Advances and Applications in Mathematical Sciences
Volume 21, Issue 2, December 2021, Pages 641-650

INTUITIONISTIC (λ, θ)- FUZZY BI-IDEALS IN NEAR-SUBTRACTION SEMIGROUPS

P. ANNAMALAI SELVI and S. JAYALAKSHMI

Department of Mathematics
Sri Parasakthi College for Women
Courtallam-627802, Tamilnadu
Affiliated to Manonmaniam Sundaranar
University Abishekapatti, Tirunelveli-627012
Tamilnadu, India
E-mail: ananya2015.as@gmail.com
jayarajkutti@gmail.com

Abstract

In this paper we introduce the concept of (λ, θ)- fuzzy bi-ideals and intuitionistic (λ, θ)- fuzzy bi-ideals in near-subtraction semi groups and give some characterizations of intuitionistic (λ, θ)- fuzzy bi-ideals in near subtraction semi groups.

Introduction

The fundamental concept of fuzzy set was first initiated by Zadeh [12]. The concept of intuitionistic fuzzy set was introduced by Atanassov [2] as a generalization of the notion of fuzzy set. Narmada et al. [9] introduced the intuitionistic fuzzy bi-ideals in near-rings. In this paper we introduce the notion of (λ, θ)- fuzzy bi-ideals and intuitionistic (λ, θ)- fuzzy bi-ideals in near-subtraction semi groups. Also we prove that intuitionistic (λ, θ)- fuzzy subset μ of X is a (λ, θ)- fuzzy bi-ideal of X if and only if the level set $U(A; t, r)$ is a fuzzy bi-ideal of X. Further we prove that, a homomorphic image of a intuitionistic (λ, θ)- fuzzy bi-ideal is a intuitionistic (λ, θ)- fuzzy bi-ideal.

2020 Mathematics Subject Classification: 03F55, 03E72, 08A72.

Keywords: Near subtraction semigroups, (λ, θ)- fuzzy bi-ideals and intuitionistic (λ, θ)- fuzzy bi-ideals.

Received May 17, 2021; Accepted June 7, 2021

2. Preliminaries

Definition 2.1. A non empty set X together with two binary operations "–" and "•" is said to be a near subtraction semigroup (right) if it satisfies the following conditions:

(i) $(X, -)$ is a subtraction algebra.

(ii) $(X, \bullet)$ is a semi group.

(iii) $(x - y)z = xz - yz$

for every $x, y, z \in X$.

It is clear that $0x = 0$, for all $x \in X$. Similarly we can define a left near-subtraction semi group. Here after a near-subtraction semigroup means only a right near-subtraction semi group.

Definition 2.2. A nonempty subset S of a subtraction semi group X is said to be a sub algebra of X, if $x - y \in S$, for all $x, y \in S$.

Definition 2.3. A fuzzy sub algebra μ of X is called a fuzzy bi-ideal of X if for all $x, y, z \in X$

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\,yz) \geq \min\{\mu(x), \mu(z)\}$

Definition 2.4. A family of fuzzy set $\{\mu i/i \in \wedge\}$ ($\wedge$- index set) is a near-subtraction semi group of X, the intersection $\bigcap_{i\in\wedge}\mu_i$ of $\{\mu i/i \in \wedge\}$ is defined by $(\bigcap_{i\in\wedge}\mu_i)(x) = \inf\{\mu_i(x)/i \in \wedge\}$ for each $x \in X$.

Definition 2.5. An intuitionistic fuzzy set (IFS) A is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x))/x \in X\}$ where the function $\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership and the degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. We use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x))/x \in X\}$.

Definition 2.6. Let $A = \langle\mu, \lambda\rangle$ be a fuzzy subset of X. Define a level set

$\cup(A; t, r) = \{x \in X / \mu(x) \geq t$ and $\gamma(x) \leq r\}$ where $t \in [0, 1]$ and $r \in [0, 1]$ is called the level set of X.

3. (λ, θ)- Fuzzy bi-ideals of Near-subtraction Semi Groups

Definition 3.1. A fuzzy sub algebra μ of X is called a (λ, θ)- fuzzy bi-ideal of X if for all $x, y, z \in X$,

(i) $\max\{\mu(x - y, \lambda) \min\{\mu(x), \mu(y), \theta\}\}$

(ii) $\max\{\mu(xyz, \lambda) \geq \min\{\mu(x), \mu(z), \theta\}\}$

where $0 \leq \lambda \leq \theta \leq 1$.

Example 3.2. Let $X = \{0, a, b, c\}$ be the Klein's four group. Define subtraction and multiplication in X as follows.

-	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	0
c	c	a	0	a

Then $(X, -, \cdot)$ is a near subtraction semi group (see [3, p. 408] Scheme (0, 7, 0, 7)). Let μ be a fuzzy subset of X, defined $\mu : X \to [0, 1]$ by $\mu(0) = 0.8$, $\mu(a) = \mu(b) = 0.7$, $\mu(c) = 0.6$, $\lambda = 0.3$, $\theta = 0.9$. Then μ is a (λ, θ)- fuzzy bi-ideal of X.

4. Intuitionistic (λ, θ)- fuzzy bi-ideals of Near-subtraction Semi Groups

Definition 4.1. A fuzzy subset μ of X is called an intuitionistic (λ, θ)- fuzzy bi-ideal of X if for all $x, y, z \in X$,

(i) $\max\{\mu(x - y, \lambda) \geq \min\{\mu(x), \mu(y), \theta\}\}$

(ii) $\max\{\mu(xyz, \lambda) \geq \min\{\mu(x), \mu(z), \theta\}\}$

(iii) $\min\{\mu(x - y, \lambda) \leq \max\{\mu(x), \mu(y), \theta\}\}$

(iv) $\min\{\mu(xyz, \lambda) \leq \max\{\mu(x), \mu(z), \theta\}\}$

where $0 \leq \lambda \leq \theta \leq 1$.

Example 4.3. Let $X = \{0, a, b, c\}$ be the Klein's four group. Define subtraction and multiplication in X as follows.

–	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	a	a	0	a

Then $(X, -, \cdot)$ is a near subtraction semi group (see [3, p. 408] Scheme (7, 7, 1, 1)). Let μ be a fuzzy subset of X, defined $\mu : X \to [0, 1]$ by $\mu(0) = 0.9$, $\mu(a) = \mu(b) = 0.8$, $\mu(c) = 0.7$, $\lambda = 0.5$, $\theta = 0.95$. Then μ is a intuitionistic (λ, θ)- fuzzy bi-ideal of X.

Theorem 4.4. *Let* $A = \langle \mu, \gamma \rangle$ *be a* (λ, θ)-*fuzzy subset of X. Then* $A = \langle \mu, \gamma \rangle$ *is a intuitionistic* (λ, θ)-*fuzzy bi-ideal of X if and only if the level set* $\cup(A; t, r)$ *is a bi-ideal of X for all* $t \in (\lambda, \theta]$ *and* $0 \leq \lambda \leq \theta \leq 1$.

Proof. Assume that $A = (\mu, \gamma)$ be an intuitionistic (λ, θ)-fuzzy bi-ideal of X.

To prove that $U(A; t; r)$ is a bi-ideal of X. First to prove that $U(A; t; r)$ is a sub algebra. Now suppose $x, y \in U(A; t; r)$ then $\mu(x) \geq t$ and $\mu(y) \geq t$ and $\gamma(x) \leq r$ and $\gamma(y) \leq r$, then by definition of intuitionistic (λ, θ)-fuzzy bi-ideal $\max\{\mu(x-y), \lambda\} \geq \min\{\mu(x), \mu(y), \theta\} \geq t, \max\{\mu(x-y), \lambda\} \geq \min\{\mu(x), \mu(y), \theta\} \geq t$ and $\min\{\gamma(x-y), \lambda\} \leq \max\{\gamma(x), \gamma(y), \theta\} \geq r$, $\min\{\gamma(x-y), \lambda\} \leq \max\{\gamma(x), \gamma(y), \theta\} \leq r$

Hence $x\,y \in U(A; t; r)$ Therefore $U(A; t; r)$ is a sub algebra.

Suppose $x\,y, z \in U(A; t; r)$ then $\mu(z) \geq t$ and $\gamma(z) \leq r$

$\max\{\mu(xyz), \lambda\} \geq \min\{\mu(x), \mu(z), \theta\} \geq t$

and $\min\{\gamma(xyz), \lambda\} \leq \max\{\gamma(x), \gamma(z), \theta\} \leq r$

Hence $xyz \in U(A; t; r)$. Therefore $U(A; t; r)$ is a bi-ideal of X.

Conversely, assume that $U(A; t; r)$ is a bi-ideal of X.

To prove that $A = (\mu, \gamma)$ is a intuitionistic (λ, θ)-fuzzy bi-ideal of X.

Suppose, we assume that $x\,y \in U(A; t; r)$ and $\max\{\mu(x-y), \lambda\} \ngeq \min\{\mu(x), \mu(y), \theta\}$ or $\min\{\mu(x-y), \lambda\} \nleq \max\{\gamma(x), \gamma(y), \theta\}$.

If $\max\{\mu(x-y), \lambda\} \ngeq \min\{\mu(x), \mu(z), \theta\}$, then there exists $t \in [0, 1]$ such that $\max\{\mu(x-y), \lambda\} < t < \min\{\mu(x), \mu(y), \theta\}$, then $x - y \notin U(A; t, r)$. This is a contradiction.

If $\min\{\gamma(x-y), \lambda\} \nleq \max\{\gamma(x), \gamma(y), \theta\}$, then there exists $r \in [0, 1]$ such that $\min\{\gamma(x-y), \lambda\} > r > \max\{\gamma(x), \gamma(y), \theta\}$.

Hence $x, y \notin U(A; t, r)$, but $x - y \notin (A; t, r)$. This is a contradiction.

Hence $\max\{\mu(x-y), \lambda\} \geq \min\{\mu(x), \gamma(y), \theta\}$ and $\min\{\gamma(x-y), \lambda\} \leq \max\{\gamma(x), \gamma(x), \gamma(y), \theta\}$

Suppose, we assume that $\max\{\mu(xyz), \lambda\} \ngeq \min\{\mu(x), \mu(z), \theta\}$ or

$$\min\{\mu(xyz), \lambda\} \nleq \max\{\mu(x), \mu(z), \theta\}.$$

If $\max\{\mu(xyz), \lambda\} \ngeq \min\{\mu(x), \mu(z), \theta\}$, then there exists $t \in [0, 1]$ such that $\max\{\mu(xyz), \lambda\} < t < \min\{\mu(x), \mu(z), \theta\}$. Hence $x, z \in U(A; t, r)$

But $xyz \notin U(A; t; r)$ This is a contradiction.

If $\min\{\gamma(xyz), \lambda\} \nleq \max\{\gamma(x), \gamma(z), \theta\}$, then there exists $r \in [0, 1]$ such that $\min\{\gamma(xyz), \lambda\} > r > \max\{\gamma(x), \gamma(z), \theta\}$.

Hence $x, z \in U(A; t; r)$, but $xyz \notin U(A; t; r)$. This is a contradiction.

Hence $\max\{\mu(xyz), \lambda\} \geq \min\{\mu(x), \mu(z), \theta\}$, and $\min\{\gamma(xyz), \lambda\} \leq \max\{\gamma(x), \gamma(z), \theta\}$

Hence $A = (\mu, \gamma)$ is an intuitionistic (λ, θ)-fuzzy bi-ideal of X.

Theorem 4.5. *Let* $\{\mu_i\}^{n_{i=1}}$ *be any family of intuitionistic* (λ, θ)-*fuzzy bi-ideals of X. Then* $\mu = \bigcap_{i=1}^{n} \mu_i$ *is an intuitionistic* (λ, θ)-*fuzzy bi-ideal of X.*

Proof. Let $\{\mu_i\}^{n_{i=1}}$ be any family of intuitionistic (λ, θ)-fuzzy bi-ideals of X.

For any $x, y, z \in X$,

(i) $\max\{\mu(x-y), \lambda\} = \max\{(\bigcap_{i=1}^{n} \mu_i)(x-y), \lambda\}$

$$\geq \min_{1 \leq i \leq n}\{\min\{\mu_i(x), \mu_i(y),\}\}$$

$$= \min\{\min_{1 \leq i \leq n}\{\mu_i(x)\}, \min_{1 \leq i \leq n}\{\mu_i(y)\}, \theta\}$$

$$= \min\{(\bigcap_{i=1}^{n} \mu_i)(x), (\bigcap_{i=1}^{n} \mu_i)(y), \theta\}$$

$$= \min\{\mu(x), \mu(y), \theta\}.$$

Therefore $\max\{\mu(x-y), \lambda\} \geq \min\{\mu(x), \mu(y), \theta\}$.

(ii) $\max\{\mu(x\,y\,z), \lambda\} = \max\{(\bigcap_{i=1}^{n} \mu_i)(x\,y\,z), \lambda\}$

$$\geq \min_{1\leq i\leq n}\{\min\{\mu_i(x), \mu_i(z),\}\}$$

$$= \min\{\min_{1\leq i\leq n}\{\mu_i(x)\}, \min_{1\leq i\leq n}\{\mu_i(z)\}, \theta\}$$

$$= \min\{(\bigcap_{i=1}^{n} \mu_i)(x), (\bigcap_{i=1}^{n} \mu_i)(z), \theta\}$$

$$= \min\{\mu(x), \mu(z), \theta\}.$$

Therefore $\max\{\mu(x\,y\,z), \lambda\} \geq \min\{\mu(x), \mu(z), \theta\}$

(iii) $\min\{\mu(x-y), \lambda\} = \min\{(\bigcap_{i=1}^{n} \mu_i)(x-y), \lambda\}$

$$\leq \max_{1\leq i\leq n}\{\max\{\mu_i(x), \mu_i(y),\}\}$$

$$= \max\{\max_{1\leq i\leq n}\{\mu_i(x)\}, \max_{1\leq i\leq n}\{\mu_i(y)\}, \theta\}$$

$$= \max\{(\bigcap_{i=1}^{n} \mu_i)(x), (\bigcap_{i=1}^{n} \mu_i)(y), \theta\}$$

$$= \min\{\mu(x), \mu(y), \theta\}$$

Therefore $\min\{\mu(x-y), \lambda\} \leq \max\{\mu(x), \mu(y), \theta\}$.

(iv) $\min\{\mu(x\,y\,z), \lambda\} = \min\{(\bigcap_{i=1}^{n} \mu_i)(x\,y\,z), \lambda\}$

$$\leq \max_{1\leq i\leq n}\{\max\{\mu_i(x), \mu_i(y),\}\}$$

$$= \max\{\max_{1\leq i\leq n}\{\mu_i(x)\}, \max_{1\leq i\leq n}\{\mu_i(z)\}, \theta\}$$

$$= \max\{(\bigcap_{i=1}^{n} \mu_i)(x), (\bigcap_{i=1}^{n} \mu_i)(z), \theta\}$$

$$= \min\{\mu(x), \mu(z), \theta\}$$

Therefore $\min\{\mu(x\,y\,z), \lambda\} \leq \max\{\mu(x), \mu(z), \theta\}$.

Hence μ is an intuitionistic (λ, θ)-fuzzy bi-ideal of X.

Theorem 4.7. *Let $f : X_1 \to X_2$ be a homomorphism of near-subtraction semi groups and let A be an intuitionistic (λ, θ)-fuzzy bi-ideal of X_1. Then $f(A)$ is an intuitionistic (λ, θ)-fuzzy bi-ideal of X_2, where $f(A)(y) = \sup_{x \in X_1}\{A(x)/f(x) = y\}$ for all $y \in X_2$.*

Proof. Let A be an intuitionistic (λ, θ)-fuzzy bi-ideal of X_1.

For $y_1, y_2, y_3 \in X_2$.

(i) $\max(f(A)(y_1 - y_2), \lambda\} = \max\{\sup\{A(x_1 - x_2)/f(x_1 - x_2) = y_1 - y_2, \lambda\}\}$

$= \max\{\sup\{A(x_1 - x_2), \lambda\}/f(x_1 - x_2) = y_1 - y_2\}$

$= \sup\{\max\{A(x_1 - x_2), \lambda\}/f(x_1 - x_2) = y_1 - y_2\}$

$\geq \sup\{\min\{A(x_1), A(x_2), \theta\}/f(x_1) = y_1, f(x_2) = y_2\}$

$= \min\{\sup\{A(x_1)\}/f(x_1) = y_1\}, \sup\{A(x_2)/f(x_2) = y_2\}, \theta\}$

$= \min\{(f(A)(y_1), f(A)(y_2), \theta\}$.

Therefore $\max\{(f(A)(y_1 - y_2), \lambda\} \geq \min\{f(A)(y_1), f(A)(y_2), \theta\}$.

(ii) $\max(f(A)(y_1\,y_2\,y_3), \lambda\} = \max\{\sup\{A(x_1\,x_2\,x_3)/f(x_1\,x_2\,x_3) = y_1\,y_2\,y_3, \lambda\}\}$

$= \max\{\sup\{A(x_1\,x_2\,x_3), \lambda\}/f(x_1\,x_2\,x_3) = y_1\,y_2\,y_3\}$

$= \sup\{\max\{A(x_1\,x_2\,x_3), \lambda\}/f(x_1\,x_2\,x_3) = y_1\,y_2\,y_3\}$

$\geq \sup\{\min\{A(x_1), A(x_3), \theta\}/f(x_1) = y_1, f(x_3) = y_3\}$

$= \min\{\sup\{A(x_1)\}/f(x_1) = y_1\}, \sup\{A(x_3)/f(x_3) = y_3\}, \theta\}$

$= \min\{(f(A)(y_1), f(A)(y_3)), \theta\}$.

Therefore $\max\{(f(A)(y_1 - y_2), \lambda\} \geq \min\{f(A)(y_1), f(A)(y_3), \theta\}$.

(iii) $\min(f(A)(y_1 - y_2), \lambda\} = \min\{\sup\{A(x_1 - x_2)/f(x_1 - x_2) = y_1 - y_2, \lambda\}\}$

$= \min\{\sup\{A(x_1 - x_2), \lambda\} / f(x_1 - x_2) = y_1 - y_2\}$

$= \sup\{\min\{A(x_1 - x_2), \lambda\} / f(x_1 - x_2) = y_1 - y_2\}$

$\leq \sup\{\max\{A(x_1), A(x_2), \theta\} / f(x_1) = y_1, f(x_2) = y_2\}$

$= \max\{\sup\{A(x_1)\} / f(x_1) = y_1\}, \sup\{A(x_2) / f(x_2) = y_2\}, \theta\}$

$= \max\{(f(A)(y_1), f(A)(y_2)), \theta\}.$

Therefore $\min\{(f(A)(y_1 - y_2), \lambda\} \leq \max\{f(A)(y_1), f(A)(y_2), \theta\}$.

(iv) $\min(f(A)(y_1 y_2 y_3), \lambda\} = \min\{\sup\{A(x_1 x_2 x_3)/f(x_1 x_2 x_3) = y_1 y_2 y_3, \lambda\}\}$

$= \min\{\sup\{A(x_1 x_2 x_3), \lambda\} / f(x_1 x_2 x_3) = y_1 y_2 y_3\}$

$= \sup\{\min\{A(x_1 x_2 x_3), \lambda\} / f(x_1 x_2 x_3) = y_1 y_2 y_3\}$

$\leq \sup\{\max\{A(x_1), A(x_3), \theta\} / f(x_1) = y_1, f(x_3) = y_3\}$

$= \max\{\sup\{A(x_1)\} / f(x_1) = y_1\}, \sup\{A(x_3) / f(x_3) = y_3\}, \theta\}$

$= \max\{(f(A)(y_1), f(A)(y_3)), \theta\}.$

Therefore $\min\{(f(A)(y_1 y_2 y_3), \lambda\} \leq \max\{f(A)(y_1), f(A)(y_3), \theta\}$.

Hence $f(A)$ is an intuitionistic (λ, θ)- fuzzy bi-ideal of X_2.

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