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DESIGN AND DEVELOPMENT OF ALGORITHM FOR *M* MODULO *N* GRACEFUL LABELING ON CYCLE AND COMPLETE GRAPH

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Abstract

A graph G with p vertices and q edges is said to be M modulo N Graceful Labeling (where N is positive integer and $M = 1$ to N) if there is a function f from the vertex set of G to $\{0, M, N + M, 2N, \ldots, N(q-1), N(q-1)+M\}$ in such a way that (i) f is 1-1, (ii) f induces a bijection f^* from edge set of G to $\{M, N+M, 2N+M, \ldots, N(q-1)+M\}$ where $f^*(u, v) = |f(u) - f(v)|$ for all $u, v \in V(G)$. In this paper we classified that existence of M modulo N Graceful labeling in cycle C_n. Further we show that every complete graph K_n, $n > 4$ is not M modulo N Graceful Labeling. Also we proposed design and development of C++ algorithm for M modulo N Graceful Labeling on cycle graph C_n.

1. Introduction

Let $G = (V, E)$ denotes a graph with p number of vertices and q number of edges. The symbols V and E will denote the vertex set and edge set of a graph G respectively. A simple graph with n vertices is said to be complete if

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there is an edge between every pair of vertices. The complete graph on n vertices is denoted by K_n. A graceful labeling of a graph G of size q is an injective assignment of labels from the set $\{0, 1, \ldots, q\}$ to the vertices of G such that when each edge of G has been assigned a label defined by the absolute difference of its end-vertices, the resulting edge labels are distinct. A graph G is said to be one modulo N graceful labeling (where N is a positive integer) if there is a function f from the vertex set of G to $\{0, 1, N, (N+1), 2N, \ldots, N(q-1), N(q-1)+1\}$ in such a way that (i) f is $1-1$ (ii) f induces a bijection f^* from the edge set of G to $\{1, N+1, 2N+1, \ldots, N(q-1)+1\}$ where $f^*(uv) = |f(u) - f(v)|$ for all $u, v \in V(G)$. A graph $G(V(G), E(G))$ with p vertices and q edges is said to be M modulo N graceful labeling (where N is positive integer and $M = 1$ to N) if there is a function f from the vertex set of G to $\{0, M, N, N+M, 2N, \ldots, N(q-1), N(q-1)+M\}$ in such a way that (i) f is 1-1, (ii) f induces a bijection f^* from edge set of G to $\{M, N+M, 2N+M, \ldots, N(q-1)+M\}$ where $f^*(u, v) = |f(u) - f(v)|$ for all $u, v \in V(G)$. A graph G satisfied M modulo N graceful labeling is known as M modulo N graceful graph. "Graceful labeling" was introduced by Rosa [7] and proved that the cycle C_n is graceful if and only if $n \equiv 0$ or $3 \pmod 4$. S. W. Golomb. [3] explained that the complete graph K_n is graceful if and only if, $n \leq 4$. J. A. Gallian [4] studied a complete survey on graph labeling. A. Elumalai, A. Anand Ephremnath [1] studied that Graceful Labeling of Arbitrary Super subdivision of Grid graph and Cyclic snake. Odd gracefulness was introduced by R. B. Gnanajothi [2]. C. Sekar [11] proved that the graph obtained by identifying an endpoint of a star with a vertex of a cycle is graceful. Maheo and Thuillier [5] have shown that cycle C_n is k-graceful if and only if either $n \equiv 0$ or $1 \pmod 4$ with k even and $k \leq n(n-1)/2$ or $n \equiv 3 \pmod 4$ with k odd and $k \leq (n^2-1)/2$, while P. Pradhan et al. [6] have shown that cycle C_n; $n \equiv 0 \pmod 4$ is k-graceful for all $k \in N$ (set of natural numbers). Sushant Kumar Rout, Debdas Mishra and Purnachandra Nayak [12] are worked on odd graceful labeling of some new type of graphs obtained by joining of cycle and star and find various result regarding odd graceful labeling. C. Sekar [11] introduced one modulo three graceful labeling. V. Ramachandran and C. Sekar [9] introduced the concept of one modulo N graceful where N is a positive integer and discussed various cycle related graphs are one modulo N graceful. V. Ramachandran and C. Sekar [10] talked about that one modulo N gracefulness of Crowns, Armed crowns and chain of even cycles. V. Ramachandran [8] proved that Cycle C_n is one modulo N graceful labeling if $n \equiv (0 \bmod 4)$. C. Velmurugan and V. Ramachandran [13] introduced M modulo N graceful labeling and proved that path and star are M modulo N graceful graph. If a graph G is M Modulo N graceful labeling, then when $M = N = 1$ the labeling is graceful labeling, when $M = 1$ and $N = 2$ the labeling is odd graceful labeling, when $M = 1$ and $N = 3$ the labeling is one modulo 3 graceful labeling and when $M = 1$ and $N = N$ the labeling is one modulo N graceful labeling.

In this paper we explain four theorems for designing M modulo N graceful Labeling of cycle graph C_n and one theorem for complete graph K_n. Also we propose $C{+}{+}$ algorithm for M modulo N graceful Labeling on cycle graph C_n.

2. Main Result

Theorem 2.1. *The cycle C_n is not M modulo N graceful labeling for all $M = 1$ to N if $n \equiv 1 (\bmod 4)$.*

Proof. Let the cycle C_n, $n \equiv 1 (\bmod 4)$. [7] Rosa proved that the cycle C_n is not graceful if $n \equiv 1 (\bmod 4)$. Since in that case $M = N = 1$.

Clearly this implies that cycle C_n, $n \equiv 1 (\bmod 4)$ is not M modulo N graceful labeling if $M = N$.

To prove that C_n, $n \equiv 1 (\bmod 4)$ is not M modulo N graceful labeling for all $M \neq N$.

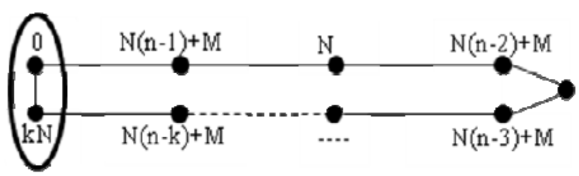
Suppose C_n, $n \equiv 1 (\bmod 4)$ is said to be M modulo N graceful labeling for all $M \neq N$.

Then there exist a function f from the vertex set of C_n to $\{0, M, N, N + M, 2N, \ldots, N(q-1), N(q-1) + M\}$ is one to one. To show that f induces a bijection f^* from edge set of C_n to $\{M, N+M, 2N+M, \ldots, N(q-1)+M\}$, where $f^*(u, v) = |f(u) - f(v)|$ for all $u, v \in V(C_n)$.

Let $u_1, u_2, \ldots, u_n$ be the vertices of C_n, since C_n is M modulo N graceful labeling, hence labeling of the two continuous vertices of C_n is either $f(u_{2i-1}) = kN$ and $f(u_{2i}) = kN + M$, or $f(u_{2i-1}) = kN + M$ and $f(u_{2i}) = kN$, for some $k, 0 \le k \le n-1, 1 \le i \le [(n-1)/2]$. Since n is odd therefore u_1 and u_n both has a labeling either kN or $kN + M$ for some $k, 0 \le k \le n-1$. In C_n, u_1 and u_n are adjacent, therefore $f^*(u_1, u_n)$ $= |f(u_n) - f(u_1)| = kN$ for some $k, 0 \le k \le n-1$ which is not in $f^*(E(C_n))$. Hence there exists a contradiction. So cycle $C_n, n \equiv 1 \pmod 4$ is not M modulo N graceful labeling for all $M \ne N$.

Hence the cycle $C_n, n \equiv 1 \pmod 4$ is not M modulo N graceful labeling for all $M = 1$ to N.

Example 1. The cycle $C_n, n \equiv 1 \pmod 4$ is not M modulo N graceful labeling. The contradiction part marked as oval shape.



Theorem 2.2. *The cycle $C_n, n \equiv 3 \pmod 4$ is M modulo N graceful labeling if $M = N$ and not M modulo N graceful labeling if and $M \ne N$.*

Proof. Let $C_n, n \equiv 3 \pmod 4$ be cycle with n vertices.

[7] Rosa proved that the cycle C_n is graceful if $n \equiv 3 \pmod 4$, in that case $M = N = 1$ and hence $C_n, n \equiv 3 \pmod 4$ is M Modulo N graceful labeling for all $M = N$.

Define

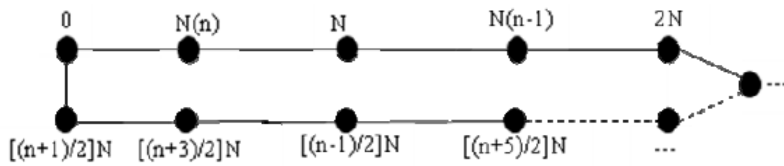
$f(u_{2i}) = (n - i)N + M,\ i = 1$ to $(n - 1)/2$.

$f(u_{2i}) = (n + 1 - i)N,\ i = 1$ to $(n - 1)/2$. Since $M = N$.

$f(u_{2i+1}) = iN,\ i = 0$ to $(n - 3)/4$.

$f(u_{2i+1}) = (i + 1)N,\ i = [(n - 3)/4 + 1]$ to $(n - 1)/2$.

Example 2. Cycle $C_n,\ n \equiv 3 \pmod 4$ is M modulo N graceful labeling if $M = N$.



To prove that $C_n,\ n \equiv 3 \pmod 4$ is not M modulo N graceful labeling for all $M \neq N$. Suppose $C_n,\ n \equiv 3 \pmod 4$ is said to be M modulo N graceful labeling for all $M \neq N$.

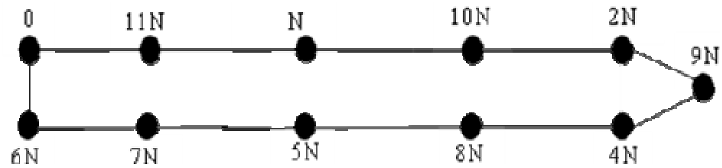
Then there exist a function f from the vertex set of C_n to $\{0, M, N, N+M, 2N, \ldots, N(q-1), N(q-1)+M\}$ is one to one. To show that f induces a bijection f^* from edge set of C_n to $\{M, N, N+M, 2N+M, \ldots, N(q-1), N(q-1)+M\}$, where $f^*(u, v) = |f(u) - f(v)|$ for all $u, v \in V(C_n)$.

Let $u_1, u_2, \ldots, u_n$ be the vertices of C_n, since C_n is M modulo N graceful labeling, hence labeling of the two continuous vertices of C_n is either $f(u_{2i-1}) = kN$ and $f(u_{2i}) = kN + M$, or $f(u_{2i-1}) = kN + M$ and $f(u_{2i}) = kN$, for some $k,\ 0 \leq k \leq n-1,\ 1 \leq i \leq [(n-1)/2]$. Since n is odd therefore u_1 and u_n both has a labeling either kN or $kN + M$ for some $k,\ 0 \leq k \leq n-1$. In C_n, u_1 and u_n are adjacent, therefore $f^*(u_1, u_n) = |f(u_n) - f(u_1)| = kN$ for some $k,\ 0 \leq k \leq n-1$ which is not in $f^*(E(C_n))$. Hence there exists a contradiction. So $C_n,\ n \equiv 3 \pmod 4$ is not M modulo N graceful labeling for all $M \neq N$.

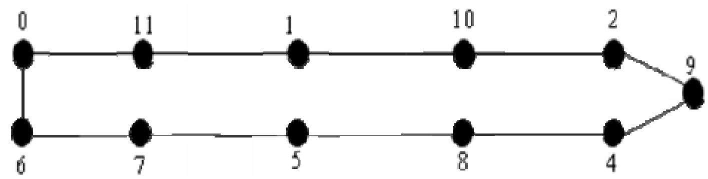
Hence the cycle $C_n,\ n \equiv 3 \pmod 4$ is M modulo N graceful labeling if

$M = N$ and not M modulo N graceful labeling if $M \neq N$.

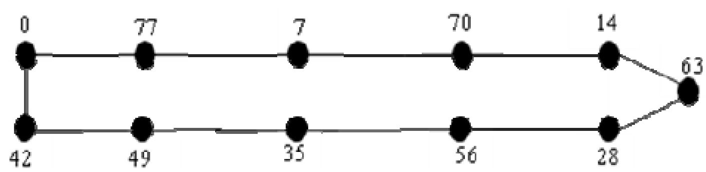
Example 3. Cycle C_{11} is M modulo N graceful labeling if $M = N$



Example 4. Cycle C_{11} is 1 modulo 1 graceful labeling



Example 5. Cycle C_{11} is 7 modulo 7 graceful labeling



Theorem 2.3. *The cycle C_n is not M modulo N graceful labeling if $n \equiv 2 (\text{mod } 4)$ except when $M = 1$ and $N = 2$.*

Proof. Let Cycle C_n, $n \equiv 2 (\text{mod } 4)$. Clearly [7] A. Rosa proved that Cycle C_n, $n \equiv 2 (\text{mod } 4)$ is not graceful, in that case $M = N = 1$. Therefore Cycle C_n, $n \equiv 2 (\text{mod } 4)$ is not M modulo N graceful Labeling for all $M = N$.

[2] Gnanajothi proved that C_n, $n \equiv 2 (\text{mod } 4)$ is odd graceful. [8] V. Ramachadran showed that C_n, $n \equiv 2 (\text{mod } 4)$ is one modulo N graceful labeling but neither graceful nor one modulo N graceful for every positive integer $N \geq 3$.

To prove that Cycle C_n, $n \equiv 2 (\text{mod } 4)$ is not M modulo N graceful labeling if $M \neq N$, except $M = 1$ and $N = 2$.

Suppose C_n, $n \equiv 2 (\text{mod } 4)$ is M modulo N graceful labeling if $M \neq N$.

Clear that [8] V. Ramachadran showed that $C_n,\ n \equiv 2\,(\mathrm{mod}\ 4)$ is one modulo N graceful labeling. Then there exist a function f from the vertex set of $C_n, n \equiv 2(\mathrm{mod}\,4)$ to $\{0,\ M,\ N,\ N+M,\ 2N,\ \ldots,\ N(q-1),\ N(q-1)+M\}$ is one to one. To show that f induces a bijection f^* from edge set of $C_n, n \equiv 2(\mathrm{mod}\,4)$ to $\{M, N+M, 2N+M, \ldots, N(q-1)+M\}$, where $f^*(u,\ v) = |\ f(u) - f(v)\ |$ for all $u,\ v \in V\{C_n,\ n \equiv 2(\mathrm{mod}\ 4)\}$.

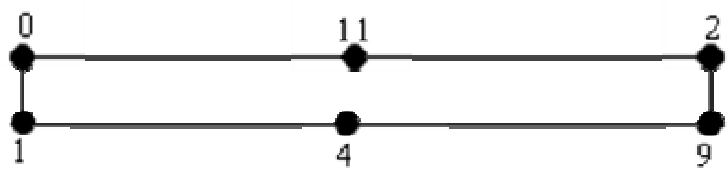
To get the edge label M either we have labels of two adjacent vertices by 0 and M or $Nk + M$ and Nk for some k.

Suppose we take the vertex adjacent to the vertex having the label M will be label as Nj for some $j > 0$ and then we get $|\ Nj = M\ | \equiv (N - M)$ is not in $f^*(E\{C_n,\ n \equiv 2(\mathrm{mod}\ 4)\})$ or duplicate labeling exits.

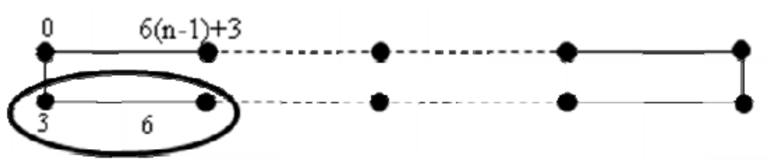
Similarly in the second situation, the second vertex adjacent to the vertex having the label Nk must be $Np + M$ where $p < k$ and in this case $|(Np+M)-Nk| = |Nk-Np-M| \equiv (N-M)$ not in $f^*(E\{C_n,\ n \equiv 2(\mathrm{mod}\ 4)\})$ or duplicate labeling exits.

Hence cycle $C_n,\ n \equiv 2\,(\mathrm{mod}\ 4)$ is not M modulo N graceful labeling except when $M = 1$ and $N = 2$.

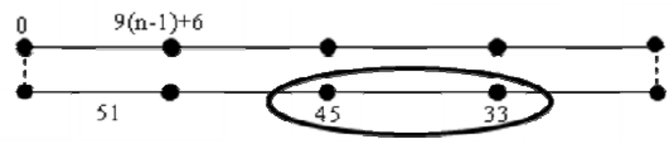
Example 6. Cycle C_6 is one modulo 2 graceful labeling



Example 7. Cycle $C_n,\ n \equiv 2\,(\mathrm{mod}\ 4)$ is not 3 modulo 6 graceful labeling, since edge label 3 create a duplicate labeling. $f^*(E\{C_n, n \equiv 2(\mathrm{mod}\,4)\}) = \{3, 9, 15, \ldots, 6(n-1)+3\}$, i.e., $M = 3,\ N = 6$ and let $j = 1$. The contradiction part marked as oval shape.



Example 8. Cycle C_n, $n \equiv 2 \pmod 4$ is not 6 modulo 9 graceful labeling, since 12 does not belongs in $f^*(E\{C_n, n \equiv 2 (\mathrm{mod}\, 4)\}) = \{6, 15, 24, 33, 42, 51, 60, \ldots, 9(n-1)+6\}$. i.e., $M = 6$, $N = 9$ and let $k = 5$, $p = 3$. The contradiction part marked as oval shape.



Theorem 2.4. *The Cycle C_n is M modulo N graceful labeling if* $n \equiv 0 (\mathrm{mod}\, 4)$.

Proof. Let C_n be cycle of length $n \equiv 0 (\mathrm{mod}\, 4)$.

To define labeling of vertices in C_n

$$f(u_{2i-1}) = (i-1)N \text{ for } i = 1 \text{ to } \frac{n}{2}$$

$$f(u_{2i}) = (n-i)N + M,\ i = 1 \text{ to } \frac{n}{2}$$

$$f(u_{[(n+4i)/2]}) = \left(\frac{3n}{4} - (i+1)\right)N + M,\ i = 1 \text{ to } \frac{n}{4}.$$

Hence the Vertices Labeling are $\{f(u_{2i+1}),\ i = 1 \text{ to } \frac{n}{2} \cup f(u_{2i}),\ i = 1 \text{ to } \frac{n}{2} \cup f(u_{[(n+4i)/2]}),\ i = 1 \text{ to } \frac{n}{2}\} = \{0, N, \ldots, \left(\frac{n}{2} - 1\right)N\} \cup \{(n-1)N + M, (n-2)N + M, \ldots, (3n/4)N + M\} \cup \{[(3n/4) - 2]N + M, [(3n/4) - 3]N + M, \ldots, [(n-2)/]N + M\} = \{0, N, 2N, \ldots, [(n-2/2)]N + M, \ldots, (n-1)N + M\}$ are distinct.

To define labeling of edges in C_n

$f^*(e_{2i-1}) = |f(u_{2i}) - f(u_{2i-1})| = |(n-i)N + M - (i-1)N| = |(n-2i+1)N + M|$, $i = 1$ to $n/4$.

$f^*(e_{2i}) = |f(u_{2i}) - f(u_{2i+1})| = |(n-i)N + M - (i)N| = |(n-2i)N + M|, i = 1$ to $n/4$.

$f^*(e_{(n+4i)/2}) = |f(u_{(n+4i)/2}) - f(u_{[(n+4i+2)/2]})| = |\left(\frac{3n}{4} - (i+1)\right)$

$N + M - [(n+4i)/4]N| = |\left(\frac{n-4i-2}{2}\right)N + M|$, $i = 1$ to $\left(\frac{n}{4} - 1\right)$.

$f^*(e_{(n+4i-2)/2}) = |f(u_{(n+4i)/2}) - f(u_{[(n+4i-2)/2]})|$

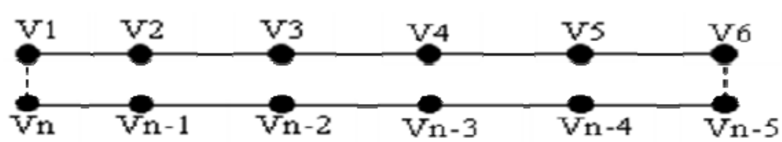
$= |\left(\frac{3n}{4} - (i+1)\right)N + M - \left(\frac{n+4i-4}{4}\right)N|$

$= \left(\frac{n-4i}{2}\right)N + M|$, $i = 1$ to $\frac{n}{4}$.

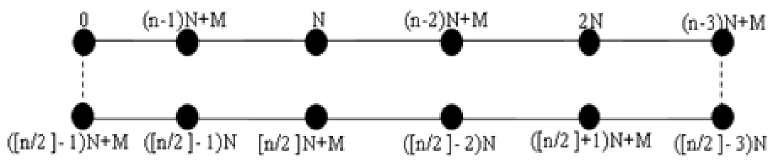
$f^*(e_n) = |f(u_n) - f(u_1)| = \left(\frac{n-2}{2}\right)N + M.$

Hence the edges Labeling are $\{f^*(e_{2i-1}) = i = 1$ to $n/4\} \cup \{f^*(e_{2i}), i = 1$ to $n/4\} \cup \{f^*(e_{(n+4i)/2}), i = 1$ to $(n/4) - 1\} \cup \{f^*(e_{(n+4i-2)/2}), i = 1$ to $(n/4)\} \cup \{f^*(e_n)\{M, N + M, 2N + M, \ldots, (n-1)N + M\}$ are distinct. Hence Cycle C_n is M modulo N graceful labeling if $n \equiv 0 \pmod 4$.

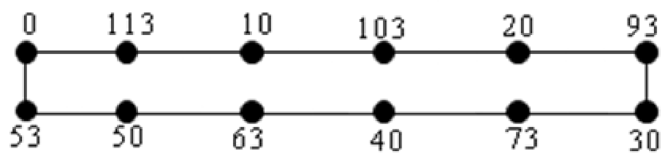
Example 9. Vertices Labeling of C_n



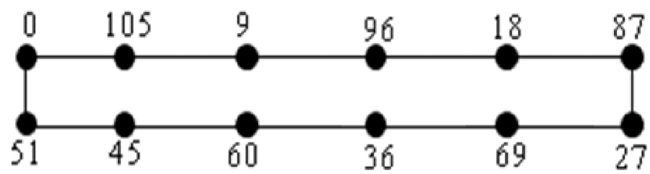
Example 10. M modulo N graceful labeling of C_n



Example 11. 3 modulo 10 graceful labeling of C_{12}



Example 12. 6 modulo 9 graceful labeling C_{12}



Algorithm 2.5. Development of C++ algorithm for M modulo N Graceful Labeling on cycle graph C_n.

```
#include<iostream.h>
#include<conio.h>
void main()
{
clrscr();
int i, j, n, N, M, Y;
cout<<"Enter n vealue for Cn: n=";
cin>>n;
cout<<endl<<"C"<<n<<" is cycle of Length "<<n<<endl;
if((n%4)==0)
```

```
{
cout<<"Enter N vealue for Cn: N=";
cin>>N;
cout<<endl<<"Want to find particular M and N Value: say Yes=1or NO
=0:Y=";
cin>>Y;
if(Y==1)
{
cout<<"Enter M vealue for Cn: M=";
cin>>M;
goto G;
}
for(M=1;M<=N;M++)
{
G:
cout<<endl<<M<<" modulo "<<N<<" graceful Labeling of Vertices of C"
<<n<<endl;
for(i=1;i<=(n/4);i++)
{
cout<<" V"<<(2*i)-1<<"="<<(i-1)*N;
cout<<" V"<<2*i<<"="<<(n-i)*N+M;
}
for(i=1;i<=(n/4);i++)
{
j=(n/4)+i;
cout<<" V"<<(2*j)-1<<"="<<(j-1)*N;
```

```
cout<<" V"<<(n+(4*i))/2<<"="<<(((3*n)/4)-(i+1))*N+M;
}
cout<<endl<<M<<" modulo "<<N<<" graceful Labeling of edges of C"<<n<<endl;
for(i=1;i<=(n/4);i++)
{
cout<<" e"<<(2*i)-1<<"="<<(n-(2*i)+1)*N+M;
cout<<" e"<<2*i<<"="<<(n-(2*i))*N+M;
}
for(i=1;i<=(n/4);i++)
{
cout<<" e"<<(n+(4*i)-2)/2<<"="<<((n-(4*i))/2)*N+M;
if(i<=((n/4)-1))
{
cout<<" e"<<(n+(4*i))/2<<"="<<((n-(4*i)-2)/2)*N+M;
}}
cout<<" e"<<n<<"="<<((n-2)/2)*N+M;
if(Y==1)
{
goto H;
}}
H:
if(Y==1)
{
cout<<endl<<"Hence C"<<n<<"is"<< M<<" modulo "<< N<<" graceful Labeling";
```

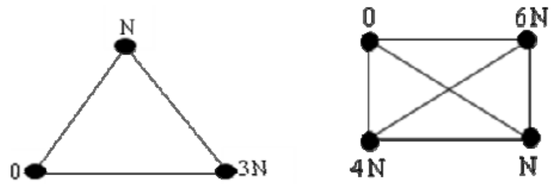
```
}
else
{
cout<<endl<<"Hence C"<<n<<" is M modulo N graceful Labeling";
}}
if((n%4)==1)
{
cout<<endl<<"Hence C"<<n<<" is not M modulo N graceful Labeling";
}
if((n%4)==2)
{
cout<<endl<<"Hence C"<<n<<" is not M modulo N graceful Labeling
except M=1 and N=2";
}
if((n%4)==3)
{
cout<<"Enter N value for Cn: N=";
cin>>N;
for(M=1;M<=N;M++)
{
if(M!=N)
{
cout<<endl<<"Hence C"<<n<<" is not M modulo N graceful Labeling
since" << M<<" not equal to "
<< N;
}
```

```
else
{
cout<<endl<<"Hence C"<<n<<" is "<< M<<" modulo "<< N <<" graceful Labeling since M = N"
<<endl;
for(i=1;i<=((n-1)/2);i++)
{
cout<<" u"<<(2*i)<<"="<<(n+1-i)*N;
}
for(i=0;i<=((n-3)/4);i++)
{
cout<<" u"<<((2*i)+1)<<"="<<i*N;
}
for(i=(((n-3)/4)+1);i<=((n-1)/2);i++)
{
cout<<" u"<<(2*i)+1<<"="<<(i+1)*N;
}}}}
getch();
}
```

Theorem: 2.6. *For any N, the Complete graph K_n is not M modulo N graceful labeling if $n > 4$ and $M = 1$ to N.*

Proof [3]**.** S. W. Golomb explained that the complete graph K_n is graceful if and only if, $n \leq 4$. Clearly K_n is *M* modulo *N* graceful labeling for all *M* and *N* if $n \leq 2$.

Also K_n, $3 \leq n \leq 4$ is M modulo N graceful labeling, when $M = N$, since K_n, $3 \leq n \leq 4$ is graceful, but not M modulo N graceful labeling, when $M \neq N$, since which contains odd cycle c_3.



To prove that complete graph K_n, $n > 4$ is M modulo N graceful labeling when $M = 1$ to N.

Suppose complete graph K_n, $n > 4$ is M modulo N graceful labeling when $M = 1$ to N.

There exist a function f from the vertex set of K_n to $\{0, M, N + M, 2N + M, \ldots, N(q-1), N(q-1) + M\}$ is one to one. Then show that f induces a bijection f^* from edge set of K_n to $\{M, N + M, 2N + M, \ldots, N(q-1) + M\}$ where $f^*(u, v) = |f(u) - f(v)|$ for all $u, v \in V(K_n)$.

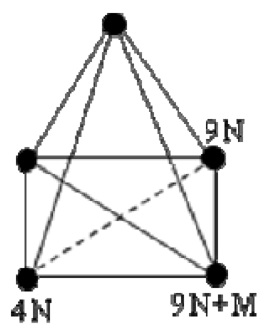
Let $v_1, v_2, \ldots, v_n$ be the vertices of K_n. Assume $f(v_1) = kN$, for some k, $0 \leq k \leq q - 1$ suppose at least two vertices other than v_1 has a labeling must be of the form $kN + M$, for some k, $0 \leq k \leq q - 1$. Since K_n is complete, i.e., each and every vertices adjacent to each and every other vertices. Hence at least one edge has labeling distinct from $\{M, N + M, 2N + M, \ldots, N(q-1) + M\}$. i.e., Let that two vertices are v_r and v_s, $1 < r, s \leq n$, and $r \neq s$, $f(v_r) = iN + M$ and $f(v_s) = jN + M$, $i \neq j$, and $q - 1 > i, j > 0$ then $f^*(v_r, v_s) = |f(v_r) - f(v_s)| = |i - j| N$ which is not in $f^*(E(K_n))$. Hence there exist a contradiction for the definition of f^*. Therefore every Complete graph K_n, $n > 4$ is not M modulo N graceful graph.

Similarly $f(v_1)$ has a labeling from any one of the following

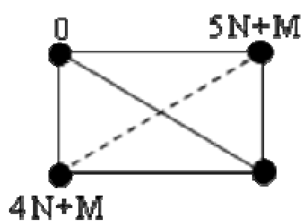
$\{M, N+M, 2N+M, \ldots, N(q-1)+M\}$ then any two vertices except v_1 has distinct labeling from $\{0, N, 2N, \ldots, N(q-1)\}$ since K_n is complete. Let $1 < r, s \le n$, and $r \ne s$, $f(v_r) = iN$ and $f(v_s) = jN, i \ne j$, and $q-1 > i. j > 0$, then $f^*(v_r, v_s) = |(v_r) - (v_s)| = |i-j|N$ which is not in $f^*(E(K_n))$. Hence there exist a contradiction for the definition of f^*.

Hence the complete graph $K_n, n > 4$ is not M modulo N Graceful Labeling for all N, where $M = 1$ to N.

Example 13. K_5 is not M modulo N graceful Labeling, since the absolute difference between vertices incident with dotted edge is not in $f^*(E(K_n))$.



Example 14. K_4 is not M modulo N graceful Labeling if $M \ne N$, since the absolute difference between vertices incident with dotted edge is not in $f^*(E(K_n))$.



Conclusion

In this paper we conclude the following results

(i) Cycle C_n, $n \equiv 0 \pmod 4$ are M modulo N graceful labeling.

(ii) Cycle C_n, $n \equiv 1 \pmod 4$ is not M modulo N graceful labeling for all N, where $M = 1$ to N.

(iii) Cycle C_n, $n \equiv 2 \pmod 4$ is not M modulo N graceful labeling for except $M = 1$ and $N = 2$.

(iv) Cycle C_n, $n \equiv 3 \pmod 4$ is M modulo N graceful labeling for all $M = N$ and not M modulo N graceful labeling for all $M \neq N$.

We design and developed C++ algorithm for M modulo N graceful labeling on cycle graph C_n. Furthermore we showed that Complete graph K_n, $n = 1$ and 2 is M modulo N graceful labeling. Complete graph K_n, $n = 3$ and 4 is M modulo N graceful labeling if $M = N$ and not M modulo N graceful labeling if $M \neq N$. Complete graph K_n, $n > 4$ is not M modulo N graceful labeling for all N, where $M = 1$ to N.

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