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# ADJOINT OF NON-SQUARE FUZZY MATRICES WITH COMPATIBLE NORM

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## Abstract

In this paper determinant theory and the adjoint of non-square fuzzy matrices have been studied. Some properties of the adjoint of non-square fuzzy matrices are discussed. A new type of compatible norm $\|\cdot\|_C$. distributive law and equivalence of the non-square fuzzy matrices.

## 1. Introduction

The concept of fuzzy set was introduced by Zadeh [10] A. Arunkumar, S. Murthy, G. Ganapathy [1] introduced determinant for non-square matrices. In 1995 Ragab. M. Z and Eman [8] introduced the determinant and Adjoint of square Fuzzy Matrix. Nagoorgani A. and Kalyani G. [5] introduced the binormed sequences in fuzzy matrices. A. Nagoorgani and A. R. Manikandan [6] introduced integral over Fuzzy Matrices. A. R. Meenakshi [3] some

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concept of matrix theory and applications in fuzzy matrices. Dennis Bernstein [2] introduced compatible norm in matrix mathematics theory, facts and Formulas. A. K Shymal and Madhumangal Pal [9] properties of triangular fuzzy matrices. Some concept of Madhumangal Pal and Rajkumar Pradhan. [4] triangular Fuzzy Matrix sNorm. A. Nagoorgani and A. Pappa [7] introduced determinant for non-square fuzzy matrices with compatible norm.

In this paper the concept Adjoint of non-square fuzzy matrices with Compatible Norm discussed. In section [4] adjoint of non-square fuzzy matrices properties are given. In section [5] distributive law of non-square fuzzy matrices with compatible norm. In section [3] equivalence of non-square fuzzy matrices using compatible norm.

## 2. Preliminaries

We consider $\mathcal{F} = [0, 1]$ the fuzzy algebra with operator $[+, \cdot]$ and the standard order "≤" where $a + b = \max\{a, b\}$, $a \cdot b = \min\{a, b\}$ for all $a, b$ in $\mathcal{F} \cdot \mathcal{F}$ is a commutative semiring with additive and multiplicative identies 0 and 1 respectively. Let $\mathcal{F}_{mn}$ denote the set of all $m \times n$ NSFM over $\mathcal{F}_{mn}$. In short $\mathcal{F}_{mn}$ is the set of all NSFM of order $m \times n$ define "+" and Scalar Multiplication in $\mathcal{F}_{mn}$ as $A + B = [a_{ij} + b_{ij}]$ where $A = [a_{ij}]$ and $B = [b_{ij}]$ and $cA = [ca_{ij}]$ where $c$ is in [0,1] with these operations $\mathcal{F}_{mn}$. Forms a linear space. NSFM Multiplication is number of columns in the first Matrix must be equal to the number of rows in the second Matrix with the operations $\mathcal{F}_{mn}$ forms a linear space.

## 3. Determinant Theory and the Equivalence of Non-Square Fuzzy Matrices

**Definition 3.1.** An $m \times n$ Matrix $A = [a_{ij}]$ whose components are in the unit interval [0, 1] is called Fuzzy Matrix.

**Definition 3.2.** The determinant $|A|$ of an $n \times n$ Fuzzy Matrix $A$ is defined as follows; $|A| = \sum_{\sigma \in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \ldots a_{n\sigma(n)}$. Where $S_n$ denotes the symmetric group of all permutations of the indices $(1, 2, \ldots, n)$.

**Definition 3.3.** A Non-Square Fuzzy Matrix [NSFM] $A = [a_{ij}]$ of order $m \times n$ over $\mathcal{F}_{mn}$. If $n > m$. Then the Matrix $A$ is called horizontal Non-Square Fuzzy Matrix. Otherwise $A$ is called Vertical Non-Square Fuzzy Matrix.

**Definition 3.4.** To every Non-Square Fuzzy Matrix [NSFM] $A = [a_{ij}]$ of order $m \times n$ over $\mathcal{F}_{mn}$ with entries as unit interval [0, 1] Determinant $|A|$ of $m \times n$ over $\mathcal{F}_{mn}$. Fuzzy Matrix $A$ is defined as follows. $|A| = \sum_{\sigma \in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \ldots a_{m\sigma(n)}$ (where $S_n$ denotes $mn$).

**Definition 3.5.** The NSFM $|A| = [a_{ij}]$ be the order $m \times n$ over $\mathcal{F}_{mn}$. If the order $m \times n \geq 3$. The minor of arbitrary element $a_{ij}$ is the determinant of the value.

**Definition 3.6.** Non-Square Fuzzy Matrices of minor:

The NSFM $A = [a_{ij}]$ be the order of $m \times n$ over $\mathcal{F}_{mn}$. The minor of an element $a_{ij}$ in det $|A|$ is the order $(m-1) \times (n-1)$ NSFM formed by deleting $i$-th row and the $j$-th column from $A = (a_{ij})$ denoted by $M_{ij}$.

**Definition 3.7.** Cofactor:

The NSFM $A = (a_{ij})$ be the order of $m \times n$ over $\mathcal{F}_{mn}$. The Cofactor of an element $a_{ij}$ is denoted by $A_{ij}$ and is defined as $A_{ij} = (1)^{i+j} M_{ij}$.

**Definition 3.8.** (Compatible Non-Square Fuzzy Matrices)**.** Compatible Fuzzy Matrices which can be multiplied for this to be possible. The number of columns in the first Non-Square Fuzzy Matrix must be equal to the number of rows in the second Non-Square Fuzzy Matrix (NSFM). The product of $m \times p$. Non-square Fuzzy Matrix and $p \times n$. Non-Square Fuzzy Matrix has order $m \times n$ Non-Square Fuzzy Matrix over $\mathcal{F}_{mn}$ we consider $\mathcal{F} = [0, 1]$.

**Definition 3.9.** (Compatible Norm $\| \cdot \|_c$). Let $\mathcal{F}_{mn}$ is the set of all $(m \times n)$ NSFM over $\mathcal{F} = [0, 1]$. Define the norms $\| \cdot \|_c, \| \cdot \|_{c'}, \| \cdot \|_{c''}$ on the order $m \times n$, $m \times p$, $p \times n$ over $\mathcal{F}_{mn}$ respectively, are compatible if for all $A \in \mathcal{F}_{mp}$

and $B \in \mathcal{F}_{pn}$. Then

$$\| AB \|_c \leq \| A \|_{c'} \| B \|_{c''}.$$

**Definition 3.10.** Let $A$ be in NSFM $A = [a_{ij}]$ be the order of $m \times n$ over $\mathcal{F}_{mn}$ defined as $A^c = [1 - a_{ij}]$, where $A = (a_{ij})$.

For all $i = 1$ to $m$, $j = 1$ to $n$. Then $A^c$ is known as the complement Matrix of $A$.

$$\text{If } A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 09 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix} \text{ then } A^c = \begin{bmatrix} 0.5 & 1.0 & 0.6 & 0.4 \\ 0.9 & 0.1 & 0.3 & 0.5 \\ 0.2 & 0.7 & 0.5 & 0.8 \end{bmatrix}$$

**Definition 3.11.** Let $A = [a_{ij}]$ and $B = [b_{ij}]$ NSFM over $\mathcal{F}_{mn} A$ is defined greater than $B$ if $\| B \|_c \leq \| A \|_c B$ is greater than $A$ if $\| A \|_c \leq \| B \|_c$. $A$ and $B$ NSFM are said to comparable if either $\| A \|_c \leq \| B \|_c$ (or) $\| B \|_c \leq \| A \|_c$.

**Example 3.12.** If $A \leq B$.

$$\text{If } A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix} B = \begin{bmatrix} 0.6 & 0.8 & 0.5 & 0.7 \\ 0.3 & 0.9 & 0.8 & 0.6 \\ 0.9 & 0.4 & 0.6 & 0.5 \end{bmatrix}$$

$\| A \|_c = 0.6$

$\| B \|_c \leq 0.8.$

**Theorem 3.13.** *Let* $A, B$ *NSFM over* $\mathcal{F}_{mn}$. *Then* $\| A \|_c \leq \| B \|_c \Leftrightarrow \| A + B \|_c = \| B \|_c$.

**Proof.** $\| A \|_c \leq \| B \|_c$ then $\| A \|_c + \| B \|_c \max \{a_{ij}, b_{ij}\} = [b_{ij}] = B$.

Conversely, if $\| A + B \|_c = \| B \|_c$ then $a_{ij} \leq b_{ij}$, that is, $\| A \|_c \leq \| B \|_c$. Thus $\| A \|_c \leq \| B \|_c \Leftrightarrow \| A + B \|_c = \| B \|_c$.

**Example 3.13.1.** If

$$A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix} B = \begin{bmatrix} 0.6 & 0.8 & 0.5 & 0.7 \\ 0.3 & 0.9 & 0.8 & 0.6 \\ 0.9 & 0.4 & 0.6 & 0.5 \end{bmatrix}$$

$$\| A + B \|_c = \begin{bmatrix} 0.6 & 0.8 & 0.5 & 0.7 \\ 0.3 & 0.9 & 0.8 & 0.6 \\ 0.9 & 0.4 & 0.6 & 0.5 \end{bmatrix} = 0.8$$

$\| A + B \|_c = \| B \|_c$.

**Theorem 3.14.** *Let* $A$, $B$ *be NSFM over* $\mathcal{F}_{mn}$. *If* $\| A \|_c \le \| B \|_c$ *then for any* $C \in \mathcal{F}_{np} \| AC \|_c \le \| BC \|_c$ *and for any* $D \in \mathcal{F}_{pm} \| DA \|_c \le \| DB \|_c$.

**Proof.** If $\| A \|_c \le \| B \|_c$ NSFM for $C$ is the compaitable NSFM then $\| AC \|_c \le \| BC \|_c$ $A = [a_{ij}] B = [b_{ij}] C = [c_{jk}]$.

Since $\| A \|_c \le \| B \|_c$, $a_{ij} \le b_{ij}$ for $i = 1$ to $m$ and $j = 1$ to $n$ by NSFM compaitable

$$a_{ij} \, c_{jk} \le b_{ij} \, c_{jk}$$

for $k = 1$ to $p$. by NSFM addition we get $\sum_k a_{ij} c_{jk} \le \sum_k b_{ij} c_{jk}$.

Thus $\| AC \|_c \le \| BC \|_c \cdot \| DA \|_c \le \| DB \|_c$ can be proved in the same manner.

**Theorem 3.15.** *If A and B are NSFM is the set of all* $m \times n$ over $\mathcal{F}_{mn}$. *We consider* $\mathcal{F} = [0, 1]$ *and any Scalar in* $[0, 1]$ *we have*

*If* $\| A\tilde{x} \|_c \le \| A \|_{c'} \| \tilde{x} \|_{c''}$

(i) $\| \tilde{y} A \|_c \le \| \tilde{y} \|_{c'} \| A \|_{c''}$

(ii) $\| \alpha A \tilde{x} \|_c \le \alpha \| A \|_{c'} \| \tilde{x} \|_{c''}$

(iii) $\| A\tilde{x} + B\tilde{x} \|_c = \| A\tilde{x} \|_c + \| B\tilde{x} \|_c$.

**Proof.**

(i) If $m = 1$ the norms $\| \cdot \|_c$, $\| \cdot \|_{c'}$, $\| \cdot \|_{c''}$, $\mathcal{F}_n$, $\mathcal{F}_p$, $\mathcal{F}_{pn}$ respectively, are compatible if for all $A \in \mathcal{F}_{pn}\, \bar{y} \in \mathcal{F}_p$. Let $\bar{y}$ be any fuzzy vector in $m \times n$ over $\mathcal{F}_{mn}$.

Then it is enough to prove that

$$\| \bar{y}A \|_c \leq \| \bar{y} \|_c, \| A \|_c$$

$$\| \bar{y}A \|_c \leq \| \bar{y} \|_c\, [a_{ij}]$$

$$\leq \| \bar{y} \|_{c'} \| A \|_{c''}.$$

(ii) If $n = 1$ the norms $\| \cdot \|_c, \| \cdot \|_{c'}, \| \cdot \|_{c''}, \mathcal{F}_m, \mathcal{F}_{mp}, \mathcal{F}_p$ respectively, are compatible if for all $A \in \mathcal{F}_{mp}, \bar{x} \in \mathcal{F}_p$.

If $\alpha$ in [0, 1] then $\alpha A = [\alpha a_{ij}]$

$$\| \alpha A\bar{x} \|_c \leq [\alpha a_{ij}] \| \bar{x} \|_c$$

$$\leq \alpha [a_{ij}] \| \bar{x} \|_c$$

$$\leq \alpha \| A \|_{c'} \| \bar{x} \|_{c'}.$$

(iii) If $n = 1$ the norms $\| \cdot \|_c, \| \cdot \|_{c'}, \| \cdot \|_{c''}, \mathcal{F}_p, \mathcal{F}_p, \mathcal{F}_{pm}$ respectively, are compatible if for all $A, B \in \mathcal{F}_{mp}$ $\bar{x} \in \mathcal{F}_p$ $\| A\bar{x} \|_c = [a_{ij}] \| \bar{x} \|_c$ and $\| B\bar{x} \|_c = [b_{ij}] \| \bar{x} \|_c$

$$\| A\bar{x} + B\bar{x} \|_c = [[a_{ij}] + [b_{ij}]] \| \bar{x} \|_c$$

$$= [a_{ij}] \| \bar{x} \|_c + [b_{ij}] \| \bar{x} \|_c$$

$$= \| A\bar{x} \|_c + \| B\bar{x} \|_c.$$

## 4. Adjoint of Non-Square Fuzzy Matrices with Compatible Norm

**Definition 4.1.** The Adjoint Matrix of an $m \times n$ NSFM over $\mathcal{F}_{mn} A$ is denoted by the $(i, j^{th})$ entry adj $A$ and is defined as

$$b_{ij} = | A_{ji} |,$$

where $| A_{ji} |$ is the determinant of the $(m-1) \times (n-1)$. Fuzzy Matrix formed by deleting row $j$ and column $i$ from $A$ and $B = adj\ A$.

**Remark 1.** We can rewrite $b_{ij}$ of $adj\ A = B = [b_{ij}]$ as follows

$$b_{ij} = \sum_{\sigma \in S_{n_j m_i}} \prod_{t \in n_j m_i} a_{t\sigma(t)},$$

**Example 4.1.1.**

If $A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix}$

$adjA = b_{ij} = | A_{ji} |$

minor of $M_{ij} = b_{ij}$

$$\begin{array}{lll} b_{11} = 0.5 & b_{21} = 0.5 & b_{31} = 0.6 \\ b_{12} = 0.7 & b_{22} = 0.6 & b_{32} = 0.5 \\ b_{13} = 0.8 & b_{23} = 0.6 & b_{33} = 0.6 \\ b_{14} = 0.8 & b_{24} = 0.5 & b_{34} = 0.5 \end{array}$$

$$adj\ A = b_{ij} = | A_{ji} | = \begin{bmatrix} 0.5 & 0.5 & 0.6 \\ 0.7 & 0.6 & 0.6 \\ 0.8 & 0.6 & 0.6 \\ 0.8 & 0.5 & 0.5 \end{bmatrix} = 0.6.$$

**Theorem 4.2.** *For an* $m \times n$ *NSFM A and B we have the following*

(i) $| A | \leq | B |$ *implies* $| adj\ A | \leq | adj\ B |$

(ii) $| A |^T \leq | B |^T$ implies $| adj\ A |^T \leq | adj\ B |^T$

(iii) $| adj\ A + adjB | \leq | adj\ (A + B) |$

(iv) $| adjA^T | = | (adjA)^T |$

(v) $| A(adj\ A) | = | (A\ adj\ A) |^T$.

**Proof.**

1. Let $C = |\, adj\ A \,|$ and $D = |\, adj\ B \,|$ that is

$$c_{ij} = \sum_{\sigma \in S_{n_j m_i}} \prod_{t \in n_j m_i} a_{t\sigma(t)},$$

and

$$d_{ij} = \sum_{\sigma \in S_{n_j m_i}} \prod_{t \in n_j m_i} b_{t\sigma(t)}.$$

It is clear that $c_{ij} \leq d_{ij}$ because $a_{t\sigma(t)} \leq b_{t\sigma(t)}$ for every $t \in n_j m_i$

2. Let $C_1 = |\, adj\ A \,|^T$ and $D_1 = |\, adj\ B \,|^T$ that is

$$c_{ji} = \sum_{\sigma \in S_{m_i n_j}} \prod_{\sigma(t) \in m_i n_j} a_{t\sigma(t)},$$

and

$$d_{ji} = \sum_{\sigma \in S_{m_i n_j}} \prod_{\sigma(t) \in m_i n_j} b_{t\sigma(t)}.$$

It is clear that $c_{ji} \leq d_{ji}$ because $a_{t\sigma(t)} \leq b_{t\sigma(t)}$ for every $t \in m_i n_j$.

3. Because $A, B \leq A + B$, it is clear that $adjA, adjB \leq adj(A + B)$ and so $|\, adj\ A + adj\ B \,| \leq |\, adj\ (A + B) \,|$.

4. Let $B = adj\ A$ and $C = adj\ A^T$, then

$$b_{ij} = \sum_{\sigma \in S_{n_j m_i}} \prod_{t \in n_j m_i} a_{t\sigma(t)},$$

and

$$c_{ij} = \sum_{\sigma \in S_{n_j m_i}} \prod_{\sigma(t) \in n_j m_i} a_{t\sigma(t)},$$

which is the element $b_{ij}$ hence $|\, (adj\ A)^T \,| = |\, adj\ A^T \,|$.

Similarly we prove following properties.

(i) If $|\, A \,| = |\, A_1 + A_2 \,|$ then $|\, adj\ A \,| = |\, adj A_1 + adj A_2 \,|$

(ii) If $c|A| = c|A_1| + c|A_2|$ then $|cadj\ A| = |cadj\ A_1| + |cadj\ A_2|$.

**Theorem 4.3.** *Let A be a NSFM and adj A of NSFM, the multiple A and adj A is equal to a Square Fuzzy Matrix.*

$$\| A(adj\ A) \|_c \neq \| (adj\ A)A \|_c.$$

**Proof.** Let $C = A(adj\ A)$ and $D = (adj\ A)A$ then

$$C_{ij} = \sum_{j=1}^{n} a_{ij} | A_{ji} | \text{ (Square Fuzzy Matrix)}$$

and

$$d_{ij} = \sum_{i=1}^{m} | A_{ji} | a_{ij}. \text{ (Square Fuzzy Matrix)}$$

**Example 4.3.1.**

If

$$A = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix}$$

$$\| adj\ A \|_c = \begin{bmatrix} 0.5 & 0.5 & 0.6 \\ 0.7 & 0.6 & 0.6 \\ 0.8 & 0.6 & 0.6 \\ 0.8 & 0.5 & 0.5 \end{bmatrix}$$

$$\| A(adj\ A) \|_c = \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0.6 \\ 0.7 & 0.6 & 0.6 \\ 0.8 & 0.6 & 0.6 \\ 0.8 & 0.5 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.5 & 0.5 \\ 0.7 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}$$

its $3 \times 3$. Square Fuzzy Matrix

$$\| A(adj\ a) \|_c = 0.6.$$

$$\| (adj\ A)A \|_c = \begin{bmatrix} 0.5 & 0.5 & 0.6 \\ 0.7 & 0.6 & 0.6 \\ 0.8 & 0.6 & 0.6 \\ 0.8 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.5 & 0.0 & 0.4 & 0.6 \\ 0.1 & 0.9 & 0.7 & 0.5 \\ 0.8 & 0.3 & 0.5 & 0.2 \end{bmatrix}$$

$$\| (adj\ A)A \|_c = \begin{bmatrix} 0.6 & 0.5 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.5 & 0.6 \end{bmatrix} \text{ its } 4 \times 4 \text{ Square Fuzzy Matrix}$$

$$\| (adj\ A)A \|_c = 0.5.$$

**Theorem 4.4.** *For any* $m \times n$ *NSFM* $A$, *the NSFM* $A(adj\ A)$ *and* $(adj\ A)A$ *is compatible.*

**Proof.**

(i) $\| A(adj\ A) \|_c \leq \| A \|_{c'} \| adj\ A \|_{c''}$

$0.6 = (0.6)(0.6)$

$0.6 \leq 0.6$

(ii) $\| (adj\ A)A \|_c \leq \| (adj\ A) \|_{c''} \| A \|_{c'}$

$0.5 \leq (0.6)(0.6)$

$0.5 \leq 0.6$

(iii) $\| (A(adj\ A))^2 \|_c \leq \| A(adj\ A) \|_c$

$$\| (A(adj\ A))^2 \|_c = \begin{bmatrix} 0.6 & 0.5 & 0.5 \\ 0.7 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.5 & 0.5 \\ 0.7 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.6 \end{bmatrix} \leq \| A(adj\ A) \|_c$$

(iv) $\| ((adj\ A)A)^2 \|_c \leq \| (adj\ A)A \|_c$

$$\| ((adj\ A)A)^2 \|_c = \begin{bmatrix} 0.6 & 0.5 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.5 & 0.6 \end{bmatrix} \begin{bmatrix} 0.6 & 0.5 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.5 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & 0.5 & 0.5 & 0.5 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.6 & 0.6 & 0.6 & 0.6 \\ 0.5 & 0.5 & 0.5 & 0.6 \end{bmatrix} \le \| (adj\ A)A \|_c.$$

## 5. Distributive Law of Non-Square Fuzzy Matrices with Compatible Norm

(i) $\| A(B+C)\|_c = \| AB\|_c + \| AC\|_c$ and $\| A(B+C) \|_c = \| A \|_{c'} + \| B+C \|_{c''}$

(ii) $\|(B+C)A\|_c = \| BA\|_c + \|CA\|_c$ and $\| (B+C)A \|_c = \| B+C \|_{c'} + \| A \|_{c''}$.

Then the NSFM are over $\mathcal{F} = [0,1] A$ and $(B+C), (B+C)$ and $A$ compatible in $\mathcal{F}_{mn}$

(i) $A, B, C$ are $m \times n, n \times p, n \times p$ respectively $\| A(B+C)\|_c = \| AB\|_c + \| AC\|_c$.

Let $| A | = [a_{ij}], | B | = [b_{jk}]$ and $| C | = [c_{jk}]$ such that the ranges $i, j, k$ are $i = 1$ to $m$, $j = 1$ to $n, k=1$ to $p$ respectively.

$$\| B + C \|_c = [b_{jk} + c_{jk}] = \max \{b_{jk}, c_{jk}\}$$

$(ik)^{\text{th}}$ element in the product of $A$ and $(B+C)$ that is $A(B+C)$ is the sum of the products of the corresponding elements in the $i^{\text{th}}$ row of $A$ and $k^{\text{th}}$ column of $B+C$

$$= \sum_{j=1}^{n} a_{ij}(b_{jk} + c_{jk})$$

$$= \sum_{j=1}^{n} \min (a_{ij} b_{jk}) + \sum_{j=1}^{n} \min (a_{ij} c_{jk})$$

= $(ik)^{\text{th}}$ entries of $AB$ + $(ik)^{th}$ entries of $AC$

$= (ik)^{th}$ entries of $(AB + AC)$

$= \| AB + BC \|_c = \| AB \|_c + \| AC \|_c$

(ii) $A, B, C$ are $p \times m, n \times p, n \times p$ respectively $\|(B+C)A\|_c = \|BA\|_c + \|CA\|_c$.

Let $|A| = [a_{ki}], |B| = [b_{jk}]$ and $|C| = [c_{jk}]$ such that the ranges $i, j, k$ are $i = 1$ to $m$, $j = 1$ to $n$, $k = 1$ to $p$ respectively.

$$\| B + C \|_c = [b_{jk} + c_{jk}] = \max\{b_{jk}, c_{jk}\}$$

$(ji)^{th}$ element in the product of $(B + C)$ and $A$ that is $(B + C)A$ is the sum of the products of the corresponding elements in the $j^{th}$ row of $B + C$ and $i^{th}$ column of $A$

$$= \sum_{j=1}^{n} (b_{jk} + c_{jk}) a_{ki}$$

$$= \sum_{j=1}^{n} \min(b_{jk} a_{ki}) + \sum_{j=1}^{n} \min(c_{jk} a_{ki})$$

$= (ji)^{th}$ entries of $BA + (ji)^{th}$ entries of $CA$

$= (ji)^{th}$ entries of $(BA + CA)$

$= \| (B + C)A \|_c = \| BA \|_c + \| CA \|_c$.

## 6. Conclusion

In this paper new definition for the Equivalence of Non-Square Fuzzy Matrices and its properties are suggested in Fuzzy environment. A numerical example is given to clarify the developed theory and the proposed Adjoint of Non-Square Fuzzy Matrices with Compatible Norm.

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