Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4399-4407

AN APPLICATION OF SOLVING FUZZY SEQUENCING PROBLEMS WITH OCTOGONAL FUZZY NUMBERS USING SIMPSON'S 1/3rd RULE

M. ANANTHANARAYANAN and R. MAHALAKSHMI

A. M. Jain College, Chennai-114
Tamil Nadu, India

Anna Adarsh College for Women
Chennai-40, Tamil Nadu, India

Abstract

In this paper, we consider the current scenario of assigning n occupations to three machines, the handling time is considered as octagonal fuzzy numbers. The fuzzy sequencing problem is changed over crisp values by utilizing Simpson's 1/3rd Rule. Hence, the optimal sequence (order) of the jobs is found by using Johnson's algorithm, by which total elapsed time and idle time for each machine is obtained. An illustrative numerical example has been provided to exhibit the viability of the fuzzy sequencing problems.

Introduction

The term fuzzy logic was introduced in 1965. The uncertainty in determining the data can be replaced by the fuzzy notions introduced by Zadeh [5] in the year 1965. Dr. S. Ramkumar, Dr. M. Ananthanarayanan [1], presented a Comparative Analysis of Fuzzy Shortest Travelling Path using Octogonal fuzzy numbers based on Measures of Dispersion. Kripa. K, Govindarajan. R. [3] presented the different methods to solve fuzzy sequencing problem using fuzzy technological values. Dr. S. U. Malini and S. Kalaivani [7], introduced a method to solve fuzzy sequencing problem using octagonal fuzzy number. P. Malini and Dr. M. Ananthanarayanan [2], solved fuzzy transportation problem using ranking of octagonal fuzzy numbers. M.

2020 Mathematics Subject Classification: Primary 03E72, 65K10, 90XX.
Keywords: Fuzzy number, Triangular fuzzy number, Trapezoidal fuzzy number, Octogonal fuzzy numbers, Fuzzy arithmetic operations, Fuzzy Sequencing problems, Simpson's 1/3rd Rule, Johnson's Algorithm.

Received January 10, 2022; Accepted March 2, 2022

Shanmugasundari [8] proposed a parametric approach on Fuzzy sequencing problem. H. J. Zimmermann [6] introduced "Fuzzy Set Theory and Its Applications".

In this paper we present the essential ideas and definitions of fuzzy numbers and manages the proposed new algorithm. To solve the procedure a suitable example is illustrated. The Fuzzy optimal solution is obtained using Fuzzy Sequencing problem. Finally, the conclusion is given.

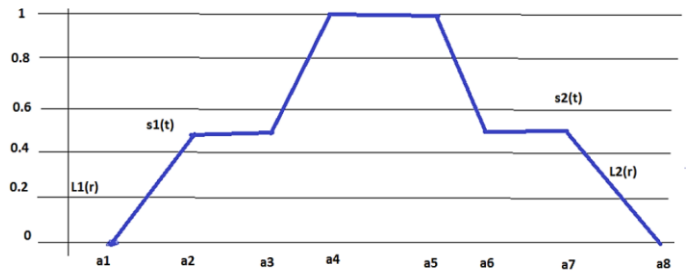
Definitions. Fuzzy Number. A fuzzy number is a generalization of a regular real number that does not relate to a single value, but rather to a connected collection of alternative values with weights ranging from 0 to 1.

A fuzzy number is a convex normalized fuzzy set on the real line R such that, there exist as least one (i) $x \in X$ with $\mu_{\underline{A}}(x) = 1$ (ii) $\mu_A(x)$ is piece wise continuous.

Octagonal Fuzzy Number.

A fuzzy number with membership function $\underline{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers and its membership function $\mu_A(x)$ is given below

$$\mu_A(x) = \begin{cases} 0.5\left[\dfrac{x - a_1}{a_2 - a_1}\right] & a_1 \le x \le a_2 \\ 0.5 & a_2 \le x \le a_3 \\ 0.5 + (1 - 0.5)\left[\dfrac{x - a_3}{a_4 - a_3}\right] & a_3 \le x \le a_4 \\ 1 & a_4 \le x \le a_5 \\ 0.5 + (1 - 0.5)\left[\dfrac{a_6 - x}{a_6 - a_5}\right] & a_5 \le x \le a_6 \\ 0.5 & a_6 \le x \le a_7 \\ 0.5\left[\dfrac{a_8 - x}{a_8 - a_7}\right] & a_7 \le x \le a_8 \\ 0 & \text{otherwise} \end{cases}$$



Fuzzy arithmetic operations.

Addition.

$$(aa_1, bb_1, cc_1, dd_1, ee_1, ff_1, gg_1, hh_1) + (aa_2, bb_2, cc_2, dd_2, ee_2, ff_2, gg_2, hh_2)$$

$$= (aa_1 + aa_2, bb_1 + bb_2, cc_1 + cc_2, dd_1 + dd_2, ee_1 + ee_2, ff_1 + ff_2, gg_1 + gg_2, hh_1 + hh_2)$$

Subtraction.

$$(aa_1, bb_1, cc_1, dd_1, ee_1, ff_1, gg_1, hh_1) - (aa_2, bb_2, cc_2, dd_2, ee_2, ff_2, gg_2, hh_2)$$

$$= (aa_1 - aa_2, bb_1 - bb_2, cc_1 - cc_2, dd_1 - dd_2, ee_1 - ee_2, ff_1 - ff_2, gg_1 - gg_2, hh_1 - hh_2)$$

Fuzzy Sequencing Problem.

It is the process of selecting an acceptable order in which a finite number of occupations can be assigned to a finite number of machines in order to improve the outcome in terms of time, cost, or profit.

Simpson's 1/3rd Rule.

This rule is used to estimate the value of a definite integral. It is a method for numerical integration. It is a technique for mathematical combination. It works by making a significantly number of intervals and fitting a parabola in each pair of intervals.

$$\int y\,dx = \frac{h}{3}[(a_1 + a_8) + 4(a_2 + a_4 + a_6) + 2(a_3 + a_5 + a_7)],\ h = 1$$

Johnson's Algorithm.

Let us consider the problem of processing n-jobs, $1, 2, 3, \ldots, n$ on 2 machines A and B under the following assumptions.

Step 1. Transform the Octogonal fuzzy numbers into values of crisp by Simpson's $1/3^{rd}$ Rule.

Step 2. The optimal sequence is found using Johnson's Bellman Algorithm.

- Determine the minimum handling time among the given work A_i's from the first Machine A and works B_i's from the second Machine B.
- Choose the least handling time in the works A_i's and B_i's then write down the optimal sequence toward the start of the arrangement if it lies in the works of A_i's, otherwise distribute towards the finish of the grouping sequence in the order $A - B$.
- If $A_s = B_t$ then the allocation will be done in the form of s^{th} work first then t^{th} work next.
- Eliminating the assigned task and then repeat the above steps with reduced handling times. The excess works will be put either to the primary work or before the last work. Rehashing this cycle until every one of the works appointed, thus optimal sequence in found.

Step 3. Calculate the idle time and total elapsed time for all the allocated machines.

Numerical example.

A development organization needs to deal with eight things through three phases of work, viz, cutting, drilling and welding process. Handling times are given in the accompanying table:

Job	Cutting (M_1)	Drilling (M_2)	Welding (M_3)
A	(0,5,5,6,7,7,9,10)	(1,2,3,4,5,6,9,10)	(6,7,8,9,10,11,12,13)
B	(1,2,3,4,5,6,9,10)	(0,0.2,1,2,3,3.5,4,5)	(0,5,5,6,7,7,9,10)

C	(3,4,4,5,6,6,7,8)	(0,2,3,4,5,6,7,7)	(8,9,10,11,12,14,15,16)
D	(0,0.2,1,2,3,3.5,4,5)	(4,4,5,6,7,7,8,9)	(5,6,7,8,12,13,14,15)
E	(0,2,3,4,5,6,7,7)	(3,4,4,5,6,6,7,8)	(7.2,8,9,10,12,12.5,13,14)

Decide a sequence for the given occupations that will limit the total elapsed time (complete passed time) and find the idle time (inactive opportunity) for each machines.

Solution. Consider the problem of 8 jobs on 3 machines (cutting, drilling and welding machines).

Job	Cutting (M_1)	Drilling (M_2)	Welding (M_3)
A	(0,5,5,6,7,7,9,10)	(1,2,3,4,5,6,9,10)	(6,7,8,9,10,11,12,13)
B	(1,2,3,4,5,6,9,10)	(0,0.2,1,2,3,3.5,4,5)	(0,5,5,6,7,7,9,10)
C	(3,4,4,5,6,6,7,8)	(0,2,3,4,5,6,7,7)	(8,9,10,11,12,14,15,16)
D	(0,0.2,1,2,3,3.5,4,5)	(4,4,5,6,7,7,8,9)	(5,6,7,8,12,13,14,15)
E	(0,2,3,4,5,6,7,7)	(3,4,4,5,6,6,7,8)	(7.2,8,9,10,12,12.5,13,14)

Octogonal fuzzy numbers has been converted into values of crisp by utilizing Simpson's 1/3rd Rule Using Simpson's 1/3rd Rule Octogonal fuzzy numbers has been converted into crisp values.

Let us illustrate by the following example:

$$\int y dx = \frac{h}{3}[(a_1 + a_8) + 4(a_2 + a_4 + a_6) + 2(a_3 + a_5 + a_7)],\ h = 1$$

$$R(0, 5, 5, 6, 7, 7, 9, 10) = \frac{1}{3}[(0+10) + 4(5+6+7) + 2(5+7+9)]$$

$$= \frac{1}{3}[10 + 72 + 42] = \frac{124}{3} = 41.3$$

$$R(1, 2, 3, 4, 5, 6, 9, 10) = \frac{1}{3}[(1+10) + 4(2+4+6) + 2(3+5+9)]$$

$$= \frac{1}{3}[11 + 48 + 34] = \frac{93}{3} = 31.0$$

Hence the processing are calculated as follows:

Job	A	B	C	D	E
M_1	41.3	31.0	35.0	14.6	28.3
M_2	31.0	14.6	28.3	40.3	35.0
M_3	62.3	41.3	78.0	64.7	70.4

The above table satisfies the condition Min of $M_3 \geq$ Max of M_2. Let us consider $M_4 = M_1 + M_2$ and $M_5 = M_1 + M_3$, we get the processing time of these Machines as follows.

Job	A	B	C	D	E
M_4	72.3	45.6	63.3	54.9	63.3
M_5	93.3	55.9	106.3	105.0	105.4

Optimal sequence is obtained by Johnson's Bellman Algorithm:

B	D	E	C	A

Method 1. To find the minimum total elapsed time using Octogonal Fuzzy Numbers.

Job	Cutting Machine (M_1)		
	Starting Time	Finishing Time	Idle Time
B	(0,0,0,0,0,0,0,0)	(1,2,3,4,5,6,9,10)	–
D	(1,2,3,4,5,6,9,10)	(1,2.2,4,6,8,9.5,13,15)	–
E	(1,2.2,4,6,8,9.5,13,15)	(1,4.2,7,10,13,15.5,20,22)	–
C	(1,4.2,7,10,13,15.5,20,22)	(4,8.2,11,15,19,21.5,27,30)	–
A	(4,8.2,11,15,19,21.5,27,30)	(4,13.2,16,21,26,28.5,36,40)	(31.2,36.2,43,50,61,67,75,82) – (4,13.2,16,21,26,28.5,36,40)
		Total	(27.2,23,27,29,35,38.5,39,42)

Job	Drilling Machine (M_2)		
	Starting Time	Finishing Time	Idle Time

B	(1,2,3,4,5,6,9,10)	(1,2.2,4,6,8,9.5,13,15)	(1,2,3,4,5,6,9,10)
D	(1,2.2,4,6,8,9.5,13,15)	(5,6.2,9,12,15,16.5,21,24)	–
E	(5,6.2,9,12,15,16.5,21,24)	(8,10.2,13,17,21,22.5,28,32)	–
C	(8,10.2,13,17,21,22.5,28,32)	(8,12.2,16,21,26,28.5,35,39)	–
A	(8,12.2,16,21,26,28.5,35,39)	(9,14.2,19,25,31,34.5,44,49)	(31.2,36.2,43,50,61,67,75,82) – (9,14.2,19,25,31,34.5,44,49)
		Total	(23.2,24,27,29,35,38.5,40,43)

Job	Welding Machine (M_3)		
	Starting Time	Finishing Time	Idle Time
B	(1,2.2,4,6,8,9.5,13,15)	(1,7.2,9,12,15,16.5,22,25)	(1,2.2,4,6,8,9.5,13,15)
D	(5,6.2,9,12,15,16.5,21,24)	(10,12.2,16,20,27,29.5,35,39)	(4,-1,0,0,0,0,-1,-1)
E	(10,12.2,16,20,27,29.5,35,39)	(17.2,20.2,25,30,39,42,48,53)	–
C	(17.2,20.2,25,30,39,42,48,53)	(25.2,29.2,35,41,51,56,63,69)	–
A	(25.2,29.2,35,41,51,56,63,69)	(31.2,36.2,43,50,61,67,75,82)	–
		Total	(5,1.2,4,6,8,9.5,12,14)

The minimum total elapsed time $= (31.2,\ 36.2,\ 43,\ 50,\ 61,\ 67,\ 75,\ 82)$ hours.

Idle time on machine $M_1 = (27.2,\ 23,\ 27,\ 29,\ 35,\ 38.5,\ 39,\ 42)$ hours.

Idle time on machine $M_2 = (23.2,\ 24,\ 27,\ 29,\ 35,\ 38.5,\ 40,\ 43)$ hours.

Idle time on machine $M_3 = (5,\ 1.2,\ 4,\ 6,\ 8,\ 9.5,\ 12,\ 14)$ hours.

Method 2. To find the minimum total elapsed time using values of crisp.

Job	Cutting Machine (M_1)		Drilling Machine (M_2)		Welding Machine (M_3)		Idle Time		
	Starting Time	Finishing Time	Starting Time	Finishing Time	Starting Time	Finishing Time	M_1	M_2	M_3
B	0	31.0	31.0	45.6	45.6	86.9	–	31.0	45.6
D	31.0	45.6	45.6	85.9	86.9	151.6	–	–	–
E	45.6	73.9	85.9	120.9	151.6	222.0	–	–	–

C	73.9	108.9	120.9	149.2	222.0	300.0	–	–	–
A	108.9	150.2	150.2	181.2	300.0	362.3	212.1	1.0+ 181.1	–
				Total:		Total:	212.1	213.1	45.6

The minimum total elapsed time $= 362.3$ hours.

Idle time on machine $M_1 = 362.3 - 150.2 = 212.1$ hours.

Idle time on machine $M_2 = 31.0 + 1.0 + 362.3 - 181.2 = 213.1$ hours.

Idle time on machine $M_3 = 45.6$ hours.

Conclusion

In this paper, fuzzy sequencing problem is solved using the simple method which was introduced and transformed into crisp sequencing problem using Simpson's 1/3rd rule. It assists with forming vulnerability in real environment and furthermore fills in as application for the decision makers in real life circumstance.

In this, the idle time and total elapsed time are evaluated by Johnson's algorithm to get ideal arrangement. By this technique we got the ideal arrangement in both fuzzy nature as well as in crisp nature.

References

[1] Dr. S. Ramkumar and Dr. M. Ananthanarayanan, A comparative analysis of fuzzy shortest travelling path using octogonal fuzzy numbers based on measures of dispersion, International Journal of Applied Engineering Research ISSN 0973-4562 13(24) (2018), 16980-16983.

[2] P. Malini and Dr. M. Ananthanarayanan, Solving fuzzy transportation problem using ranking of octagonal fuzzy numbers, International Journal of Pure and Applied Mathematics ISSN: 1311-8080 110(2) (2016), 275-282.

[3] K. Kripa and R. Govindarajan, Fuzzy Sequencing Problem, International Journal of Scientific Research 5(5) ISSN No 2277-8179[IF:3.508] IC Value: 69.48.

[4] K. Selvakumari and S. Sowmiya, Fuzzy network problems using job sequencing technique in hexagonal fuzzy numbers, International Journal of Advance Engineering and Research Development 4(9) (2017).

[5] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965), 338-353.

[6] H. J. Zimmermann, Fuzzy Set Theory and Its Applications, Boston: Kulwer, (1991).

[7] Dr. S. U. Malini and S. Kalaivani, Solving fuzzy sequencing problem using octagonal fuzzy number, International Journal of Mathematical Archive 9(9) (2018), 109-114.

[8] M. Shanmugasundari, Fuzzy sequencing problem: A Novel Approach, International Journal of Pure and Applied Mathematics 113(13) (2017), 56-64.

[9] VS Jadhav and OS Jadhav, Solving flow-shop scheduling problem to minimize total elapsed time using fuzzy approach, International Journal of Statistics and Applied Mathematics 4(5) (2019), 130-133.

[10] Laxminarayan Sahoo, Solving job sequencing problems with fuzzy processing times, IJARIIE-ISSN(O)-2395-4396, 3(4) (2017).

[11] C. H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95 (1998), 307-317.

[12] K. Selvakumari and S. Santhi, An approach for solving fuzzy sequencing problems with octogonal fuzzy numbers using robust ranking techniques, International Journal of Mathematics Trends and Technology (IJMTT), ISSN:2231-5373, V56(3) (2018), 148-152.