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APPROXIMATE ANALYTICAL SOLUTIONS FOR CHEMICAL ENTROPY GENERATION AND MHD EFFECTS ON THE UNSTEADY HEAT AND FLUID FLOW THROUGH A POROUS MEDIUM USING MODIFIED HOMOTOPY ANALYSIS METHOD

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Abstract

In this paper, Modified Homotopy analysis method (MHAM) is proposed to solve non-linear differential equations with boundary conditions for chemical entropy generation and Magneto hydrodynamic (MHD) effects on the unsteady heat and fluid flow through a porous medium. The effect of various parameter on the velocity, temperature, concentration profiles are discussed graphically. Analytical solutions with graphical representation and comparative tables, explicitly reveal the complete reliability of the proposed method. The accuracy and convergence of MHAM are validated with previous work that reveal an excellent agreement.

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1. Introduction and Mathematical formulation

The study of entropy generation plays on important role in characterizing the reliability of electronic or mechanical components. We can predict the life time of component systematically by characterizing the defects of electronic and mechanical components. Because of its wide range of applications, It became a emerging topic in recent years. Spasojevic [1] has given numerical approach to entropy production minimization in diabatic distillation column with trays. Anderson et al. [2] reduced and solved numerically the system of time-dependent boundary layer equations to ordinary differential equations governing momentum and heat transfer in a laminar liquid film on a horizontal stretching sheet. Tsai et al. [3] have studied the influence of the parameter such as unsteadiness parameter, space dependent parameter and temperature dependent parameter for non-uniform heat source/sink effect on the flow and heat transfer. The unsteady heat, mass and fluid transfer over a horizontal stretching sheet has been investigated by stanford shateyi et al. [4]. Entropy generation due to heat and mass transfer, fluid friction and magnetic effect has been analysized by bouabid et al. [5]. Flow of a thin liquid film of a power-law fluid over the unsteady stretching of a surface has been discussed by andersson et al. [6]. Abel et al. [7] analysed numerical perspective view of the influence of radiation on the heat and fluid flow caused by an unsteady stretching surface.

The main aim of the article is to successfully employed Modified Homotopy analysis method for governing boundary value problem of chemical entropy generation and MHD effects on the unsteady heat and fluid flow which was modelled by abdel et al. [8]

$$f''' - A\left(\frac{nf''}{2} + f'\right) + f\ f'' - f'^2(M + S)f' = 0 \tag{1}$$

$$(3R + 4)\theta'' + 3R\,\mathrm{Pr}\left(f\theta' - 2f'\theta - \left(\frac{A}{2}\right)(3\theta + \eta\theta') + H\theta\right) = 0 \tag{2}$$

$$\varphi'' + \mathrm{Pr}\,Le\left(f\varphi' - 2f'\varphi - \left(\frac{A}{2}\right)(3\varphi + \eta\varphi') - \gamma\varphi\right) = 0 \tag{3}$$

The Mathematical formula for entropy generation is as follows:

$$Ns = Br\,\mathrm{Re}\left[\theta'^2 + f''2 + \frac{\Omega_1}{\Omega}\varphi'^2 + \Omega_1\theta'\varphi' + \frac{1}{\Omega}(M+S)f'^2\right] \tag{4}$$

The corresponding boundary conditions are as follows:

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1,\ \varphi(0) = 1,\ f'(\infty) = 0,\ \varphi(\infty) = 0. \tag{5}$$

2. Approximate Analytical Expressions via Modified Homotopy Analysis Method

Consider the differential equation $N[u(t)] = 0$ (6)

Where N is the non-linear operator, t denotes a time independent variable, $u(t)$ is the unknown function depended on the time t. Here we have unknown functions are $f'''(\eta)$, $\theta''(\eta)$ and $\varphi''(\eta)$. By using basic Liao Homotopy analysis method [11-22] we construct the Homotopy for (1-3) eqns.

$$(1-p)L[f'''(\eta, p) - f_0'''(\eta)] = phN[f'''(\eta, p)] \tag{7}$$

$$(1-p)L[\theta''(\eta, p) - \theta_0''(\eta)] = phN[\theta'''(\eta, p)] \tag{8}$$

$$(1-p)L[\varphi''(\eta, p) - \varphi_0''(\eta)] = phN[\varphi''(\eta, p)] \tag{9}$$

Applying equations (1)-(3) in equations (7)-(9)

$$(1-p)(f''' + (M+S)f''') = hp\left(f''' - A\left(\frac{\eta f''}{2} + f'\right) + f\ f'' + f'^2 - (M+S)f'\right) \tag{10}$$

$$(1-p)(\theta'' + \theta') = hp\left((3R+4)\theta'' + 3R\,\mathrm{Pr}\left(f\theta' - 2f'\theta - \left(\frac{A}{2}\right)(3\theta + \eta\theta') + H\theta\right)\right) = 0 \tag{11}$$

$$(1-p)(\varphi'' + \varphi') = hp\left(\varphi'' + \mathrm{Pr}\,Le\left(f\varphi' - 2f'\varphi - \left(\frac{A}{2}\right)(3\varphi + \eta\varphi') - \gamma\varphi\right)\right) = 0 \tag{12}$$

The approximate analytical solutions of the equations (1)-(3) are as follows:

$$f = f_0 + pf_1 + p^2 f_2 + \ldots \tag{13}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots \tag{14}$$

$$\varphi = \varphi_0 + p\varphi_1 + p^2\varphi_2 + \ldots \tag{15}$$

The initial approximations for the equations (10)-(12) are as follows:

$$f_0 = 0,\ f_0' = 1, \varphi_0 = 1 \text{ and } \theta_0 = 1 \text{ when } \eta \to 0 \tag{16}$$

$$f_0' = 0,\ \varphi_0 = 0 \text{ and } \theta_0 = 0 \text{ when } \eta \to \infty \tag{17}$$

$$f_i = 0,\ f_i' = 0, \varphi_i = 0 \text{ and } \theta_i = 0 \text{ when } \eta \to 0 \tag{18}$$

$$f_i' = 0,\ \varphi_i = 0 \text{ and } \theta_i = 0 \text{ when } \eta \to \infty\ i = 1, 2, 3, \ldots \tag{19}$$

Substituting the equations (13)-(15) into the equations (10)-(12) and comparing the coefficients of the powers of p, we get the following eqns.

$$p^0 : f_0''' + (M + S)\, f_0'' = 0 \tag{20}$$

$$p^0 : \theta'' + \theta' = 0 \tag{21}$$

$$P^0 : \varphi'' + \varphi' = 0 \tag{22}$$

$$p^1 : \begin{bmatrix} f_1''' + (M + S) f_1'' = (M + S) f_0'' \\ + A\eta \dfrac{f_0''}{2} + Af_0' - f_0 f_0'' - f_0'^2 + (M + S) f_0' \end{bmatrix} \tag{23}$$

$$p^1 : \begin{bmatrix} \theta_1'' + \theta_1' = -3\theta_0'' - 3R\theta_0'' + \theta_0' - 3R \Pr f_0\theta_0' \\ + 6R \Pr f_0'\theta_0 + \left(\dfrac{3A}{2}\right) R \Pr(3\theta_0 + \eta\theta_0') - 3R \Pr H\theta_0 \end{bmatrix} \tag{24}$$

$$p^1 : \begin{bmatrix} \varphi_1'' + \varphi_1' = \varphi_1' - \Pr Le\ f_0\varphi_0' + 2 \Pr Le\, f_0'\varphi_0 \\ + \left(\dfrac{3A}{2}\right) Le \Pr \varphi_0' + \dfrac{\Pr LeA}{2} \eta\varphi_0' + \Pr Le\gamma\varphi_0 \end{bmatrix} \tag{25}$$

According to HAM, for $-1 \le h \le 1$, as $p \to 1$,

$$f = \lim_{p \to 1} f(\eta) = f_0 - hf_1 \tag{26}$$

$$\theta = \lim_{p \to 1} \theta(\eta) = \theta_0 - h\theta_1 \tag{27}$$

$$\varphi = \lim_{p \to 1} \varphi(\eta) = \varphi_0 - h\varphi_1 \tag{28}$$

Substituting the equations (20)-(25) into the equations (26)-(28), we obtain the solution in the text equations (32)-(34)

$$f = \left[\frac{1}{M+S} - \frac{1}{M+S}e^{-(M+S)\eta} - h\left(\begin{array}{l} C_1 + C_2 e^{-(M+S)\eta} + \left(\dfrac{1-(M+S)^2}{4(M+S)^2}\right)e^{-2(M+S)\eta} \\ + \left(\dfrac{A+M+S+1-(M+S)^2}{(M+S)^2}\right)\eta e^{-(M+S)\eta} \\ - \dfrac{Ae^{-(M+S)\eta}}{2(M+S)}\left(\dfrac{\eta^2}{2} + \dfrac{2\eta}{M+S}\right) \end{array}\right)\right] \tag{29}$$

$$\theta = \left[e^{-\eta} - h\left(\begin{array}{l} C_3 e^{-\eta} + \left(4 + 3R - \dfrac{3R\Pr}{M+S} - \dfrac{9AR\Pr}{2} + 3R\Pr H\right)\eta e^{-\eta} \\ + \left(\dfrac{6R\Pr}{(M+S)(M+S+1)} - \dfrac{3R\Pr}{(M+S)^2(M+S+1)}\right)e^{-(M+S+1)\eta} \\ + \dfrac{3AR\Pr}{2}e^{-\eta}\left(\dfrac{\eta^2}{2} + \eta\right) \end{array}\right)\right] \tag{30}$$

$$\varphi = \left[e^{-\eta} - h\left(\begin{array}{l} C_4 e^{-\eta} + \left(1 - \dfrac{\Pr Le}{M+S} + \dfrac{3RLeA}{2} - \Pr Le\gamma\right)\eta e^{-\eta} \\ + \left(\dfrac{2\Pr Le}{(M+S)(M+S+1)} - \dfrac{\Pr Le}{(M+S)^2(M+S+1)}\right)e^{-(M+S+1)\eta} \\ + \dfrac{\Pr LeA}{2}e^{-\eta}\left(\dfrac{\eta^2}{2} + \eta\right) \end{array}\right)\right] \tag{31}$$

where

$$C_1 = \frac{M+S+1}{(M+S)^3} - \frac{1}{M+S} - \frac{1}{2(M+S)^2} + \frac{1}{2} \text{ and } C_2 = \frac{(M+S)^2 - 1}{4(M+S)^2} - C_1 \tag{32}$$

$$C_3 = \frac{3R\Pr}{(M+S)^2(M+S+1)} - \frac{6R\Pr}{(M+S)(M+S+1)} \tag{33}$$

$$C_4 = \frac{\Pr Le}{(M+S)^2(M+S+1)} - \frac{2RLe}{(M+S)(M+S+1)} \tag{34}$$

3. Comparison of Analytical Solution with Previous Work

The approximate analytical expressions of (29)-(31) of non-linear differential eqns. (1)-(3) are compared with previous numerical expressions and excellent agreement is noticed. Numerical expressions are obtained by central volume finite element method. Our analytical expressions of the boundary value problem are derived by Modified Homotopy analysis method. In Table 1, we have showed the Mass transfer $-\theta'(0)$ for various values of A and $\Pr$ it also indicates that accuracy of the analytical solutions which is derived by using MHAM. Expression for Entropy generation is also derived and excellent agreement is noted.

4. Results and Discussions

The system of non-linear differential eqns. (1)-(3) with boundary conditions (5) for the velocity, the temperature and the concentration profiles for the fluid flow were considered and solved analytically using Modified Homotopy analysis method. Analytical expressions for the velocity, the temperature and the concentration profiles have been used for obtaining the entropy generation profile expression. Figures (1)-(5) shows that influence of unsteadiness, magnetic field, porosity, heat generation/absorption and chemical reaction parameters on the velocity, the temperature, the concentration and entropy generation profiles analysed graphically. In Table 1, mass transfer $-\theta'(0)$ showed for different values of unsteadiness parameter A, Prandtl number Pr.

Table 1. Comparison of the Mass transfer value $-\theta'(0)$ when $h = -0.13267$.

A	Pr	Abd El-Aziz [7]	Shateyi and motsa [4]	Abdel-Rahman [8]	Present Study [MHAM]
0.8	0.1	0.4517	0.45149	0.4518	0.45154
	1.0	1.6728	1.67285	1.67275	1.67288

	10.0	5.70503	5.07598	5.70573	5.705528
1.2	0.1	0.5087	0.50850	0.50841	0.5087
	1.0	1.818	1.81801	1.81793	1.81798
	10.0	6.12067	6.2102	6.12012	6.1203
2.0	0.1	0.606013	0.60352	0.60341	0.603408
	0.1	2.07841	2.07841	2.07830	2.07832
	10.0	6.88506	6.88615	6.88586	6.88585

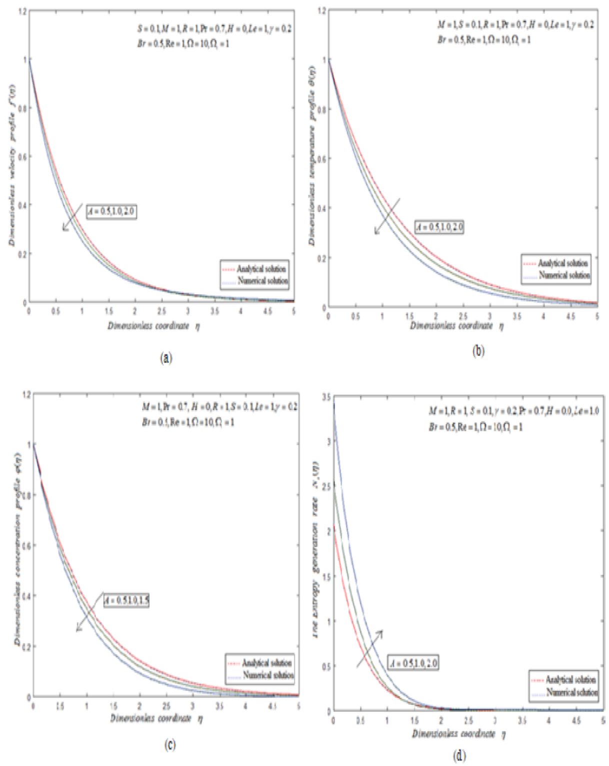

Figure 1. (a)-(d) Effect of unsteadiness parameter A on the velocity, the temperature, the concentration and the entropy generation rate profile.

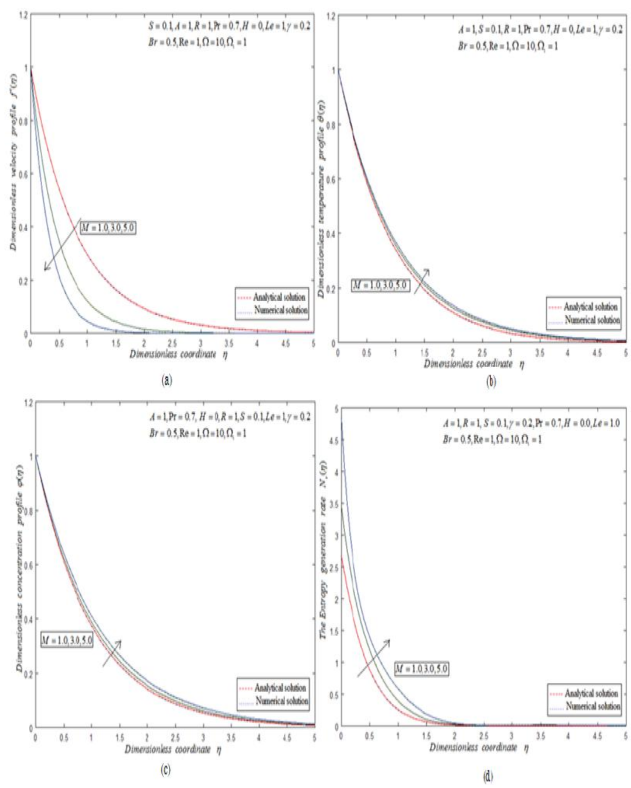

Figure 2 (a)-(d)**.** Effect of Magnetic field parameter M on the velocity, the temperature, the concentration and the entropy generation rate profile.

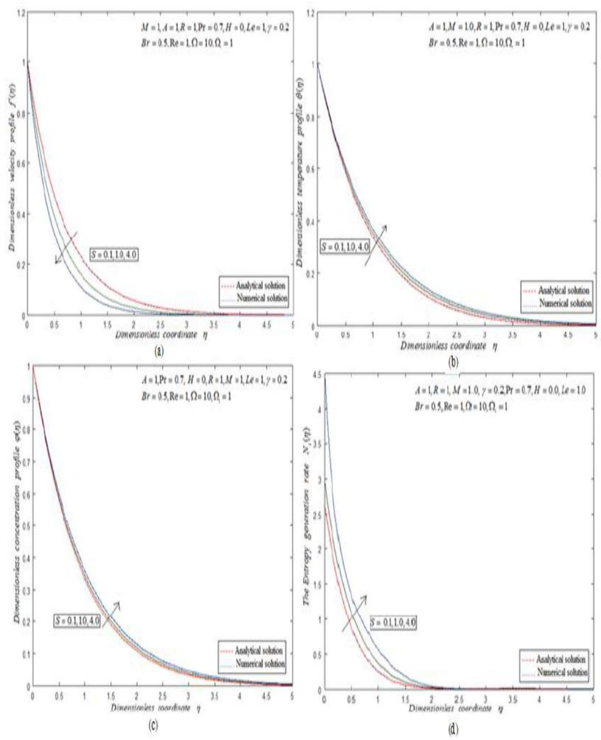

Figure 3. (a)-(d) Effect of porosity parameter S on the velocity, the temperature, the concentration and the entropy generation rate profile.

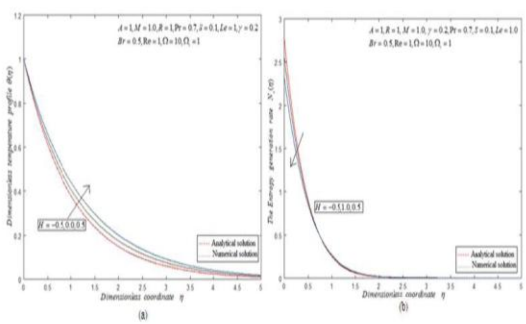

Figure 4. (a)-(b) Effect of Heat generation/absorption parameter H on the temperature and the entropy generation rate profile.

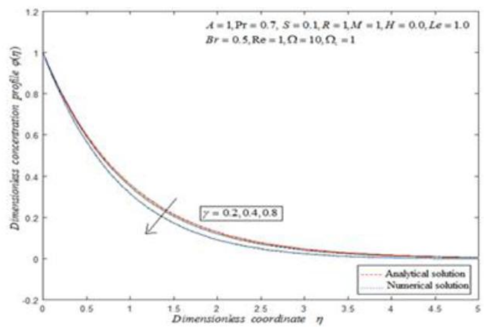

Figure 5. Effect of the chemical reaction parameter γ on the concentration profile.

5. Conclusion

In this article, an approximate but very accurate solution of the velocity, the temperature, the concentration profile and entropy generation rates were successfully obtained using a powerful analytic method called the modified homotopy analysis method. By MHAM solutions we can predicted the influence of unsteadiness parameter, magnetic field parameter, porosity parameter, Lewis number parameter, heat generation/absorption parameter, chemical reaction parameter and Brinkman number parameter on the velocity profile, the temperature profile, the concentration profile and entropy generation rates profile. This paper suggested that this method (MHAM) is an efficient tool for calculating fluid flow through a porous medium. Also this study indicates that the modified homotopy analysis method greatly improves HAM's truncated series solution in rate of convergence and that it often yields the true analytic solution.

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