{18}(1024\beta_1+4825\beta_2+7371\beta_3)$$
$$+z^{23}(740\beta_1+5120\beta_2+7552\beta_3)+z^6(960\beta_1+4780\beta_2+7804\beta_3)+4z^{17}(1168\beta_1$$

$$+5314\beta_2+8221\beta_3)+4z^{13}(1339\beta_1+5536\beta_2+8830\beta_3)$$

$$+4z^{16}(1314\beta_1+5883\beta_2+9071\beta_3)$$

$$+z^{7}(1363\beta_1+6176\beta_2+10528\beta_3)+2z^{11}(2057\beta_1+8700\beta_2+14132\beta_3)$$

$$+z^{15}(5415\beta_1+23584\beta_2+37024\beta_3)),$$

$$D_z=16(\beta_1+\beta_3)+16z(4\beta_2+5\beta_3)+20z^{29}(6\beta_1+13\beta_3)+16z^2(13\beta_1+17\beta_3)$$

$$+16z^3(\beta_1+32\beta_3+44\beta_3)+z^{30}(44\beta_1+92\beta_3)+16z^4(4\beta_1+67\beta_3+95\beta_3)$$

$$+16z^5(12\beta_1+124\beta_2+179\beta_3)+16z^6(25\beta_1+207\beta_2+303\beta_3)+4z^{27}(35\beta_1$$

$$+201\beta_2+371\beta_3)+4z^{26}(90\beta_1+457\beta_2+763\beta_3)+z^{28}(48\beta_1+484\beta_2+844\beta_3)$$

$$+4z^{25}(134\beta_1+642\beta_2+1133\beta_3)+8z^{22}(281\beta_1+1215\beta_2+1951\beta_3)$$

$$+4z^{8}(294\beta_1+1851\beta_2+2759\beta_3)+8z^{20}(462\beta_1+1969\beta_2+3123\beta_3)$$

$$+4z^{9}(438\beta_1+2526\beta_2+3793\beta_3)+8z^{18}(636\beta_1+2669\beta_2+4191\beta_3)$$

$$+8z^{16}(721\beta_1+3051\beta_2+4747\beta_3)+4z^{21}(695\beta_1+2880\beta_2+4774\beta_3)$$

$$+4z^{10}(614\beta_1+3305\beta_2+4995\beta_3)+4z^{11}(811\beta_1+4041\beta_2+1663\beta_3)$$

$$+4z^{12}(1024\beta_1+4825\beta_2+7371\beta_3)+z^{7}(740\beta_1+5120\beta_2+7552\beta_3)$$

$$+z^{24}(960\beta_1+4780\beta_2+7804\beta_3)$$

$$+4z^{13}(1168\beta_1+5314\beta_2+8221\beta_3)+4z^{17}(1339\beta_1+5536\beta_2+8830\beta_3)$$

$$+4z^{14}(1314\beta_1+5883\beta_2+9071\beta_3)+z^{23}(1363\beta_1+6176\beta_2+10528\beta_3)$$

$$+2z^{19}(2057\beta_1+8700\beta_2+14132\beta_3)+z^{15}(5415\beta_1+23584\beta_2+37024\beta_3)$$

and for $\beta_1=0,\ \beta_2=1,\ \beta_3=0$

$$N_z=11+30z+99z^2+141z^3+226z^4+270z^5+402z^6+461z^7+519z^8$$

$$+ 494z^9 + 471z^{10} + 416z^{11} + 344z^{12} + 256z^{13} + 168z^{14} + 96z^{15} + 44z^{16} + 16z^{17} + 4z^{18}, \quad (10)$$

$$D_z = 4 + 16z + 44z^2 + 96z^3 + 168z^4 + 256z^5 + 344z^6 + 416z^7 + 471z^8 + 494z^9 + 519z^{10} + 461z^{11} + 402z^{12} + 270z^{13} + 226z^{14} + 141z^{15} + 99z^{16} + 30z^{17} + 11z^{18}. \quad (11)$$

Theorem 3.3 (Kung-Traub Technique, KTT8). *Rational map* $R_p(z)$ *emerging from the technique (3) implemented on* $p(z) = (z - a_1)(z - a_2)$, $a_1 \neq a_2$, $R_p(z)$ *is conjugate via the Mobius transformation given by* $M(z) = \frac{z - a_1}{z - a_2}$ *to*

$$S(z) = z^8 \frac{N_z}{D_z},$$

where

$$N_z = 10 + 74z + 289z^2 + 760z^3 + 1506z^4 + 2382z^5 + 3116z^6 + 3432z^7 + 3214z^8 + 2568z^9 + 1749z^{10} + 1006z^{11} + 479z^{12} + 182z^{13} + 52z^{14} + 10z^{15} + z^{16}. \quad (12)$$

$$D_z = 1 + 10z + 52z^2 + 182z^3 + 479z^4 + 1006z^5 + 1749z^6 + 2568z^7 + 3214z^8 + 3432z^9 + 3116z^{10} + 2382z^{11} + 1506z^{12} + 760z^{13} + 289z^{14} + 74z^{15} + 10z^{16}. \quad (13)$$

Theorem 3.4 (Liu-Wang Technique, LWT8). *Rational map* $R_p(z)$ *emerging from the technique* (4) *implemented on* $p(z) = (z - a_1)(z - a_2)$, $a_1 \neq a_2$, $R_p(z)$ *is conjugate via the Mobius transformation given by* $M(z) = \frac{z - a_1}{z - a_2}$ *to*

$$S(z) = -z^8 \frac{N_z}{D_z},$$

where

$$N_z = 13 + 4z^{13} + z^{14} - z^{12}(-13 + \alpha_1) - 2z(-10 + \alpha_1) - \alpha_1 + z^{11}(32 - 4\alpha_1 + \alpha_2)$$

$$+ z^{10}(63 - 11\alpha_1 + 2\alpha_2) + z^2(73 - 16\alpha_1 + 4\alpha_2) + z^5(188 + 58\alpha_1 + 14\alpha_2 - 5\alpha_1\alpha_2)$$

$$-2z^7(-96 - 7\alpha_2 + 2\alpha_1(14 + \alpha_2)) + z^3(96 + 5\alpha_2 - \alpha_1(24 + \alpha_2))$$

$$+ z^9(6(18 + \alpha_2) - \alpha_1(24 + \alpha_2)) + z^4(171 + 14\alpha_2 - 2\alpha_1(25 + 2\alpha_2))$$

$$+ z^8(-2\alpha_1(20 + \alpha_2) + 5(31 + 2\alpha_2)) + z^6(215 + 18\alpha_2 - \alpha_1(65 + 6\alpha_2)),$$

$$D_z = -1 - 4z + z^2(-13 + \alpha_1) + z^{14}(-13 + \alpha_1) + 2z^{13}(-10 + \alpha_1) + z^{12}(-73 + 16\alpha_1 - 4\alpha_2)$$

$$+ z^4(-63 + 11\alpha_1 - 2\alpha_2) + z^3(-32 + 4\alpha_1 - \alpha_2) + 2z^7(-96 - 7\alpha_2 + 2\alpha_1(14 + \alpha_2))$$

$$+ z^{11}(-96 - 5\alpha_2 + \alpha_1(24 + \alpha_2)) + z^5(-6(18 + \alpha_2) + \alpha_1(24 + \alpha_2))$$

$$+ z^6(2\alpha_1(20 + \alpha_2) - 5(31 + 2\alpha_2)) + z^{10}(-171 - 14\alpha_2 + \alpha_1(50 + 4\alpha_2))$$

$$+ z^8(-125 - 18\alpha_2 + \alpha_1(65 + 6\alpha_3)) + z^9(\alpha_1(58 + 5\alpha_2) - 2(94 + 7\alpha_2))$$

and for $\beta_1 = 0$, $\beta_2 = 0$

$$N_z = 13 + 20z + 21z^2 + 16z^3 + 9z^4 + 4x^5 + z^6, \tag{14}$$

$$D_z = 1 + 4z + 9z^2 + 16z^3 + 21z^4 + 20z^5 + 13z^6. \tag{15}$$

Theorem 3.5 (Sharma-Kumar Technique, SKT8). *Rational map* $R_p(z)$ *emerging from the technique* (5) *implemented on* $p(z) = (z - a_2)(z - a_2)$, $a_1 \neq a_2$, $R_p(z)$ *is conjugate via the Mobius transformation given by* $M(z) = \frac{z - a_1}{z - a_2}$ *to*

$$S(z) - z^8 \frac{N_z}{D_z},$$

where

$$N_z = 6 + 42z + 159z^2 + 400z^3 + 742z^4 + 1067z^5 + 1238z^6 + 1184z^7$$

$$+ 942z^8 + 619z^9 + 330z^{10} + 138z^{11} + 43z^{12} + 9z^{13} + z^{14}, \tag{16}$$

$$D_z = 1 + 9z + 43z^2 + 138z^3 + 330z^4 + 619z^5 + 942z^6 + 1184z^7 + 1238z^8 + 1067z^9 + 742z^{10} + 400z^{11} + 159z^{12} + 42z^{13} + 6z^{14}. \quad (17)$$

3.1. Extraneous Fixed Points

It can be seen that all these techniques can be written as

$$l_{k+1} = l_k - v_k(l_k)H_g(l_k, w_k, t_k),$$

where $v_k(l_k) = \frac{g(l_k)}{g'(l_k)}$. Obviously the root α_1 is a fixed point of the technique, since $v_k(\alpha_1) = 0$. The points $\xi \neq \alpha_1$ at which $H_g(\xi) = 0$ are also fixed points of the technique, whereas the second part on the right vanishes. These points are termed extraneous fixed points (see [14]). In this section, we will discuss the extraneous fixed points of each technique for the polynomial $z^2 - 1$. Now the point ξ is attractive if $|R'_p(\xi)|$ is less than one, indifferent if $|R'_p(\xi)|$ is equal to one or repulsive if $|R'_p(\xi)|$ is greater than one, where $R_p(z) = z - v(z)H_g(z, w(z), t(z))$ is the iteration function.

Theorem 3.6. *For extraneous fixed points of Bi-Wu-Ren Technique* (1) *are at* $z = -15.0093$, $z = -0.3521 \pm 1.3794i$, $z = -0.3040 \pm 0.1209i$, $z = -0.2770 \pm 0.9331i$, $z = -0.2630 \pm 0.2870i$, $z = -0.1955 \pm 0.0062i$, $z = 0.1934$, $z = 0.2064$, $z = 0.2322 \pm 0.5800i$, $z = 0.2954 \pm 0.2238i$, $z = 0.3405$, $z = 0.3799$, $\pm 1.244i$, $z = 0.6021 \pm 2.8228i$. *All extraneous points are repulsive*.

Theorem 3.7 *For extraneous fixed points of Cordero-Torregrosa-Vassileva Technique* (2) *are at* $z = -0.4583 \pm 1.3187i$, $z = -0.3773 \pm 0.8680i$, $z = -0.2525 \pm 0.1236i$, $= -0.1254 \pm 0.5986i$, $z = -0.1227 \pm 0.4495i$, $z = -0.1110 \pm 2.3441i$, $z = 0.1110 \pm 2.3441i$, $z = 0.1227 \pm 0.4495i$, $z = 0.1254 \pm 0.5986i$, $z = 0.2525 \pm 0.1236i$, $z = 0.3773 \pm 0.8680i$, $z = 0.4583 \pm 1.3187i$. All extraneous points are repulsive.

Theorem 3.8 *For extraneous fixed points of Kung-Traub Technique* (3) *are at* $z = -0.3233 \pm 0.3908i$, $z = -0.3193 \pm 1.0204i$, $z = -0.2137 \pm 0.4196i$, $z = -0.1694 \pm 0.3502i$, $z = 0. \pm 0.2937i$, $z = 0. \pm 1.0404i$, $z = 0. \pm 3.2450i$,

$z = 0.1694 \pm 0.3502i$, $z = 0.2137 \pm 0.4196i$, $z = 0.3193 \pm 1.0204i$, $z = 0.3233 \pm 0.3908i$. *All extraneous points are repulsive, except* $z = 0. \pm 3.2450i$ *is attractive.*

Theorem 3.9. *For extraneous fixed points of Liu-Wang Technique* (4) *are at* $z = -0.5144 \pm 1.1826i$, $z = -0.3343 \pm 0.6291i$, $z = -0.3055 \pm 0.1453i$, $z = 0.3055 \pm 0.1453i$, $z = 0.3343 \pm 0.6291i$, $z = 0.5144 \pm 1.1826i$. *All extraneous points are repulsive.*

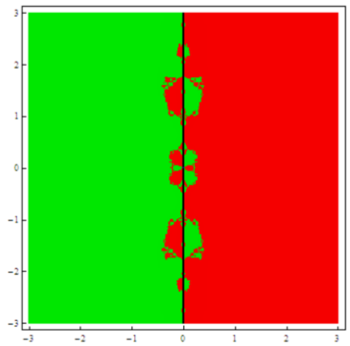
Theorem 3.10. *For extraneous fixed points of Sharma-Kumar Technique* (5) *are at* $z = -0.3299 \pm 0.6029i$, $z = 0.2696 \pm 0.4848i$, $z = -0.2536 \pm 0.2529i$ $z = -0.2261 \pm 1.3769i$, $z = -0.2156 \pm 0.3851i$, $z = 0.2156 \pm 0.3851i$, $z = 0.2261 \pm 1.3769i$, $z = 0.2536 \pm 0.2529i$, $z = 0.2696 \pm 0.4848i$, $z = 0.3299 \pm 0.6029i$. *All extraneous points are repulsive.*

4. Basins of Attraction

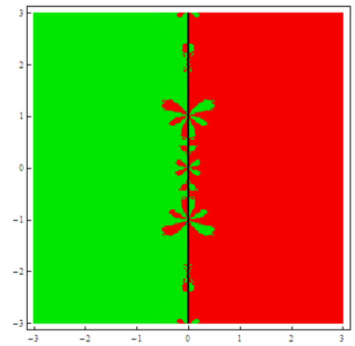
Example 1. In the first trial, we have test every one of the techniques to get the simple zeros of the quadratic polynomial $z^2 = 1$. The result for the basins of attraction are given in Figures (a)-(e).

Example 2. In our second trial, we have run every one of the techniques to get the simple zeros for the cubic polynomial $z^3 = 1$. The result of the basins of attraction are given in Figures (f)-(j).

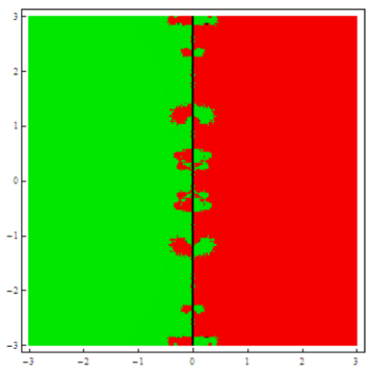
Example 3. In our third trial, we have run every one of the techniques to get the simple zeros for the biquadratic polynomial $z^4 = 1$. The result of the basins of attraction are given in Figures (k)-(o).



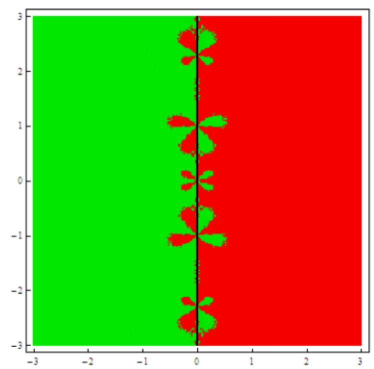
(a) $BWRT_8$



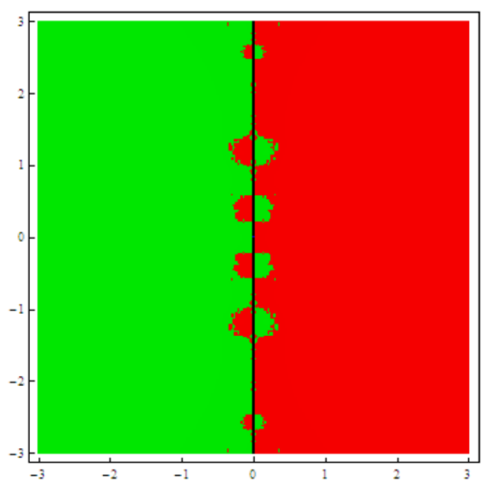
(b) $CTVT_8$



(c) KTT_8

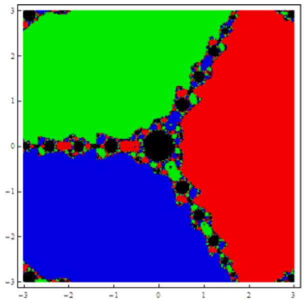


(d) LWT_8

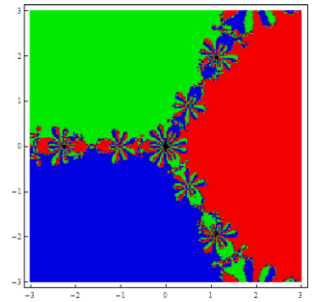


(e) SKT_8

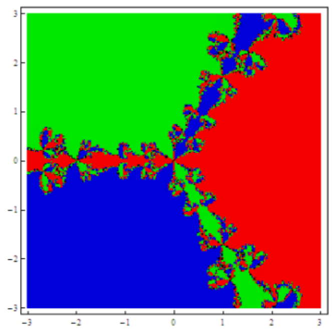
Figures (a)-(e). Basins of attraction for polynomial $f(z) = z^2 - 1$.



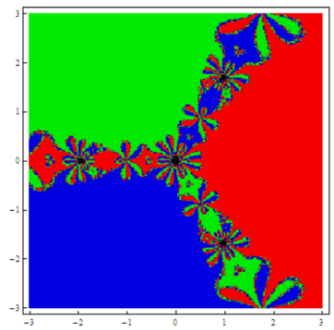
(f) $BWRT_8$



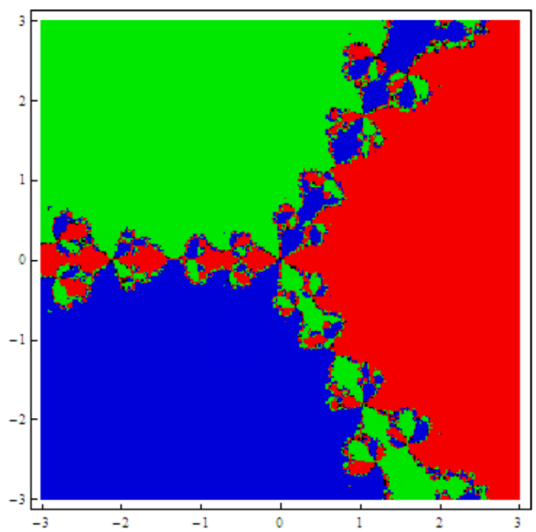
(g) $CTVT_8$



(h) KTT_8

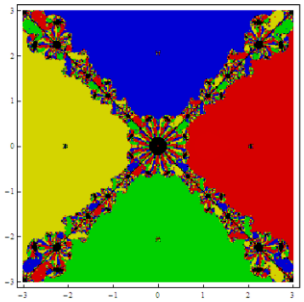


(i) LWT_8

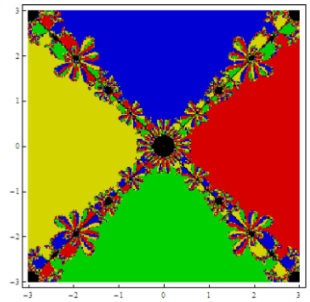


(j) SKT_8

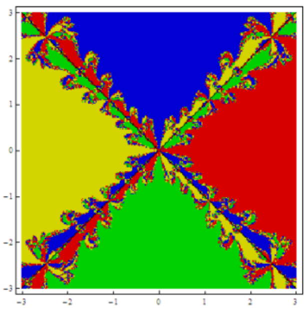
Figures (f)-(j). Basins of attraction for polynomial $f(z) = z^3 - 1$.



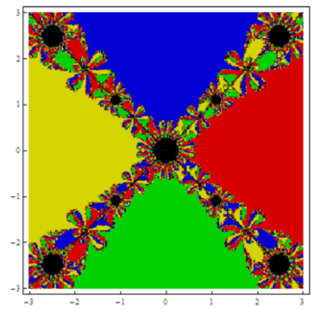
(k) $BWRT_8$



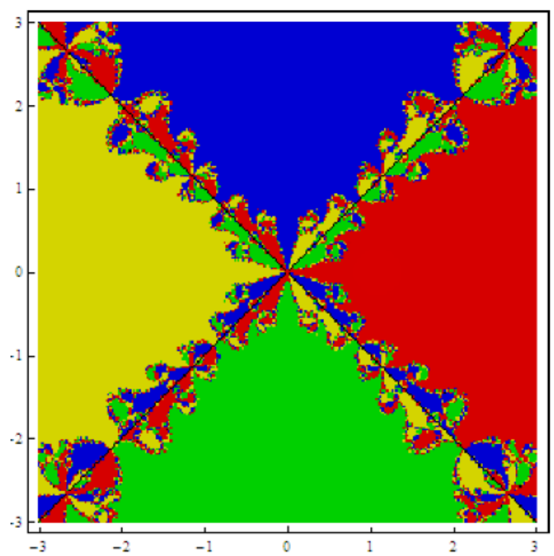
(l) $CTVT_8$



(m) KTT_8



(n) LWT_8



(o) SKT_8

Figures (k)-(o). Basins of attraction for polynomial $f(z) = z^4 - 1$.

In order to draw the above figures the strategy taken into account is the following. A color is assigned to each basin of attraction of a zero. We mark with black the points of the figure for which the corresponding iterations starting at them do not reach any root.

Notice from figures (a)-(e) that all methods perform well since there is no black region in any figure. This means corresponding to quadratic polynomial (example 1) every method possesses similar convergence behavior. In case of cubic polynomial (example 2) the Kung-Traub and Sharma-Kumar methods perform well since other figures have black spot in their basins. In the case of bi-quadratic polynomial (example 3) again Kung-Traub and Sharma-Kumar methods have better convergence behavior.

5. Conclusion

We have considered some eighth order techniques and checked their performance by comparing the complex dynamics in terms of basins of attraction. We have found that Kung-Traub and Sharma-Kumar techniques are better than the other ones. Bi-Wu-Ren Technique performance poorly in example 2 and in the example 3 the Bi-Wu-Ren Technique, Cordero-Torregrosa-Vassileva Technique and Liu-Wang Technique perform very weak (divergence area is more) than Kung-Traub and Sharma-Kumar techniques.

6. Acknowledgments

Authors would like to thank to Department of Mathematics, SLIET, Punjab for the excellent lab facility and for continuous support.

References

[1] S. Amat, S. Busquier and S. Plaza, Aeq. Math. 69 (2005), 212-223.

[2] W. Bi, Q. Wu and H. Ren, Appl. Math. Comput. 214 (2009), 236-245.

[3] F. Chicharro, A. Cordero, J. M. Gutierrez and J. R. Torregrosa, Appl. Math. Com. 219 (2013), 7023-7035.

[4] C. Chun, M. Y. Lee, B. Neta and J. Dzunic, Appl. Math. Com. 218 (2012), 6423-6438.

[5] A. Cordero, J. Garcia-Maimo, J. R. Torregrosa, M. P. Vassileva and P. Vindel, Appl. Math. Lett. 26 (2013), 842-848.

[6] A. Cordero, J. R. Torregrosa and M. P. Vassileva, J. Comput. Appl. Math. 23 (2011), 3189-3194.

[7] J. M. Gutierrez, A. A. Magrenan and J. L. Varona, Appl. Math. Comp. 218 (2011), 2467-2479.

[8] H. T. Kung and J. F. Traub, J.ACM 21 (1974), 643-651.

[9] L. Liu and X. Wang, J. Comput. Appl. Math. 215 (2010), 3449-3454.

[10] B. Neta, M. Scott and C. Chun, Appl. Math. Comp. 218 (2012), 10548-10557.

[11] M. Scott, B. Neta and C. Chun, Appl. Math. Comput. 218 (2011), 2584-2599.

[12] J. R. Sharma and S. Kumar, SeMA., DOI 10.1007/s40324-017-0131-3, 390{403 (2017).

[13] J. F. Traub, Iterative Methods for the Solution of Equations, Prentice-Hall, Englewood Clis, New Jersey 1964.

[14] E. R. Vrscay and W. J. Gilbert, Nume. Math. 52 (1988), 1-16.