Advances and Applications in Mathematical Sciences
Volume 21, Issue 4, February 2022, Pages 1861-1868

APPLICATION OF NEUTROSOPHIC BASIC PROBABILITY ASSIGNMENT IN WATER ANALYSIS PROCESS

R. IRENE HEPZIBAH and D. LEONITA

PG and Research
Department of Mathematics
T. B. M. L. College
Porayar, Tamil Nadu, India
Affiliated to Bharathidasan University
E-mail: ireneraj74@gmail.com

PG and Research
Department of Mathematics
Idhaya College for Women
Kumbakonam, Tamil Nadu, India
Affiliated to Bharathidasan University
E-mail: leodominic59@gmail.com

Abstract

Dempster-Shafer theory of evidence has been widely used in many data fusion application systems. The determination of Basic Probability Assignment, which is the main and the first step in evidence theory, is still an open issue. Compared with traditional BPA, the NBPA considers the truth membership, indeterminacy membership and falsity membership of focal elements. This technique combines the features of evidence theory and Neutrosophic set can handle more flexible, more information and more reasonable NBPA. In this paper, the effect of the parameters which are investigated by two experts are combined by using rule of combination of Neutrosophic Basic Probability Assignment.

1. Introduction

The neutrosophy is a new branch of philosophy. To deal with incomplete, inconsistent and indeterminate information, Florentin Smarandache

2020 Mathematics Subject Classification: 03F55.
Keywords: Water analysis, parameters, DS-Theory, Neutrosophic Basic Probability Assignment.
Received October 30, 2021; Accepted November 10, 2021

introduced the notion of neutrosophic sets. Neutrosophic set is a useful mathematical tool which is the generalization of the classic sets, conventional fuzzy set and intuitionistic fuzzy set. In neutrosophic logic, each proposition has a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F), where T, I, F are standard or non standard subsets of $\left]^{-}0, 1^{+}\right[$.

The Dempster-Shafer theory of evidence, one of the most popular uncertainty theories used in many areas. Dempster-Shafer Theory is a mathematical theory of evidence. It is also called the theory of probable or evidential reasoning. The Dempster-Shafer theory is a powerful theoretical tool which can be applied for the representation of incomplete knowledge, belief updating. It is based on milestones on the lower and upper bounds of belief assignment to the hypothesis. The Dempster-Shafer model of representation and processing of uncertainty has led to a huge number of practical applications in a wide range of domains. In Dempster-Shafer theory, basic probability assignment plays a vital role. All other measures can be defined in the terms of BPA. In actual practice the BPA is usually provided by experts subjectivity.

In Dempster-Shafer theory (DST), evidence can be associated with multiple possible events. Further, Evidence theory is based on two dual non additive measures, namely Belief measure and Plausibility measure. Belief and Plausibility measures can conveniently be characterized by a function $m : \rho(X) \to [0, 1]$ such that $m(\emptyset) = 0$ and $\sum_{A \in \rho(X)} m(A) = 1$.

The function is known as a Basic Probability Assignment (BPA). Every set $A \in \rho(X)$ for which $m(A) > 0$ is usually called a focal element of m. The Dempster Rule of combination is critical to the original conception of Dempster-Shafer Theory. The measure of Belief and Plausibility are derived from the combined basic assignments. Dempster's Rule combines multiple belief functions through their basic probability assignment (m). These belief functions are defined on the same frame of discernment, but are based on independent assignment or bodies of evidence. The Dempster Rule of combination is purely a conjunctive operation. The combination rules results in a belief function based on conjunctive pooled evidence.

The standard way of combining evidence is expressed by the formula,

$$m_{1,2}(A) = \frac{\sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B) \cdot m_2(C)}, \text{ for all } A \neq \emptyset \text{ and } m_{1,2}(\emptyset) = 0.$$

The paper is organized as follows: Section 1 introduces Dempster-Shafer theory and Neutrosophic set theory. Section 2 deals with the basic definitions about Neutrosophic sets and Neutrosophic Basic Probability Assignment. Combination of Neutrosophic Basic Probability Assignment and the effectiveness of the proposed method is illustrated by means of an example in section 3. Finally, some concluding remarks are given in section 4.

2. Preliminaries

Definition 2.1. Let X be a space of points, with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth membership function T_A, indeterminacy membership function I_A and falsity membership function $F_A \cdot T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non standard subsets of $]0^-, 1^+[$.

That is, $T_A : X \to]0^-, 1^+[$,

$I_A : X \to]0^-, 1^+[$,

$F_A : X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. Assume a frame of discernment $\ominus = \{k_1, k_2, k_3, \ldots, k_n\}$. The power set of $\ominus$ is $2^{\ominus} = \{\emptyset, \{k_1\}, \{k_2\}, \ldots, \{k_n\}, \{k_1, k_2\}, \ldots, \{k_1, k_2, k_3, \ldots, k_i\}, \ldots, \ominus\}$. A Neutrosophic basic probability assignment (NBPA) of $2^{\ominus}$ is defined as

$$m(A) = \langle m^T(A), m^I(A), m^F(A) \rangle$$

where $m^T(A) : 2^{\ominus} \to [0, 1]$, $m^I(A) : 2^{\ominus} \to [0, 1]$ and $m^F(A) : 2^{\ominus} \to [0, 1]$ are truth membership, indeterminacy membership and falsity membership respectively.

A is any element of $2^{\ominus}$. They must satisfy the following conditions:

(i) $m^T(\emptyset) = 0$, $m^I(\emptyset) = 0$ and $m^F(\emptyset) = 0$.

(ii) $\sum_{A \in 2^{\ominus}} m^T(A) = 1$.

(iii) For all $A \neq \ominus$, $m^T(A) + m^I(A) + m^F(A) \leq 1$.

Definition 2.3. For NBPA m on $\ominus$, each subset A of $\ominus$ such that $m^T(A) > 0$ or $m^I(A) > 0$ or $m^F(A) > 0$ is called a focal element of m.

Definition 2.4. Given two NBPA m_1 and m_2 on $\ominus$, the combination result is denoted as $m_1 \otimes m_2$ is given by

$$m_1 \otimes m_2(A) = \langle m^T(A), m^I(A), m^F(A) \rangle$$

where

$$\begin{cases} m^T(\emptyset) = 0 \\ m^T(A) = \dfrac{\sum_{B \cap C = A} m_1^T(B) m_2^T(C)}{1 - \sum_{B \cap C = \emptyset} m_1^T(B) m_2^T(C)} \end{cases}$$

$$\begin{cases} m^I(\emptyset) = 0 \\ m^I(A) = \dfrac{\sum_{B \cap C = A} m_1^I(B) m_2^I(C)}{1 - \sum_{B \cap C = \emptyset} m_1^I(B) m_2^I(C)} \end{cases}$$

$$\begin{cases} m^F(\emptyset) = 0 \\ m^F(A) = \dfrac{\sum_{B \cap C = A} m_1^F(B) m_2^F(C)}{1 - \sum_{B \cap C = \emptyset} m_1^F(B) m_2^F(C)} \end{cases}$$

3. Experimental Analysis

Water is a dynamic medium and its quality varies spatially and temporally. In order to characterize any water body, studies on the major components, hydrology, physicochemical and biological characteristic should be carried out. A through knowledge of the hydrological properties of the water body must be acquired before an effective water quality monitoring system is established. Each of the inland water body is characterized by unique hydrological features such as Rivers, Lakes, Ground water and Reservoirs.

The parameter analysed to assess the water quality are broadly divided into Physical parameters, Chemical parameters, Heavy metals and Biological parameters. Colour, Temperature, Transparency, Turbidity and odour are known as Physical parameters. pH, Hardness, Magnesium hardness, Nitrates, Phosphates, Sulphates, chlorides, Dissolved Oxygen, Biological Oxygen Demand, Chemical Oxygen Demand, Fluorides, Free Carbon-di-oxide, Potassium and sodium are known as Chemical parameters. Lead, Copper, Nickel, Iron, Chromium, Cadmium and Zinc are known as Heavy metals. The biological parameters involved the qualitative analyses of planktons.

The effect of pH on the chemical and biological properties of liquids makes its determination very important. It is one of the most important parameter in water chemistry and is defined as log $[H +]$ and measured as intensity of acidity or alkalinity on a scale ranging from 0-14. If free H^+ are more it is expressed acidic (i.e., $pH < 7$), while more OH^- is expressed as alkaline (i.e., $pH > 7$).

The colour and turbidity are two water quality parameters that detract from the appearance of water, making it unpleasing to drink for aesthetic reasons. Colour is organic material that has dissolved into solution, while turbidity consists of tiny particles suspended in the water column.

Turbidity is the critical parameter in drinking water because bacteria, viruses and parasites can attach themselves to the suspended particles. The Turbidity, which is measured and reported in nephelometric turbidity units (NTU), is an optical measurements of water's ability to scatter and absorb light rather than transmit it in straight lines.

Total Dissolved Solids are the inorganic matters and small amounts of organic matter, which are present as solution in water. It is an important indicator of overall water quality. High TDS concentration may cause a bad odour or taste to drinking water, as well as cause scaling of pipes and corrosion. TDS reduces potability of water for drinking purpose.

Alkalinity is not a chemical in water, but, rather, it is a property of water that is dependent on the presence of certain chemicals in the water, such as a bicarbonates, carbonates and hydroxides. It is measure of the ability of the water body to neutralize acids and bases and thus maintain a fairly stable pH level.

By the TWAD reports, the parameter values of ground water of Thanjavur District is given in the following Table.

Parameters	Acceptable Limits	Permissible Limit in the absence of Alternate source
pH	6.5-8.5	No relaxation
Colour (Pt/co Scale)	5	5
Turbidity (NTU)	1	5
Total dissolved Solids(mg/L)	500	2000
Alkalinity(mg/L)	200	600
Iron (mg/L)	1.0	No relaxation
Lead(mg/L)	0.01	No relaxation
Zinc(mg/L)	5	15

An NBPA can be regarded as a 3-tuples BPAs, the first BPA denotes the truth membership, the second BPA denotes the indeterminacy membership and the third BPA denotes the falsity membership. The basic values of neutrosophic sets are singletons, but the focal element of NBPA can be multiple subset of the frame of discernment. The proposed neutrosophic evidence *s* provides a more flexible way to model uncertainty in decision making.

The NBPA value of two experts are given in the following table:

Parameters	NBPA		Combined NBPA
	Expert 1	Expert 2	
pH	⟨0.13, 0.10, 0.10⟩	⟨0.12, 0.10, 0.10⟩	⟨0.15, 0.13, 0.12⟩
Colour (Pt/co Scale)	⟨0.13, 0.15, 0.20⟩	⟨0.14, 0.14, 0.18⟩	⟨0.18, 0.19, 0.13⟩
Turbidity NTU	⟨0.14, 0.13, 0.12⟩	⟨0.12, 0.12, 0.11⟩	⟨0.20, 0.15, 0.10⟩
Total dissolved Solids(mg/L)	⟨0.13, 0.10, 0.11⟩	⟨0.14, 0.12, 0.12⟩	⟨0.18, 0.13, 0.13⟩
Alkalinity(mg/L)	⟨0.10, 0.13, 0.13⟩	⟨0.12, 0.10, 0.10⟩	⟨0.12, 0.12, 0.36⟩
Iron (mg/L)	⟨0.12, 0.10, 0.10⟩	⟨0.10, 0.10, 0.12⟩	⟨0.12, 0.10, 0.13⟩
Lead(mg/L)	⟨0.13, 0.14, 0.11⟩	⟨0.11, 0.14, 0.14⟩	⟨0.14, 0.19, 0.15⟩
Zinc(mg/L)	⟨0.12, 0.13, 0.13⟩	⟨0.12, 0.10, 0.10⟩	⟨0.14, 0.10, 0.13⟩

Conclusion

The estimation of BPA plays a very important role in the application of DS theory in complex uncertain problems. Compared with Neutrosophic set, the NBPA assigns the truth, indeterminacy and falsity membership to not only singletons but also multiple subsets of the frame of discernment. The experimental value have correlated with Neutrosophic Evidences and the combined NBPA of parameters of water was obtained. From this calculation, it is observed that this method have advantage of simple calculations and high accuracy. Those results are promising and interesting as it being addressed for the first time.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20(1) (1986), 87-96.

[2] S. Benferhat, A. Saffiotti and P. Smets, Belief functions and default reasoning, Artificial Intelligence 122(1-2) (2000), 1-69.

[3] E. F. Boran, S. Genc, M. Kurt and D. Akay, A muliti criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Syst. Appl. 36(8) (2009), 11363-11368.

[4] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Annals of Mathematical Statistics 38 (1967), 325-339.

[5] Y. Dong, J. Zhang, Z. Li and Y. Deng, Combination of evidential sensor reports with distance function and belief entropy in fault diagnosis, Int. J. Comput. Common. Control 14(3) (2019), 329-343.

[6] P. Dutta, An uncertainty measure and fusion rule for conflict evidences of big data via Dempster-Shafer theory, Int. J. Image Data Fusion 9(2) (2018), 152-169.

[7] Florentin Smarandache, Collected papers, vol. II, University of Kishinev, Kishinev, 1997.

[8] C. Fu, J. B. Yang and S. L. Yang, A group evidential reasoning approach based on expert reliability, Eur. J. Oper. Res. 246(3) (2015), 886-893.

[9] Z. He and W. Jiang, An evidential Markov decision making model, Inf. Sci. 467 (2018), 357-372.

[10] Y. Li and Y. Deng, TDBF: Two dimensional belief function, Int. J. Intell. Syst. 34(8) (2019), 1968-1982.

[11] Z. G. Liu, Q. Pan and J. Dezert, Evidential classifier for imprecise data based on belief functions, Knowledge-Based Systems 52 (2013), 246-257.

[12] C. K. Murphy, Combining belief functions when evidence conflicts, Decis. Support Syst. 29(1) (2000), 1-9.

[13] H. Seiti, A. Hafezalkotob and L. Martinez, R-numbers, A new risk modeling associated with fuzzy numbers and its application to decision making, Inf. Sci. l483 (2019), 206-231.

[14] G. Shafer, A mathematical theory of evidence, Princeton University Press, Princeton, NJ, USA, (1976).

[15] Y. Song, X. Wang, L. Lei and A. Xue, Combination of interval-valued belief structures based on intuitionstic fuzzy set, Knowledge-Based Systems 67 (2014), 61-70.

[16] Y. Song, X. Wang, L. Lei and A. Xue, Uncertainty measure for interval valued belief structures, Measurement 80 (2016), 241-250.

[17] F. Xiao and B. Qin, A weighted combination method for conflicting evidence in multi sensor data fusion, Sensors 18(5) (2018), 1487.

[18] R. R. Yager, P. Elmore and F. Petry, Soft likelihood functions in combination evidence, Inf. Fusion 36 (2017), 185-190.

[19] Yangxue Li and Yong Deng, IEEE Access, Digital Object Identifier 10, 1109? Access (2019), 2932763.

[20] J. Yen, Generalizing the Dempster-Shafer theory to fuzzy sets, IEEE Transactions on Systems, Man and Cybernetics 20(3) (1990), 559-570.