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LEXICOGRAPHIC PRODUCT AND STRONG PRODUCT ON INTERVAL VALUED PICTURE FUZZY GRAPHS

R. KAMALI and S. JAYALAKSHMI

Department of Mathematics
VISTAS, Pallavaram
Chennai, 600117, India
E-mail: kamali_1883@yahoo.ac.in

Research Scholar
Department of Mathematics
VISTAS, Pallavaram
Chennai, 600117, India
E-mail: jayasriraghav72@gmail.com

Abstract

In this paper, we explore and propose the new definition of lexicographic product, strong product and join two of interval valued picture fuzzy graph (IVPFG). Also, we investigate some results related to their properties.

1. Introduction

In 1975, Zadeh introduced the notion of interval valued fuzzy sets as a generalization of fuzzy sets. In 1986, Atanassov proposed intuitionistic fuzzy sets. Fuzzy graph is an important tool for handling uncertain real decision-making problems in various field. In fuzzy graph theory has numerous applications in modern science and engineering, networks, medical, etc. Ahmed Mostafa Khalil [1] introduced basic properties of interval valued fuzzy soft graphs. Mishra [4] proposed the concept of a regular interval valued intuitionistic graph and discussed some other properties. Mohamed ismayil [5] proved the several results of product interval valued fuzzy graphs. Said

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Broumi [6] discussed some operations of regular and irregular interval valued neutrosophic graphs. Wael Ahmad Al Zoubi [8] modified the product of two bipolar interval valued fuzzy graphs and also the concept in homomorphism in bipolar fuzzy sets. Yahya Mohamed [9] studied the two strong interval valued Pythagorean fuzzy graphs of some operations. In this paper we explore the new definition of lexicographic product, strong product, and join of two interval valued picture fuzzy graphs, Further, investigate of their properties with suitable example.

2. Preliminaries

Definition 2.1 [3]. Let A be a picture fuzzy set. A in X is defined by $A = \{x, \mu_A(x), \eta_A(x), \gamma_A(x)/x \in X\}$ Where $\mu_A(x), \eta_A(x)$ and $\gamma_A(x)$ follow the condition $0 \leq \mu_A(x) + \eta_A(x) + \gamma_A(x) \leq 1$. The $\mu_A(x), \eta_A(x), \gamma_A(x) \in [0, 1]$, denotes respectively the positive membership degree, neutral membership degree and negative membership degree of the element in the set A.

Definition 2.2 [6]. A fuzzy graph is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subsets of a non-empty set V and μ is a symmetric fuzzy relation on σ. i.e. $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, such that $\mu(v, u) = \sigma(u) \wedge \sigma(v)$. For all $u, v \in V$ where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. σ is called fuzzy vertex set V and μ is called the fuzzy edge set E.

Definition 2.3 [6]. An intuitionistic fuzzy graph is if the form $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_i : V \to [0, 1]$ and $\gamma_i : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively, and $0 \leq \mu_i(x) + \gamma_i(x) \leq 1$ for all $v_i \in V, (i = 1, 2, \ldots, n)$ and $E \subseteq V \times V$ such that $\mu_i : V \times V \to [0, 1]$; and $\gamma_j : V \times V \to [0, 1]$ are defined by $0 \leq \mu_j(x, y) \leq \min(\mu_i(x), \mu_i(y))$ and $\gamma_i(x, y) \geq \max(\gamma_i(x), \gamma_i(y))$ such that $0 \leq \mu_j(x, y) + \gamma_j(x, y) \leq 1$ for all $(x, y) \in E, (i, j = 1, 2, \ldots, n)$.

Definition 2.4 [2]. An arc (u, v) in an interval valued fuzzy graph, G is called a strong arc if it is either Type I or Type II strong. If (u, v) is strong we say that u and v are strong.

Definition 2.5 [5]**.** A fuzzy set V is a mapping σ from V to $[0, 1]$. A fuzzy graph G is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, (i.e.) $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. The underlying crisp graph $G = (\sigma, \mu)$ is denoted by $G^* = (V, E)$ where $E \subseteq V \times V$.

Let $D[0, 1]$ be the set of all closed subintervals of the interval $[0, 1]$ and element of this set are denoted by uppercase letters. If $N \in D[0, 1]$ then it can be represented as $N = [N_L, N_U]$, N_L and N_U are the lower and upper limits of N.

3. Operations on Interval Valued Picture Fuzzy Graphs

Definition 3.1. An interval valued picture fuzzy graph (IVPFG) with underlying set V is defined to be a pair $\mathcal{G} = (\mathcal{A}, B)$ where $\mathcal{A} = \{(\mu_{\mathcal{A}L}, \eta_{\mathcal{A}L}\gamma_{\mathcal{A}L}; \mu_{\mathcal{A}U}, \eta_{\mathcal{A}U}, \gamma_{\mathcal{A}U})\}$ is an interval valued picture fuzzy set on V and $\mathcal{B} = \{(\mu_{\mathcal{B}L}, \eta_{\mathcal{B}L}, \gamma_{\mathcal{B}L}; \mu_{\mathcal{B}U}, \eta_{\mathcal{B}U}, \gamma_{\mathcal{B}U})\}$ is an interval valued picture fuzzy set on E satisfies the following conditions: (i) the function $\mu_{\mathcal{A}} : V \to [0, 1]; \eta_{\mathcal{A}} : V \to [0, 1]$ and $\gamma_{\mathcal{A}} : V \to [0, 1]$ indicates positive membership degree, neutral membership degree and negative membership degree of the component $x \in V$, respectively, such that $0 \leq \mu_{\mathcal{A}}(x) + \eta_{\mathcal{A}}(x) + \gamma_{\mathcal{A}}(x) \leq 1$ for all $x \in V$, and (ii) the function $\mu_{\mathcal{B}} : V \times V \to [0, 1]$; $\eta_{\mathcal{B}} : V \times V \to [0, 1]$ and $\gamma_{\mathcal{B}} : V \times V \to [0, 1]$ are defined by $\mu_{\mathcal{B}L}(uv) \leq \min(\mu_{\mathcal{A}L}(u), \mu_{\mathcal{A}L}(v)); \eta_{\mathcal{B}L}(uv) \leq \min(\eta_{\mathcal{A}L}(u), \eta_{\mathcal{A}L}(v))$ and $\gamma_{\mathcal{B}L}(uv) \leq \max(\gamma_{\mathcal{A}L}(u), \mu_{\mathcal{A}L}(v))$ and $\mu_{\mathcal{B}U}(uv) \leq \min(\mu_{\mathcal{A}U}(u), \mu_{\mathcal{A}U}(v))$; $\eta_{\mathcal{B}U}(uv) \leq \min(\eta_{\mathcal{A}U}(u), \eta_{\mathcal{A}U}(v))$ and $\gamma_{\mathcal{B}U}(uv) \geq \max(\gamma_{\mathcal{A}U}(u), \gamma_{\mathcal{A}U}(v))$ such that $0 \leq \mu_{\mathcal{A}U}(uv) + \eta_{\mathcal{B}U}(uv) + \gamma_{\mathcal{B}U}(uv) \leq 1$ for all $uv \in E$.

Example 3.1. Consider a graph $\mathcal{G}^* = (V, E)$ where the vertex set $\{a, b, c\}$ and the edge set $\{ab, bc, ca\}$. Let $\mathcal{P}$ be an interval valued picture fuzzy set of V and let $\mathcal{Q}$ be interval valued picture fuzzy set of $E \subseteq V \times V$ defined by

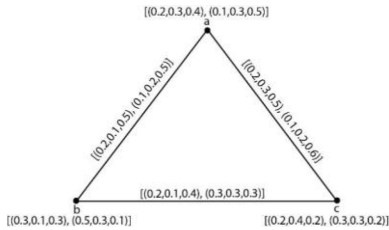

Figure 1. Interval valued picture fuzzy graph.

$[ab, (0.2, 0.1, 0.5), (0.1, 0.2, 0.5)]$; $[bc, (0.2, 0.1, 0.4), (0.3, 0.3, 0.3)]$;

$[ac, (0.2, 0.3, 0.5), (0.1, 0.2, 0.6)]$.

Definition 3.2. The lexicographic product of interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_2 = (\mathcal{A}_2, \mathcal{B}_2)$ with underlying crisp graphs $\mathcal{G}_1^* = (V_1, E_1)$ and $\mathcal{G}_2^* = (V_2, E_2)$ is denoted by $\mathcal{G}_1 \boxtimes \mathcal{G}_2 = (\mathcal{A}_1 \boxtimes \mathcal{A}_2, \mathcal{B}_1 \boxtimes \mathcal{B}_2)$ and is defined as follows

(i) $(\mu_{\mathcal{A}_1 L} \boxtimes \mu_{\mathcal{A}_2 L}), (u_1, u_2) = \max\{\mu_{\mathcal{A}_1 L}(u_1), \mu_{\mathcal{A}_2 L}(u_1)\}$

$(\mu_{\mathcal{A}_1 U} \boxtimes \mu_{\mathcal{A}_2 U}), (u_1, u_2) = \max\{\mu_{\mathcal{A}_1 U}(u_1), \mu_{\mathcal{A}_2 U}(u_2)\}$

$(\eta_{\mathcal{A}_1 L} \boxtimes \eta_{\mathcal{A}_2 L}), (u_1, u_2) = \max\{\eta_{\mathcal{A}_1 L}(u_1), \eta_{\mathcal{A}_2 L}(u_2)\}$

$(\eta_{\mathcal{A}_1 U} \boxtimes \eta_{\mathcal{A}_2 U}), (u_1, u_2) = \max\{\eta_{\mathcal{A}_1 U}(u_1), \eta_{\mathcal{A}_2 U}(u_2)\}$

$(\gamma_{\mathcal{A}_1 L} \boxtimes \gamma_{\mathcal{A}_2 L}), (u_1, u_2) = \max\{\gamma_{\mathcal{A}_1 L}(u_1), \gamma_{\mathcal{A}_2 L}(u_2)\}$

$(\gamma_{\mathcal{A}_1 U} \boxtimes \gamma_{\mathcal{A}_2 U}), (u_1, u_2) = \max\{\gamma_{\mathcal{A}_1 U}(u_1), \gamma_{\mathcal{A}_2 U}(u_2)\}$, for every $u_1 \in V_1$ and $u_2 \in V_2$

(ii) $(\mu_{\mathcal{B}_1 L} \boxtimes \mu_{\mathcal{B}_2 L}), (u, u_2)(u, v_2) = \max\{\mu_{\mathcal{A}_1 L}(u)\ \mu_{\mathcal{B}_2 L}(u_2, v_2)\}$

$(\mu_{\mathcal{B}_1 U} \boxtimes \mu_{\mathcal{B}_2 U}), (u, u_2)(u, v_2) = \max\{\mu_{\mathcal{A}_1 U}(u)\ \mu_{\mathcal{B}_2 U}(u_2, v_2)\}$

$(\eta_{\mathcal{B}_1 L} \boxtimes \gamma_{\mathcal{B}_2 L}), (u, u_2)(u, v_2) = \max\{\eta_{\mathcal{A}_1 L}(u)\ \eta_{\mathcal{B}_2 L}(u_2, v_2)\}$

$(\eta_{\mathcal{B}_1 U} \boxtimes \eta_{\mathcal{B}_2 U}), (u, u_2)(u, v_2) = \max\{\eta_{\mathcal{A}_1 U}(u)\ \eta_{\mathcal{B}_2 U}(u_2, v_2)\}$

$(\gamma_{\mathcal{B}_1 L} \boxtimes \gamma_{\mathcal{B}_2 L}), (u, u_2)(u, v_2) = \min\{\gamma_{\mathcal{A}_1 L}(u)\ \gamma_{\mathcal{B}_2 L}(u_2, v_2)\}$

$(\gamma_{\mathcal{B}_1 U} \boxtimes \gamma_{\mathcal{B}_2 U}), (u, u_2)(u, v_2) = \min\{\gamma_{\mathcal{A}_1 U}(u)\ \gamma_{\mathcal{B}_2 U}(u_2, v_2)\}$ for every $u_1 \in V_1$ and $(u_2, v_2) \in E_2$

(iii) $(\mu_{\mathcal{B}_1 L} \boxtimes \mu_{\mathcal{B}_2 L}), (u_1, u_2)(v_1, v_2) = \max\{\mu_{\mathcal{A}_2 L}(u_2)\ \mu_{\mathcal{A}_2 L}(v_1), \mu_{\mathcal{B}_2 L}(u_1, v_1)\}$

$(\mu_{\mathcal{B}_1 U} \boxtimes \mu_{\mathcal{B}_2 U}), (u_1, u_2)(v_1, v_2) = \max\{\mu_{\mathcal{A}_2 U}(u_2)\ \mu_{\mathcal{A}_2 U}(v_1), \mu_{\mathcal{B}_2 U}(u_1, v_1)\}$

$(\eta_{\mathcal{B}_1 L} \boxtimes \eta_{\mathcal{B}_2 L}), (u_1, u_2)(v_1, v_2) = \max\{\eta_{\mathcal{A}_2 L}(u_2), \eta_{\mathcal{A}_2 L}(v_1), \eta_{\mathcal{B}_2 L}(u_1, v_1)\}$

$(\eta_{\mathcal{B}_1 U} \boxtimes \eta_{\mathcal{B}_2 U}), (u_1, u_2)(v_1, v_2) = \max\{\eta_{\mathcal{A}_2 U}(u_2), \eta_{\mathcal{A}_2 U}(v_1), \eta_{\mathcal{B}_2 U}(u_1, v_1)\}$

$(\gamma_{\mathcal{B}_1 L} \boxtimes \gamma_{\mathcal{B}_2 L}), (u_1, u_2)(v_1, v_2) = \min\{\gamma_{\mathcal{A}_2 L}(u_2), \gamma_{\mathcal{A}_2 L}(v_1), \gamma_{\mathcal{B}_2 L}(u_1, v_1)\}$

$(\gamma_{\mathcal{B}_1 U} \boxtimes \gamma_{\mathcal{B}_2 U}), (u_1, u_2)(v_1, v_2) = \min\{\gamma_{\mathcal{A}_2 U}(u_2), \gamma_{\mathcal{A}_2 U}(v_1), \gamma_{\mathcal{B}_2 U}(u_1, v_1)\}$

for every $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$

Example 3.2. Consider two interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_2 = (\mathcal{A}_2, \mathcal{B}_2)$ with underlying crisp graphs $\mathcal{G}_1^* = (V_1, E_1)$ and $\mathcal{G}_2^* = (V_2, E_2)$ such that the vertices $V_1 = \{u_1, v_1, w_1\}$ and $V_2 = \{u_2, v_2\}$ and the edge sets $E_1 = \{u_1, v_1, v_1, w_1\}$ and $E_2 = \{u_2, v_2\}$ respectively, where

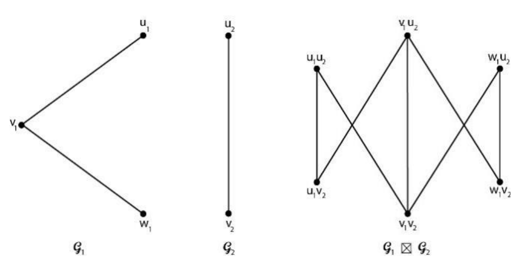

Figure 2. Lexicographic Product of Two interval valued Picture Fuzzy Graphs.

The vertex sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{A}_1 = \begin{cases} \langle u_1; (0.2,\ 0.3,\ 0.4), (0.4,\ 0.1,\ 0.4)\rangle \\ \langle v_1; (0.3,\ 0.3,\ 0.3), (0.3,\ 0.4,\ 0.1)\rangle \\ \langle w_1; (0.4,\ 0.3,\ 0.2), (0.3,\ 0.1,\ 0.4)\rangle \end{cases}$$

$$\mathcal{A}_2 = \begin{cases} \langle u_2; (0.1,\ 0.2,\ 0.5), (0.2,\ 0.3,\ 0.3)\rangle \\ \langle v_2; (0.3,\ 0.5,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \end{cases}$$

The edge sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{B}_1 = \begin{cases} \langle u_1v_1; (0.2,\ 0.2,\ 0.5), (0.2,\ 0.1,\ 0.6)\rangle \\ \langle v_2w_1; (0.1,\ 0.2,\ 0.4), (0.3,\ 0.1,\ 0.5)\rangle \end{cases}$$

$$\mathcal{B}_2 = \{\langle u_2v_2; (0.1,\ 0.1,\ 0.6), (0.2,\ 0.2,\ 0.4)\rangle$$

The lexicographic product of $\mathcal{G}_1 \boxtimes \mathcal{G}_2$

$$\mathcal{A}_1 \boxtimes \mathcal{A}_2 = \begin{cases} \langle u_1u_2; (0.2,\ 0.3,\ 0.4), (0.4,\ 0.3,\ 0.3)\rangle \\ \langle u_1v_2; (0.3,\ 0.5,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \\ \langle v_1u_2; (0.3,\ 0.3,\ 0.3), (0.3,\ 0.4,\ 0.1)\rangle \\ \langle v_1v_2; (0.3,\ 0.5,\ 0.1), (0.3,\ 0.4,\ 0.1)\rangle \\ \langle w_1u_2; (0.4,\ 0.3,\ 0.2), (0.3,\ 0.3,\ 0.3)\rangle \\ \langle w_1v_2; (0.4,\ 0.5,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \end{cases}$$

$$\mathcal{B}_1 \boxtimes \mathcal{B}_2 = \begin{cases} \langle (u_1u_2,\ u_1v_2); (0.3,\ 0.3,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \\ \langle (u_1u_2,\ v_1v_2); (0.3,\ 0.5,\ 0.1), (0.4,\ 0.4,\ 0.1)\rangle \\ \langle (w_1u_2,\ w_1v_2); (0.4,\ 0.5,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \\ \langle (u_1v_2,\ v_1u_2); (0.3,\ 0.5,\ 0.1), (0.4,\ 0.4,\ 0.1)\rangle \\ \langle (v_1u_2,\ v_1v_2); (0.3,\ 0.5,\ 0.1), (0.3,\ 0.4,\ 0.1)\rangle \\ \langle (v_1u_2,\ w_1v_2); (0.4,\ 0.5,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \\ \langle (v_1v_2,\ w_1u_2); (0.4,\ 0.5,\ 0.1), (0.4,\ 0.4,\ 0.2)\rangle \end{cases}$$

Theorem 3.1. *The lexicographic product* $\mathcal{G}_1 \boxtimes \mathcal{G}_2 = (\mathcal{A}_1 \boxtimes \mathcal{A}_2,\ \mathcal{B}_1 \boxtimes \mathcal{B}_2)$ *of two interval valued picture fuzzy graphs* $\mathcal{G}_1 = (\mathcal{A}_1,\ \mathcal{B}_1)$ *and* $\mathcal{G}_2 = (\mathcal{A}_2,\ \mathcal{B}_2)$ *with underlying crisp graphs* $\mathcal{G}_1^* = (V_1,\ E_1)$ *and* $\mathcal{G}_2^* = (V_2,\ E_2)$ *is an interval valued picture fuzzy graph.*

Proof. Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two interval valued picture fuzzy graphs.

Let $E = \{(x,\ x_2)(x,\ y_2)/x \in V_1\ \&\ (x_2,\ y_2) \in E_2\}$

$\cup \{(x_1, x_2)(y_1, y_2) / (x_1, y_1) \in E_2 \,\&(x_2, y_2) \in E_2\}$

Consider $(x, y_1)(x, y_2) \in E$, by definition 3.1 and 3.2 we have

$$(\mu_{\mathcal{B}_1L} \boxtimes \mu_{\mathcal{B}_2L})(x, x_2)(x, y_2) = \max\{\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{A}_2L}(x_2), \mu_{\mathcal{A}_1L}(y_2)\}$$

$$= \max(\min(\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{A}_2L}(x_2)), \min((\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{A}_2L}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1L} \boxtimes \mu_{\mathcal{A}_2L})(x, x_2), (\mu_{\mathcal{A}_1L} \boxtimes \mu_{\mathcal{A}_2L})(x, \mathrm{y}_2)\}$$

$$(\mu_{\mathcal{B}_1U} \boxtimes \mu_{\mathcal{B}_2U})(x, x_2)(x, y_2) = \max\{\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{\mu_{\mathcal{A}_1U}(x), [\mu_{\mathcal{A}_2U}(x_2), \mu_{\mathcal{A}_1U}(y_2)]\}$$

$$= \max(\max(\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{A}_2U}(x_2)), \max((\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1U} \boxtimes \mu_{\mathcal{A}_2U})(x, x_2), (\mu_{\mathcal{A}_1U} \boxtimes \mu_{\mathcal{A}_2U})(x, \mathrm{y}_2)\}$$

$$(\eta_{\mathcal{B}_1L} \boxtimes \eta_{\mathcal{B}_2L})(x, x_2)(x, y_2) = \max\{\eta_{\mathcal{A}_1L}(x), \eta_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{\eta_{\mathcal{A}_1L}(x), [\eta_{\mathcal{A}_2L}(x_2), \eta_{\mathcal{A}_1L}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1L}(x), \eta_{\mathcal{A}_2L}(x_2)), \min((\eta_{\mathcal{A}_1L}(x), \eta_{\mathcal{A}_1L}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1L} \boxtimes \mu_{\mathcal{A}_2L})(x, x_2), (\mu_{\mathcal{A}_1L} \boxtimes \mu_{\mathcal{A}_2L})(x, \mathrm{y}_2)\}$$

$$(\eta_{\mathcal{B}_1U} \boxtimes \eta_{\mathcal{B}_2U})(x, x_2)(x, y_2) = \max\{\eta_{\mathcal{A}_1U}(x), \eta_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\leq \max\{\eta_{\mathcal{A}_1U}(x), [\eta_{\mathcal{A}_2U}(x_2), \eta_{\mathcal{A}_1U}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1U}(x), \eta_{\mathcal{A}_2U}(x_2)), \max((\eta_{\mathcal{A}_1U}(x), \eta_{\mathcal{A}_1U}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1U} \boxtimes \eta_{\mathcal{A}_2U})(x, x_2), (\eta_{\mathcal{A}_1U} \boxtimes \mu_{\mathcal{A}_2U})(x, \mathrm{y}_2)\}$$

$$(\gamma_{\mathcal{B}_1L} \boxtimes \gamma_{\mathcal{B}_2L})(x, x_2)(x, y_2) = \min\{\gamma_{\mathcal{A}_1L}(x), \gamma_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\leq \min\{\gamma_{\mathcal{A}_1L}(x), [\gamma_{\mathcal{A}_2L}(x_2), \gamma_{\mathcal{A}_1L}(y_2)]\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1L}(x), \gamma_{\mathcal{A}_2L}(x_2)), \min((\gamma_{\mathcal{A}_1L}(x), \gamma_{\mathcal{A}_2L}(y_2)))$$

$$= \min\{(\gamma_{\mathcal{A}_1L} \boxtimes \gamma_{\mathcal{A}_2L})(x, x_2), (\gamma_{\mathcal{A}_1L} \boxtimes \gamma_{\mathcal{A}_2L})(x, \mathrm{y}_2)\}$$

$$(\gamma_{\mathcal{B}_1U} \boxtimes \gamma_{\mathcal{B}_2U})(x, x_2)(x, y_2) = \min\{\gamma_{\mathcal{A}_1U}(x), \gamma_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\leq \min\{\gamma_{\mathcal{A}_1U}(x), [\gamma_{\mathcal{A}_2U}(x_2), \gamma_{\mathcal{A}_2U}(y_2)]\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1U}(x), \gamma_{\mathcal{A}_2U}(x_2)), \min((\gamma_{\mathcal{A}_1U}(x), \gamma_{\mathcal{A}_2U}(y_2)))$$

$$= \min\{(\gamma_{\mathcal{A}_1U} \boxtimes \gamma_{\mathcal{A}_2U})(x, x_2), (\gamma_{\mathcal{A}_1U} \boxtimes \gamma_{\mathcal{A}_2U})(x, \mathrm{y}_2)\}$$

Consider $(x, x_2)(y, y_2) \in E$, we have

$$(\mu_{\mathcal{B}_1L} \boxtimes \mu_{\mathcal{B}_2L})(x_1, x_2)(y_1, y_2) = \max\{\mu_{\mathcal{B}_1L}(x_1, y_1), \mu_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{[\mu_{\mathcal{A}_1L}(x_1), \mu_{\mathcal{A}_1L}(y_1)], [\mu_{\mathcal{A}_2L}(x_2), \mu_{\mathcal{A}_2L}(y_2)]\}$$

$$= \max(\max(\mu_{\mathcal{A}_1L}(x_1), \mu_{\mathcal{A}_2L}(x_2)), \max(\mu_{\mathcal{A}_1L}(y_1), \mu_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1L} \boxtimes \mu_{\mathcal{A}_2L})(x, x_2), (\mu_{\mathcal{A}_1L} \boxtimes \gamma_{\mathcal{A}_2L})(y_1, \mathrm{y}_2)\}$$

$$(\mu_{\mathcal{B}_1U} \boxtimes \mu_{\mathcal{B}_2U})(x_1, x_2)(y_1, y_2) = \max\{\mu_{\mathcal{B}_1U}(x_1, y_1), \mu_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{[\mu_{\mathcal{A}_1U}(x_1), \mu_{\mathcal{A}_1U}(y_1)], [\mu_{\mathcal{A}_2U}(x_2), \mu_{\mathcal{A}_2U}(y_2)]\}$$

$$= \max(\max(\mu_{\mathcal{A}_1U}(x_1), \mu_{\mathcal{A}_2U}(x_2)), \max(\mu_{\mathcal{A}_1U}(y_1), \mu_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1U} \boxtimes \mu_{\mathcal{A}_2U})(x_1, x_2), (\mu_{\mathcal{A}_1U} \boxtimes \gamma_{\mathcal{A}_2U})(y_1, \mathrm{y}_2)\}$$

$$(\eta_{\mathcal{B}_1L} \boxtimes \eta_{\mathcal{B}_2L})(x_1, x_2)(y_1, y_2) = \max\{\eta_{\mathcal{B}_1L}(x_1, y_1), \eta_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{[\eta_{\mathcal{A}_1L}(x_1), \eta_{\mathcal{A}_1L}(y_1)], [\eta_{\mathcal{A}_2L}(x_2), \eta_{\mathcal{A}_2L}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1L}(x_1), \eta_{\mathcal{A}_2L}(x_2)), \max(\eta_{\mathcal{A}_1U}(y_1), \eta_{\mathcal{A}_2L}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1L} \boxtimes \eta_{\mathcal{A}_2L})(x, x_2), (\eta_{\mathcal{A}_1L} \boxtimes \eta_{\mathcal{A}_2L})(y_1, \mathrm{y}_2)\}$$

$$(\eta_{\mathcal{B}_1U} \boxtimes \eta_{\mathcal{B}_2U})(x_1, x_2)(y_1, y_2) = \max\{\eta_{\mathcal{B}_1U}(x_1, y_1), \eta_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{[\eta_{\mathcal{A}_1U}(x_1), \eta_{\mathcal{A}_1U}(y_1)], [\eta_{\mathcal{A}_2U}(x_2), \eta_{\mathcal{A}_2U}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1U}(x_1), \eta_{\mathcal{A}_2U}(x_2)), \max(\eta_{\mathcal{A}_1U}(y_1), \eta_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1 U} \boxtimes \eta_{\mathcal{A}_2 U})(x, x_2), (\eta_{\mathcal{A}_1 U} \boxtimes \eta_{\mathcal{A}_2 U})(y_1, y_2)\}$$

$$(\gamma_{\mathcal{B}_1 L} \boxtimes \gamma_{\mathcal{B}_2 L})(x_1, x_2)(y_1, y_2) = \min\{\gamma_{\mathcal{B}_1 L}(x_1, y_1), \gamma_{\mathcal{B}_2 L}(x_2, y_2)\}$$

$$\leq \min\{[\gamma_{\mathcal{A}_1 L}(x_1), \gamma_{\mathcal{A}_1 L}(y_1)], \gamma_{\mathcal{A}_2 L}(x_2), \gamma_{\mathcal{A}_2 L}(y_2)\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1 L}(x_1), \gamma_{\mathcal{A}_2 L}(x_2)), \min(\gamma_{\mathcal{A}_1 L}(y_1), \gamma_{\mathcal{A}_2 L}(y_2)))$$

$$= \min\{(\mu_{\mathcal{A}_1 L} \boxtimes \mu_{\mathcal{A}_2 L})(x_1, x_2), (\mu_{\mathcal{A}_1 L} \boxtimes \mu_{\mathcal{A}_2 L})(y_1, y_2)\}$$

$$(\gamma_{\mathcal{B}_1 U} \boxtimes \gamma_{\mathcal{B}_2 U})(x_1, x_2)(y_1, y_2) = \min\{\gamma_{\mathcal{B}_1 U}(x_1, y_1), \gamma_{\mathcal{B}_2 U}(x_2, y_2)\}$$

$$\leq \min\{[\gamma_{\mathcal{A}_1 U}(x_1), \gamma_{\mathcal{A}_1 U}(y_1)], \gamma_{\mathcal{A}_2 U}(x_2), \gamma_{\mathcal{A}_2 U}(y_2)\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1 U}(x_1), \gamma_{\mathcal{A}_2 U}(x_2)), \min(\gamma_{\mathcal{A}_1 U}(y_1), \gamma_{\mathcal{A}_2 U}(y_2)))$$

$$= \min\{(\gamma_{\mathcal{A}_1 U} \boxtimes \gamma_{\mathcal{A}_2 U})(x, x_2), (\gamma_{\mathcal{A}_1 U} \boxtimes \gamma_{\mathcal{A}_2 U})(y_1, y_2)\}$$

Definition 3.3. The strong product of interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_1 = (\mathcal{A}_2, \mathcal{B}_2)$ with underlying crisp graphs $\mathcal{G}_1^* = (V_1, E_1)$ and $\mathcal{G}_1^* = (V_2, E_2)$ is denoted by $\mathcal{G}_1 \cdot \mathcal{G}_2 = (\mathcal{A}_1 \cdot \mathcal{A}_2, \mathcal{B}_1 \cdot \mathcal{B}_2)$ and is defined as follows

(i) $(\mu_{\mathcal{A}_1 L} \cdot \mu_{\mathcal{A}_2 L})(u_1, u_2) = \max\{\mu_{\mathcal{A}_1 L}(u_1), \mu_{\mathcal{A}_2 L}(u_2)\}$

$(\mu_{\mathcal{A}_1 U} \cdot \mu_{\mathcal{A}_2 U})(u_1, u_2) = \max\{\mu_{\mathcal{A}_1 U}(u_1), \mu_{\mathcal{A}_2 U}(u_2)\}$

$(\eta_{\mathcal{A}_1 L} \cdot \eta_{\mathcal{A}_2 L})(u_1, u_2) = \max\{\eta_{\mathcal{A}_1 L}(u_1), \eta_{\mathcal{A}_2 L}(u_2)\}$

$(\eta_{\mathcal{A}_1 U} \cdot \eta_{\mathcal{A}_2 U})(u_1, u_2) = \max\{\eta_{\mathcal{A}_1 U}(u_1), \eta_{\mathcal{A}_2 U}(u_2)\}$

$(\gamma_{\mathcal{A}_1 L} \cdot \gamma_{\mathcal{A}_2 L})(u_1, u_2) = \min\{\gamma_{\mathcal{A}_1 L}(u_1), \gamma_{\mathcal{A}_2 L}(u_2)\}$

$(\gamma_{\mathcal{A}_1 U} \cdot \gamma_{\mathcal{A}_2 U})(u_1, u_2) = \min\{\gamma_{\mathcal{A}_1 U}(u_1), \gamma_{\mathcal{A}_2 U}(u_2)\}$, for every $u_1 \in V_1$, and $u_2 \in V_2$

(ii) $(\mu_{\mathcal{B}_1 L} \cdot \mu_{\mathcal{B}_2 L})(u, u_2)(u, v_2) = \max\{\mu_{\mathcal{A}_1 L}(u), \mu_{\mathcal{B}_2 L}(u_2, v_2)\}$

$(\mu_{\mathcal{B}_1 U} \cdot \mu_{\mathcal{B}_2 U})(u, u_2)(u, v_2) = \max\{\mu_{\mathcal{A}_1 U}(u), \mu_{\mathcal{B}_2 U}(u_2, v_2)\}$

$(\eta_{\mathcal{B}_1L} \cdot \eta_{\mathcal{B}_2L})(u, u_2)(u, v_2) = \max\{\eta_{\mathcal{A}_1L}(u), \eta_{\mathcal{B}_2L}(u_2, v_2)\}$

$(\eta_{\mathcal{B}_1U} \cdot \eta_{\mathcal{B}_2U})(u, u_2)(u, v_2) = \max\{\eta_{\mathcal{A}_1U}(u), \eta_{\mathcal{B}_2U}(u_2, v_2)\}$

$(\gamma_{\mathcal{B}_1L} \cdot \gamma_{\mathcal{B}_2L})(u, u_2)(u, v_2) = \min\{\gamma_{\mathcal{A}_1L}(u), \gamma_{\mathcal{B}_2L}(u_2, v_2)\}$

$(\gamma_{\mathcal{B}_1U} \cdot \gamma_{\mathcal{B}_2U})(u, u_2)(u, v_2) = \min\{\gamma_{\mathcal{A}_1U}(u), \gamma_{\mathcal{B}_2U}(u_2, v_2)\}$, for every $u \in V_1$, and $(u_2, v_2) \in E_2$

(iii) $(\mu_{\mathcal{B}_1L} \cdot \mu_{\mathcal{B}_2L})(u_1, u_2)(v_1, v_2) = \max\{\mu_{\mathcal{A}_2L}(u_2), \mu_{\mathcal{A}_2L}(v_2), \mu_{\mathcal{B}_1L}(u_1, v_1)\}$

$(\mu_{\mathcal{B}_1U} \cdot \mu_{\mathcal{B}_2U})(u_1, u_2)(v_1, v_2) = \max\{\mu_{\mathcal{A}_2U}(u_2), \mu_{\mathcal{A}_2U}(v_2), \mu_{\mathcal{B}_1U}(u_1, v_1)\}$

$(\eta_{\mathcal{B}_1L} \cdot \eta_{\mathcal{B}_2L})(u_1, u_2)(v_1, v_2) = \max\{\eta_{\mathcal{A}_2L}(u_2), \eta_{\mathcal{A}_2L}(v_2), \eta_{\mathcal{B}_2L}(u_1, v_1)\}$

$(\eta_{\mathcal{B}_1U} \cdot \eta_{\mathcal{B}_2U})(u_1, u_2)(v_1, v_2) = \max\{\eta_{\mathcal{A}_2U}(u_2), \eta_{\mathcal{A}_2U}(v_2), \eta_{\mathcal{B}_1U}(u_1, v_1)\}$

$(\gamma_{\mathcal{B}_1L} \cdot \gamma_{\mathcal{B}_2L})(u_1, u_2)(v_1, v_2) = \min\{\gamma_{\mathcal{A}_2L}(u_2), \gamma_{\mathcal{A}_2L}(v_2), \gamma_{\mathcal{B}_1L}(u_1, v_1)\}$

$(\gamma_{\mathcal{B}_1U} \cdot \gamma_{\mathcal{B}_2U})(u_1, u_2)(v_1, v_2) = \min\{\gamma_{\mathcal{A}_2U}(u_2), \gamma_{\mathcal{A}_2U}(v_2), \gamma_{\mathcal{B}_1U}(u_1, v_1)\}$

for every $(u_1, v_1) \in E_1$ and $(u_2, v_2) \in E_2$

(iv) $(\mu_{\mathcal{B}_1L} \cdot \mu_{\mathcal{B}_2L})(u_1, w)(v_1, w) = \max\{\mu_{\mathcal{A}_2L}(w), \mu_{\mathcal{B}_1L}(u_1, v_1)\}$

$(\mu_{\mathcal{B}_1U} \cdot \mu_{\mathcal{B}_2U})(u_1, w)(v_1, w) = \max\{\mu_{\mathcal{A}_2U}(w), \mu_{\mathcal{B}_1U}(u_1, v_1)\}$

$(\eta_{\mathcal{B}_1L} \cdot \eta_{\mathcal{B}_2L})(u_1, w)(v_1, w) = \max\{\eta_{\mathcal{A}_2L}(w), \eta_{\mathcal{B}_1L}(u_1, v_1)\}$

$(\eta_{\mathcal{B}_1U} \cdot \eta_{\mathcal{B}_2U})(u_1, w)(v_1, w) = \max\{\eta_{\mathcal{A}_2U}(w), \eta_{\mathcal{B}_1U}(u_1, v_1)\}$

$(\gamma_{\mathcal{B}_1L} \cdot \gamma_{\mathcal{B}_2L})(u_1, w)(v_1, w) = \min\{\gamma_{\mathcal{A}_2L}(w), \gamma_{\mathcal{B}_1L}(u_1, v_1)\}$

$(\gamma_{\mathcal{B}_1U} \cdot \gamma_{\mathcal{B}_2U})(u_1, w)(v_1, w) = \min\{\gamma_{\mathcal{A}_2U}(w), \gamma_{\mathcal{B}_1U}(u_1, v_1)\}$ for every $w \in V_2$ and $(u_1, v_1) \in E_1$

Example 3.3. Consider two interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_1 = (\mathcal{A}_2, \mathcal{B}_2)$ with underlying crisp $\mathcal{G}_1^* = (V_1, E_1)$ and $\mathcal{G}_2^* = (V_2, E_2)$ such that the vertices $\mathcal{G}_1^* = (V_1, E_1)$ and $V_1 = \{x_1, y_1\}$ and

$V_2 = \{x_2, y_2\}$ and the edge sets $E_1 = \{x_1, y_1\}$ and $E_2 = \{x_2, y_2\}$ respectively, where

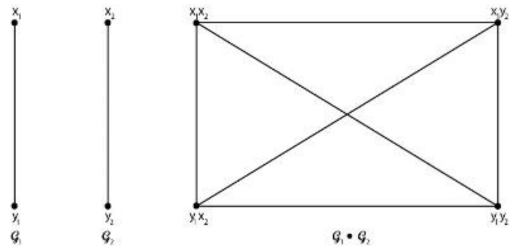

Figure 3. Strong Product of Two interval valued Picture Fuzzy Graphs. The vertex sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{A}_1 = \begin{cases} \langle x_1; (0.2,\ 0.3,\ 0.4), (0.3,\ 0.1,\ 0.4)\rangle \\ \langle y_1; (0.3,\ 0.3,\ 0.3), (0.3,\ 0.4,\ 0.1)\rangle \end{cases}$$

$$\mathcal{A}_2 = \begin{cases} \langle x_2; (0.1,\ 0.2,\ 0.5), (0.2,\ 0.4,\ 0.3)\rangle \\ y_2; \langle(0.3,\ 0.6,\ 0.1), (0.4,\ 0.3,\ 0.2)\rangle \end{cases}$$

The edge sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{B}_1 = \{\langle x_1y_1; (0.2,\ 0.2,\ 0.5), (0.2,\ 0.1,\ 0.6)\rangle$$

$$\mathcal{B}_2 = \{\langle x_2y_2; (0.1,\ 0.1,\ 0.6), (0.2,\ 0.2,\ 4)\rangle$$

The strong product of $\mathcal{G}_1 \cdot \mathcal{G}_2$

$$\mathcal{A}_1 \cdot \mathcal{A}_2 = \begin{cases} \langle x_1x_2; (0.2,\ 0.3,\ 0.4), (0.3,\ 0.4,\ 0.3)\rangle \\ \langle x_1y_2; (0.3,\ 0.6,\ 0.1), (0.4\ 0.3,\ 0.2)\rangle \\ \langle y_1x_2; (0.3,\ 0.3,\ 0.3), (0.3,\ 0.4,\ 0.1)\rangle \\ \langle y_1y_2; (0.3,\ 0.6,\ 0.1), (0.4\ 0.4,\ 0.1)\rangle \end{cases}$$

$$\mathcal{B}_1 \cdot \mathcal{B}_2 = \begin{cases} \langle(x_1x_2,\ x_1y_2); (0.3,\ 0.6,\ 0.1), (0.4,\ 0.4,\ 0.2)\rangle \\ \langle(y_1x_2,\ y_1y_2); (0.3,\ 0.6,\ 0.1), (0.4,\ 0.4,\ 0.1)\rangle \\ \langle(x_1x_2,\ y_1x_2); (0.3,\ 0.3,\ 0.3), (0.3,\ 0.4,\ 0.1)\rangle \\ \langle(x_1y_2,\ y_1y_2); (0.3,\ 0.6,\ 0.1), (0.4,\ 0.4,\ 0.1)\rangle \\ \langle(x_1x_2,\ y_1y_2); (0.3,\ 0.6,\ 0.1), (0.4,\ 0.4,\ 0.1)\rangle \\ \langle(x_1y_2,\ y_1x_2); (0.3,\ 0.6,\ 0.1), (0.4,\ 0.4,\ 0.1)\rangle \end{cases}$$

Theorem 3.2. *The strong product* $\mathcal{G}_1 \cdot \mathcal{G}_2 = (\mathcal{A}_1 \cdot \mathcal{A}_2,\ \mathcal{B}_1 \cdot \mathcal{B}_1)$ *of interval* *valued picture fuzzy graphs* $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ *and* $\mathcal{G}_2 = (\mathcal{A}_2, \mathcal{B}_2)$ *with underlying crisp graphs* $\mathcal{G}_1^* = (V_1, E_1)$ *and* $\mathcal{G}_2^* = (V_2, E_2)$ *is an interval valued picture fuzzy graph.*

Proof. Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two interval valued picture fuzzy graphs. Let $E = \{(x, x_2)(x, y_2)/x \in V_1$ and $(x_2, y_2) \in E_2\} \cup \{(x_1, x_2)(y_1, y_2)/(x_1, y_1) \in E_1$ and $(x_2, y_2) \in E_2\} \cup \{(x_1, z)(y_2, z)/z \in V_2$ and $(x_1, y_1) \in E_1\}$

Consider $(x, x_2)(x, y_2) \in E$, by definition: 3.1 and 3.3 we have

$$(\mu_{\mathcal{B}_1L} \cdot \mu_{\mathcal{B}_2L})(x, y_2)(x, y_2) = \max\{\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{A}_2L}(x_2), \mu_{\mathcal{A}_2L}(y_2)\}$$

$$= \max(\min(\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{A}_2L}(x_2)), \min(\mu_{\mathcal{A}_2L}(x), \mu_{\mathcal{A}_2L}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(x, x_2), (\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(x, y_2)\}$$

$$(\mu_{\mathcal{B}_1U} \cdot \mu_{\mathcal{B}_2U})(x, x_2)(x, y_2) = \max\{\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{A}_2U}(x_2), \mu_{\mathcal{A}_2U}(y_2)\}$$

$$= \max(\max(\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{A}_2U}(x_2)), \max((\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1U} \cdot \mu_{\mathcal{A}_2U})(x, x_2), (\mu_{\mathcal{A}_1U} \cdot \mu_{\mathcal{A}_2U})(x, y_2)\}$$

$$(\eta_{\mathcal{B}_1L} \cdot \eta_{\mathcal{B}_2L})(x, x_2)(x, y_2) = \max\{\eta_{\mathcal{A}_1L}(x), \eta_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{\eta_{\mathcal{A}_1L}(x), [\eta_{\mathcal{A}_2L}(x_2), \eta_{\mathcal{A}_2L}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1L}(x), \eta_{\mathcal{A}_2L}(x_2)), \max((\eta_{\mathcal{A}_2L}(x), \eta_{\mathcal{A}_2L}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1L} \cdot \eta_{\mathcal{A}_2L})(x, x_2), (\eta_{\mathcal{A}_1L} \cdot \eta_{\mathcal{A}_2L})(x, y_2)\}$$

$$(\eta_{\mathcal{B}_1U} \cdot \eta_{\mathcal{B}_2U})(x, x_2)(x, y_2) = \max\{\eta_{\mathcal{A}_1U}(x), \eta_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{\eta_{\mathcal{A}_1U}(x), [\eta_{\mathcal{A}_2U}(x_2), \eta_{\mathcal{A}_2U}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1U}(x), \eta_{\mathcal{A}_2U}(x_2)), \max((\eta_{\mathcal{A}_2U}(x), \eta_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1U} \cdot \eta_{\mathcal{A}_2U})(x, x_2), (\eta_{\mathcal{A}_1U} \cdot \eta_{\mathcal{A}_2U})(x, y_2)\}$$

$$(\gamma_{\mathcal{B}_1L} \cdot \gamma_{\mathcal{B}_2L})(x, x_2)(x, y_2) = \min\{\gamma_{\mathcal{A}_1L}(x), \gamma_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\leq \min\{\gamma_{\mathcal{A}_1L}(x), [\gamma_{\mathcal{A}_2L}(x_2), \gamma_{\mathcal{A}_2L}(y_2)]\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1L}(x), \gamma_{\mathcal{A}_2L}(x_2)), \min((\gamma_{\mathcal{A}_2L}(x), \gamma_{\mathcal{A}_2L}(y_2)))$$

$$= \min\{(\gamma_{\mathcal{A}_1L} \cdot \gamma_{\mathcal{A}_2L})(x, x_2), (\gamma_{\mathcal{A}_1L} \cdot \gamma_{\mathcal{A}_2L})(x, y_2)\}$$

$$(\gamma_{\mathcal{B}_1U} \cdot \gamma_{\mathcal{B}_2U})(x, x_2)(x, y_2) = \min\{\gamma_{\mathcal{A}_1U}(x), \gamma_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\leq \min\{\gamma_{\mathcal{A}_1U}(x), [\gamma_{\mathcal{A}_2U}(x_2), \gamma_{\mathcal{A}_2U}(y_2)]\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1U}(x), \gamma_{\mathcal{A}_2U}(x_2)), \min((\gamma_{\mathcal{A}_2U}(x), \gamma_{\mathcal{A}_2U}(y_2)))$$

$$= \min\{(\gamma_{\mathcal{A}_1U} \cdot \gamma_{\mathcal{A}_2U})(x, x_2), (\gamma_{\mathcal{A}_1U} \cdot \gamma_{\mathcal{A}_2U})(x, y_2)\}$$

Consider $(x_1, x_2)(y_1, y_2) \in E$, we have

$$(\mu_{\mathcal{B}_1L} \cdot \mu_{\mathcal{B}_2L})(x_1, x_2)(y_1, y_2) = \max\{\mu_{\mathcal{B}_1L}(x_1, y_1), \mu_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{[\mu_{\mathcal{A}_1L}(x_1), \mu_{\mathcal{A}_1L}(y_1)], [\mu_{\mathcal{A}_2L}(x_2), \mu_{\mathcal{A}_2L}(y_2)]\}$$

$$= \max(\max(\mu_{\mathcal{A}_1L}(x_1), \mu_{\mathcal{A}_2L}(x_2)), \max(\mu_{\mathcal{A}_2L}(y_1), \mu_{\mathcal{A}_2L}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(x_1, x_2), (\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(y_1, y_2)\}$$

$$(\mu_{\mathcal{B}_1U} \cdot \mu_{\mathcal{B}_2U})(x_1, x_2)(y_1, y_2) = \max\{\mu_{\mathcal{B}_1U}(x_1, y_1), \mu_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{[\mu_{\mathcal{A}_1U}(x_1), \mu_{\mathcal{A}_1U}(y_1)], [\mu_{\mathcal{A}_2U}(x_2), \mu_{\mathcal{A}_2U}(y_2)]\}$$

$$= \max(\max(\mu_{\mathcal{A}_1U}(x_1), \mu_{\mathcal{A}_2U}(x_2)), \max(\mu_{\mathcal{A}_2U}(y_1), \mu_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\mu_{\mathcal{A}_1U} \cdot \mu_{\mathcal{A}_2U})(x_1, x_2), (\mu_{\mathcal{A}_1U} \cdot \mu_{\mathcal{A}_2U})(y_1, y_2)\}$$

$$(\eta_{\mathcal{B}_1L} \cdot \eta_{\mathcal{B}_2L})(x_1, x_2)(y_1, y_2) = \max\{\eta_{\mathcal{B}_1L}(x_1, y_1), \eta_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\geq \max\{[\eta_{\mathcal{A}_1L}(x_1), \eta_{\mathcal{A}_1L}(y_1)], [\eta_{\mathcal{A}_2L}(x_2), \eta_{\mathcal{A}_2L}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1L}(x_1), \eta_{\mathcal{A}_2L}(x_2)), \max(\eta_{\mathcal{A}_2L}(y_1), \eta_{\mathcal{A}_2L}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1L} \cdot \eta_{\mathcal{A}_2L})(x_1, x_2), (\eta_{\mathcal{A}_1L} \cdot \eta_{\mathcal{A}_2L})(y_1, y_2)\}$$

$$(\eta_{\mathcal{B}_1U} \cdot \eta_{\mathcal{B}_2U})(x_1, x_2)(y_1, y_2) = \max\{\eta_{\mathcal{B}_1U}(x_1, y_1), \eta_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\geq \max\{[\eta_{\mathcal{A}_1U}(x_1), \eta_{\mathcal{A}_1U}(y_1)], [\eta_{\mathcal{A}_2U}(x_2), \eta_{\mathcal{A}_2U}(y_2)]\}$$

$$= \max(\max(\eta_{\mathcal{A}_1U}(x_1), \eta_{\mathcal{A}_2U}(x_2)), \max(\eta_{\mathcal{A}_2U}(y_1), \eta_{\mathcal{A}_2U}(y_2)))$$

$$= \max\{(\eta_{\mathcal{A}_1U} \cdot \eta_{\mathcal{A}_2U})(x_1, x_2), (\eta_{\mathcal{A}_1U} \cdot \eta_{\mathcal{A}_2U})(y_1, y_2)\}$$

$$(\gamma_{\mathcal{B}_1L} \cdot \gamma_{\mathcal{B}_2L})(x_1, x_2)(y_1, y_2) = \min\{\gamma_{\mathcal{B}_1L}(x_1, y_1), \gamma_{\mathcal{B}_2L}(x_2, y_2)\}$$

$$\leq \min\{[\gamma_{\mathcal{A}_1L}(x_1), \gamma_{\mathcal{A}_1L}(y_1)], \gamma_{\mathcal{A}_2L}(x_2), \gamma_{\mathcal{A}_2L}(y_2)]\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1L}(x_1), \gamma_{\mathcal{A}_2L}(x_2)), \min(\gamma_{\mathcal{A}_2L}(y_1), \gamma_{\mathcal{A}_2L}(y_2)))$$

$$= \min\{(\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(x_1, x_2), (\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(y_1, y_2)\}$$

$$(\gamma_{\mathcal{B}_1U} \cdot \gamma_{\mathcal{B}_2U})(x_1, x_2)(y_1, y_2) = \min\{\gamma_{\mathcal{B}_1U}(x_1, y_1), \gamma_{\mathcal{B}_2U}(x_2, y_2)\}$$

$$\leq \min\{[\gamma_{\mathcal{A}_1U}(x_1), \gamma_{\mathcal{A}_1U}(y_1)], \gamma_{\mathcal{A}_2U}(x_2), \gamma_{\mathcal{A}_2U}(y_2)]\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1U}(x_1), \gamma_{\mathcal{A}_2U}(x_2)), \min(\gamma_{\mathcal{A}_2U}(y_1), \gamma_{\mathcal{A}_2U}(y_2)))$$

$$= \min\{(\gamma_{\mathcal{A}_1U} \cdot \gamma_{\mathcal{A}_2U})(x_1, x_2), (\gamma_{\mathcal{A}_1U} \cdot \gamma_{\mathcal{A}_2U})(y_1, y_2)\}$$

Consider $(x_1, z)(y_1, z)/z \in V_2$ and $(x_1, y_1) \in E_1$ we have

$$(\mu_{\mathcal{B}_1L} \cdot \mu_{\mathcal{B}_2L})(x_1, z)(y_1, z) = \max\{\mu_{\mathcal{B}_1L}(x_1, y_1), (\mu_{\mathcal{A}_2L}(z)\}$$

$$\geq \max\{\mu_{\mathcal{A}_1L}(x_1), \mu_{\mathcal{A}_1L}(y_1), (\mu_{\mathcal{A}_2L}(z)\}$$

$$= \max(\mu_{\mathcal{A}_1L}(x_1), \max(\mu_{\mathcal{A}_2L}(z)), \max(\mu_{\mathcal{A}_1L}(y_1), \mu_{\mathcal{A}_2L}(z)))$$

$$= \max\{(\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(x_1, z), (\mu_{\mathcal{A}_1L} \cdot \mu_{\mathcal{A}_2L})(y_1, z)\}$$

$$(\mu_{\mathcal{B}_1U} \cdot \mu_{\mathcal{B}_2U})(x_1, z)(y_2, z) = \max\{\mu_{\mathcal{B}_1U}(x_1, y_1), (\mu_{\mathcal{A}_2U}(z)\}$$

$$\geq \max\{\mu_{\mathcal{A}_1U}(x_1), \mu_{\mathcal{A}_1U}(y_1), (\mu_{\mathcal{A}_2U}(z)\}$$

$$= \max(\mu_{\mathcal{A}_1U}(x_1), (\mu_{\mathcal{A}_2U}(z)), \max(\mu_{\mathcal{A}_1U}(y_1), \mu_{\mathcal{A}_2U}(z)))$$

$$= \max\{(\mu_{\mathcal{A}_1 U} \cdot \mu_{\mathcal{A}_2 L})(x_1, z), (\mu_{\mathcal{A}_1 U} \cdot \mu_{\mathcal{A}_2 L})(y_1, z)\}$$

$$(\eta_{\mathcal{B}_1 L} \cdot \eta_{\mathcal{B}_2 L})(x_1, z)(y_2, z) = \max\{\eta_{\mathcal{B}_1 L}(x_1, y_1), (\eta_{\mathcal{A}_2 L}(z)\}$$

$$\geq \max\{\eta_{\mathcal{A}_1 L}(x_1), \eta_{\mathcal{A}_1 L}(y_1), (\eta_{\mathcal{A}_2 L}(z)\}$$

$$= \max(\eta_{\mathcal{A}_1 L}(x_1), \max(\eta_{\mathcal{A}_2 L}(z)), \max(\eta_{\mathcal{A}_1 L}(y_1), \eta_{\mathcal{A}_2 L}(z)))$$

$$= \max\{(\eta_{\mathcal{A}_1 L} \cdot \eta_{\mathcal{A}_2 L})(x_1, z), (\eta_{\mathcal{A}_1 L} \cdot \eta_{\mathcal{A}_2 L})(y_1, z)\}$$

$$(\eta_{\mathcal{B}_1 U} \cdot \eta_{\mathcal{B}_2 U})(x_1, z)(y_2, z) = \max\{\eta_{\mathcal{B}_1 U}(x_1, y_1), (\eta_{\mathcal{A}_2 U}(z)\}$$

$$\geq \max\{\eta_{\mathcal{A}_1 U}(x_1), \eta_{\mathcal{A}_1 U}(y_1), (\eta_{\mathcal{A}_2 U}(z)\}$$

$$= \max(\eta_{\mathcal{A}_1 U}(x_1), (\eta_{\mathcal{A}_2 U}(z)), \max(\eta_{\mathcal{A}_1 U}(y_1), (\eta_{\mathcal{A}_2 U}(z)))$$

$$= \max\{(\eta_{\mathcal{A}_1 U} \cdot \eta_{\mathcal{A}_2 L})(x_1, z), (\eta_{\mathcal{A}_1 U} \cdot \eta_{\mathcal{A}_2 L})(y_1, z)\}$$

$$(\gamma_{\mathcal{B}_1 L} \cdot \gamma_{\mathcal{B}_2 L})(x_1, z)(y_2, z) = \min\{\gamma_{\mathcal{B}_1 L}(x_1, y_1), \gamma_{\mathcal{A}_2 L}(z)\}$$

$$\leq \min\{[\gamma_{\mathcal{A}_1 L}(x_1), \gamma_{\mathcal{A}_1 L}(y_1)], \gamma_{\mathcal{A}_2 L}(z)\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1 L}(x_1), (\gamma_{\mathcal{A}_2 L}(z)), \min((\gamma_{\mathcal{A}_1 L}(y_1), \gamma_{\mathcal{A}_2 L}(z)))$$

$$= \min\{(\gamma_{\mathcal{A}_1 L} \cdot \gamma_{\mathcal{A}_2 L})(x_1, z), (\gamma_{\mathcal{A}_1 L} \cdot \gamma_{\mathcal{A}_2 L})(y_2, z)\}$$

$$(\gamma_{\mathcal{B}_1 U} \cdot \gamma_{\mathcal{B}_2 U})(x_1, z)(y_2, z) = \min\{\gamma_{\mathcal{B}_1 U}(x_1, y_1), \gamma_{\mathcal{A}_2 U}(z)\}$$

$$\leq \min\{[\gamma_{\mathcal{A}_1 U}(x_1), \gamma_{\mathcal{A}_1 U}(y_1)], \gamma_{\mathcal{A}_2 U}(z)\}$$

$$= \min(\min(\gamma_{\mathcal{A}_1 U}(x_1), (\gamma_{\mathcal{A}_2 U}(z)), \min((\gamma_{\mathcal{A}_1 U}(y_1), \gamma_{\mathcal{A}_2 U}(z)))$$

$$= \min\{(\gamma_{\mathcal{A}_1 U} \cdot \gamma_{\mathcal{A}_2 U})(x_1, z), (\gamma_{\mathcal{A}_1 U} \cdot \gamma_{\mathcal{A}_2 U})(y_2, z)\}$$

Definition 3.4. The join of two interval valued picture fuzzy graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ with the crisp graphs $\mathcal{G}_1^*$ and $\mathcal{G}_2^*$ is denoted by $\mathcal{G}_1 + \mathcal{G}_2$ and is defined as follows

(i) $(\mu_{\mathcal{A}_1 L} + \mu_{\mathcal{A}_2 L})(x) = (\mu_{\mathcal{A}_1 L} \cup \mu_{\mathcal{A}_2 L})(x)$

$(\eta_{\mathcal{A}_1L} + \eta_{\mathcal{A}_2L})(x) = (\eta_{\mathcal{A}_1L} \cup \eta_{\mathcal{A}_2L})(x)$

$(\gamma_{\mathcal{A}_1L} + \gamma_{\mathcal{A}_2L})(x) = (\gamma_{\mathcal{A}_1L} \cup \gamma_{\mathcal{A}_2L})(x)$, if $x \in V_1 \cup V_2$

$(\mu_{\mathcal{A}_1U} + \mu_{\mathcal{A}_2U})(x) = (\mu_{\mathcal{A}_1U} \cup \mu_{\mathcal{A}_2U})(x)$

$(\eta_{\mathcal{A}_1U} + \eta_{\mathcal{A}_2U})(x) = (\eta_{\mathcal{A}_1U} \cup \eta_{\mathcal{A}_2U})(x)$

$(\gamma_{\mathcal{A}_1U} + \gamma_{\mathcal{A}_2U})(x) = (\gamma_{\mathcal{A}_1U} \cup \gamma_{\mathcal{A}_2U})(x)$, if $x \in V_1 \cup V_2$

(ii) $(\mu_{\mathcal{B}_1L} + \mu_{\mathcal{B}_2L})(xy) = (\mu_{\mathcal{B}_1L} \cup \mu_{\mathcal{B}_2L})(xy)$

$(\mu_{\mathcal{B}_1U} + \mu_{\mathcal{B}_2U})(xy) = (\mu_{\mathcal{B}_1U} \cup \mu_{\mathcal{B}_2U})(xy)$, if $xy \in E_1 \cap E_2$

$(\eta_{\mathcal{B}_1L} + \eta_{\mathcal{B}_2L})(xy) = (\eta_{\mathcal{B}_1L} \cup \eta_{\mathcal{B}_2L})(xy)$

$(\eta_{\mathcal{B}_1U} + \eta_{\mathcal{B}_2U})(xy) = (\eta_{\mathcal{B}_1U} \cup \eta_{\mathcal{B}_2U})(xy)$ if $xy \in E_1 \cap E_2$

$(\gamma_{\mathcal{B}_1L} + \gamma_{\mathcal{B}_2L})(xy) = (\gamma_{\mathcal{B}_1L} \cup \gamma_{\mathcal{B}_2L})(xy)$

$(\gamma_{\mathcal{B}_1U} + \gamma_{\mathcal{B}_2U})(xy) = (\gamma_{\mathcal{B}_1U} \cup \gamma_{\mathcal{B}_2U})(xy)$, if $xy \in E_1 \cap E_2$

(iii) $(\mu_{\mathcal{B}_1L} + \mu_{\mathcal{B}_2L})(x, y) = \min\{\mu_{\mathcal{A}_1L}(x), \mu_{\mathcal{A}_2L}(y)\}$

$(\mu_{\mathcal{B}_1U} + \mu_{\mathcal{B}_2U})(x, y) = \min\{\mu_{\mathcal{A}_1U}(x), \mu_{\mathcal{A}_2U}(y)\}$

$(\eta_{\mathcal{B}_1L} + \eta_{\mathcal{B}_2L})(x, y) = \min\{\eta_{\mathcal{A}_1L}(x), \eta_{\mathcal{A}_2L}(y)\}$

$(\eta_{\mathcal{B}_1U} + \eta_{\mathcal{B}_2U})(x, y) = \min\{\eta_{\mathcal{A}_1U}(x), \eta_{\mathcal{A}_2U}(y)\}$

$(\gamma_{\mathcal{B}_1L} + \gamma_{\mathcal{B}_2L})(x, y) = \max\{\gamma_{\mathcal{A}_1L}(x), \gamma_{\mathcal{A}_2L}(y)\}$

$(\gamma_{\mathcal{B}_1U} + \gamma_{\mathcal{B}_2U})(x, y) = \max\{\gamma_{\mathcal{A}_1U}(x), \gamma_{\mathcal{A}_2U}(y)\}$, for every $xy \in E'$, where E' is the set of all edges joining the vertices of V_1 and V_2.

Example 3.4. Consider two interval valued picture fuzzy graphs $\mathcal{G}_1 = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_2 = (\mathcal{A}_2, \mathcal{B}_2)$ with underlying crisp graphs $\mathcal{G}_1^* = (\mathcal{A}_1, \mathcal{B}_1)$ and $\mathcal{G}_2^* = (\mathcal{A}_2, \mathcal{B}_2)$ such that the vertices $V_1 = \{k, l\}$ and $V_2 = \{t, u, v\}$ and the edge sets $E_1 = \{kl\}$ and $E_2 = \{tu, uv\}$ respectively, where

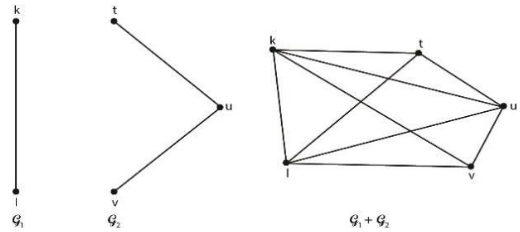

Figure 4. Join of two interval valued picture fuzzy graphs.

The vertex sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{A}_1 = \begin{cases} \langle k; (0.2,\ 0.3,\ 0.4),\ (0.5,\ 0.2,\ 0.3)\rangle \\ \langle l; (0.3,\ 0.4,\ 0.1),\ (0.1,\ 0.3,\ 0.4)\rangle \end{cases}$$

$$\mathcal{A}_2 = \begin{cases} \langle t; (0.3,\ 0.4,\ 0.3),\ (0.3,\ 0.2,\ 0.4)\rangle \\ \langle u; (0.2,\ 0.3,\ 0.4),\ (0.3,\ 0.2,\ 0.2)\rangle \\ \langle v; (0.1,\ 0.3,\ 0.4),\ (0.3,\ 0.2,\ 0.4)\rangle \end{cases}$$

The edge sets of $\mathcal{G}_1$ and $\mathcal{G}_2$

$$\mathcal{B}_1 = \{\langle kl; (0.2,\ 0.3,\ 0.4),\ (0.1,\ 0.2,\ 0.4)\rangle$$

$$\mathcal{B}_2 = \begin{cases} \langle tu; (0.2,\ 0.4,\ 0.3),\ (0.3,\ 0.2,\ 0.4)\rangle \\ \langle uv; (0.1,\ 0.3,\ 0.4),\ (0.3,\ 0.2,\ 0.4)\rangle. \end{cases}$$

The join of $\mathcal{G}_1 + \mathcal{G}_2$

$$\mathcal{B}_1 + \mathcal{B}_2 = \begin{cases} \langle kt; (0.2,\ 0.3,\ 0.4),\ (0.3,\ 0.2,\ 0.4)\rangle \\ \langle ku; (0.2,\ 0.3,\ 0.4),\ (0.3,\ 0.2,\ 0.3)\rangle \\ \langle kv; (0.1,\ 0.3,\ 0.4),\ (0.3,\ 0.2,\ 0.4)\rangle \\ \langle lt; (0.3,\ 0.4,\ 0.3),\ (0.1,\ 0.2,\ 0.4)\rangle \\ \langle lu; (0.2,\ 0.4,\ 0.3),\ (0.1,\ 0.2,\ 0.4)\rangle \\ \langle lv; (0.1,\ 0.3,\ 0.4),\ (0.1,\ 0.2,\ 0.4)\rangle. \end{cases}$$

Conclusion

In this paper, we define new operations of interval valued picture fuzzy graphs. The basic properties of interval valued picture fuzzy graph of lexicographic product and strong product are obtained. Also, define the join of two interval valued picture graphs and discussed with some examples.

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