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EDGE INDEPENDENT FUNCTION ON INTUITIONISTIC FUZZY GRAPHS

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Abstract

In this paper, μ_2-edge independent function, υ_2-edge independent function, edge independent function, maximal edge independent function, boundary set and the positive set of a strong intuitionistic fuzzy graph are defined. Convex combination of two edge dominating functions, two edge independent functions and two maximal edge independent functions of a strong intuitionistic fuzzy graph are also discussed with suitable illustration.

1. Introduction

In 1978, Berge has given a lecture on fractional graph at the Indian Statistical Institute and he also analysed the concepts of fractional matching number and fractional edge chromatic number [2]. In 1989, Berge devotes a chapter of his monograph to hypergraphs and combinatorics of finite sets [3] to fractional transversals of hypergraphs, which includes an exploration of fractional matchings of graphs. Functional generalizations for vertex subsets have been extensively studied in literature by Cockayne, McGillivray, Mynhardt and Yu [4, 5, 13]. Many of the fractional invariants in [12] can be defined by taking a definition of a standard graph invariant verbatim and inserting the word "fuzzy" in an appropriate place. One can speculate what might be meant by a fuzzy or fractional graph. This could mean a pair (V, E) in which V is a finite set and E is a fuzzy set of 2-element subsets of V.

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Alternatively, one might allow V to be a fuzzy set as well. In general, a fractional graph means a pair $G = \langle V, E \rangle$ in which V is a finite set and E is a fuzzy set or an intuitionistic fuzzy set of 2-element subsets of V. Alternatively, one might allow V to be fuzzy or an intuitionistic fuzzy set as well. i.e., There are nine ways (environments) to interpret a fractional graph $G = \langle V, E \rangle$ as follows:

Let $G = \langle V, E \rangle$ be a fractional graph, where V be the vertex set and E be the edge set. Then the following are the environments of V and E of G.

Environment	Vertex Set (V)	Edge Set (E)
1	Crisp	Crisp(Crisp Graph)
2	Crisp	Fuzzy
3	Crisp	Intuitionistic Fuzzy
4	Fuzzy	Crisp
5	Fuzzy	Fuzzy(Fuzzy Graph)
6	Fuzzy	Intuitionistic Fuzzy
7	Intuitionistic Fuzzy	Crisp
8	Intuitionistic Fuzzy	Fuzzy
9	Intuitionistic Fuzzy	Intuitionistic Fuzzy (Intuitionistic Fuzzy Graph)

As a result, fuzzy graph is nothing but a fractional graph $G = \langle V, E \rangle$ in which V is a fuzzy set and E is a 2-element set of V which is also fuzzy. An intuitionistic fuzzy graph is a fractional graph $G = \langle V, E \rangle$ in which the vertex V is an intuitionistic fuzzy set and an edge set E is also an intuitionistic fuzzy set. In a fractional graph, the above two environments are termed to be fuzzy and intuitionistic fuzzy graphs.

Let $G = (V, E)$ be a graph. The open neighbourhood $N(e_i)$ and the closed neighbourhood $N[e_i]$ of e_i are defined by $N(e_i) = \{e_j \in E : e_j$ is adjacent to $e_i\}$ and $N[e_i] = N(e_i) \cup \{e_i\}$. A function $f : E \to \mathcal{P}$ is called a $\mathcal{P}$-edge independent function of a graph G if the sum of its function values over any closed neighbourhood is at most 1 [6,7]. That is, for ever $e \in E$, $f(N(e) \cup \{e\}) = 1$.

- ❖ When $\mathcal{P} = \{0, 1\}$ we get the standard edge independent function of a fractional graph in $G = (V, E)$, where the vertex and edge sets are both crisp environments.
- ❖ When $\mathcal{P} = [0, 1]$ we obtain the fractional edge independent function of a fractional graph in $G = (V, E)$, where the vertex set is crisp and edge set is fuzzy environment.

In 2009, Arumugam et al. [1] introduced concept of edge independent function, maximal edge independent function, edge dominating function, convex combination of two edge dominating functions, edge independent functions, boundary set and the positive set of a fractional graph $G = (V, E)$, where the vertex set V is crisp and the edge set E is fuzzy. A function $f : E \to [0, 1]$ is called an edge independent function if for every e with $f(e) > 0$, we have $\sum_{x \in N[e]} f(x) = 1$ [1]. An edge independent function f is called maximal edge independent function (MEIF) if for every $e \in E$ with $f(e) = 0$, we have $\sum_{x \in N[e]} f(x) \geq 1$. A function $f : E(G) \to [0, 1]$ is called an edge dominating function (EDF) of G if $f(N[e]) = \sum_{x \in N[e]} f(x) \geq 1$ for every $e \in E(G)$. If and g be two EDFs of a graph G and $0 < \lambda < 1$, then $h_{\lambda} \lambda f + (1 - \lambda) g$ is called a convex combination of f and g [1]. If f and be two MEIFs, then either all convex combinations of f and g are MEIFs or none of them is an MEIFs. Let f be any edge dominating function of G and the boundary set B'_f and the positive set P'_f of f are defined by $B'_f = \{e : \sum_{x \in N[e]} f(x) = 1\}$ and $P'_f = \{e : f(e) > 0\}$ [1].

In [10], Nagoorgani et al. introduced the concept of an effective edge, the neighbourhood of a vertex, the closed neighbourhood degree of a vertex. Let $G = \langle V, E \rangle$ be an IFG. An edge $e = (x, y)$ of an IFG G is called an effective edge if $\mu_2(x, y) = \mu_1(x) \wedge \mu_1(y)$ and $\nu_2(x, y) = \nu_1(x) \vee \nu_1(y)$ [10]. The neighbourhood of any vertex v is defined as $N(v) = (N_\mu(v), N_\nu(v))$ where $N_\mu(v) = \{w \in V; \mu_2(v, w) = \mu_1(v) \wedge \mu_1(w)\}$ and $N_\nu(v) = \{w \in V; \nu_2(v, w) = \nu_1(v) \vee \mu_1(w)\}$ and $N[V] = N(v) \cup \{v\}$ is called the closed neighbourhood of v [10]. The neighbourhood degree of a vertex in an IFG G, is defined as $d_N(v) = (d_{N_\mu}(v), d_{N_\nu}(v))$ where $d_{N_\mu}(v) = \sum_{w \in N(v)} \mu_1(w)$ and $d_{N_\nu}(v) = \sum_{w \in N(v)} \nu_1(w)$. The closed neighbourhood degree of a vertex 'v' in an IFG G, is defined as $d_N[v] = (d_{N_\mu}[v], d_{N_\nu}[v])$ where $d_{N_\mu}[v] = [\sum_{w \in N(v)} \mu_1(u)] + \mu_1(v)$ and $d_{N_\nu}[v] = [\sum_{w \in N(v)} \nu_1(u)] + \nu_1(v)$ [10].

A strong intuitionistic fuzzy graph was proposed by Parvathi et al. [11]. An IFG $G = \langle V, E \rangle$, is said to be a strong IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\nu_{2ij} = \max(\nu_{1i}, \nu_{1j})$ for every $(v_i, v_j) \in E$. In [8], Karunambigai et al. introduced the concept of an edge dominating function, minimal edge dominating function of a fractional graph with an intuitionistic fuzzy environment. Let $G = (V, E)$ be a strong intuitionistic fuzzy graph and $f : E \to [0, 1]$ is called an edge dominating function (EDF) of $G = (V, E)$ if $d_{N_\mu}[e_i] \geq 1, d_{N_\nu}[e_i] < 1$ with $0 \leq \mu_2(e_i) + \nu_2(e_i) \leq 1$ for every $e_i \in E$ where $\mu_2(e_i) \geq 0, \nu_2(e_i) \neq 1, i = 1, 2, \ldots, n$. An edge dominating function f is called a minimal edge dominating function (MEDF) [8], if there exist an edge $w \in N[e_i]$ such that $d_{N_\mu}[e_i] = 1, d_{N_\nu}[e_i] \neq 1$ for every $e_i \in E$ where $\mu_2(e_i) \geq 0, \nu_2(e_i) \neq 1, i = 1, 2, \ldots, n$. An edge dominating function f is called a minimal edge dominating function (MEDF), if there does not exist a dominating function $g \neq f$ for which $g(e) \leq f(e)$ for all $e_i \in E$ [8].

These observations motivate us to investigate the concept of the neighbourhood of an edge, the neighbourhood degree of an edge, the closed neighbourhood degree of an edge, μ_2-edge independent function, ν_2-edge independent function, an edge independent function and maximal edge independent function, boundary set and the positive set of a fractional graph with intuitionistic fuzzy environment. Furthermore, convex combinations of two edge dominating functions, two edge independent functions and two maximal edge independent functions of a strong intuitionistic fuzzy graph and several results based on these concepts are discussed.

2. Edge Independent Function on Intuitionistic Fuzzy Graphs

Definition 2.1. Let $G = (V, E)$ be an IFG where $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. The neighbourhood of an edge e_i of an IFG $G = \langle V, E \rangle$, is defined as $N(e_i) = (N_\mu(e_i), N_\nu(e_i))$ where $(N_\mu(e_i), N_\nu(e_i)) = \{e_j \in E; e_j$ is an effective edge in G and adjacent to $e_i\}$ and $N[e_i] = N(e_i) \cup \{e_i\}$ is called the closed neighbourhood of e_i, $i, j = 1, 2, \ldots, m$.

Definition 2.2. Let $G = (V, E)$ be an IFG where $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. The neighbourhood degree of an edge $e_i \in E$ of an IFG $G = \langle V, E \rangle$, is defined as $d_N(e_i) = (d_{N_\mu}(e_i), d_{N_\nu}(e_i))$ where $d_{N_v}(e_i) = \sum_{e_j \in N(e_i)} \mu_2(e_j)$ and $d_{N_\nu}(e_i) = \sum_{e_j \in N(e_i)} \nu_2(e_j)$, $e_i, e_j \in E$, $i, j = 1, 2, \ldots, m$.

Definition 2.3. Let $G = (V, E)$ be an IFG where $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$. The closed neighbourhood degree of an edge $e_i \in E$ of an IFG $G = \langle V, E \rangle$, is defined as $d_N[e_i] = (d_{N_\mu}[e_i], d_{N_\nu}[e_i])$ where $d_{N_\mu}[e_i] = [\sum_{e_j \in N(e_i)} \mu_2(e_j)]$ and $d_{N_\nu}[e_i] = [\sum_{e_j \in N(e_i)} \nu_2(e_j)] + \nu_2(e_j)$, $e_i, e_j \in E(i, j = 1, 2, \ldots, m)$.

Definition 2.4. Let $G = (V, E)$ be an IFG and a vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$ edge set $E = \{e_1, e_2, e_3, e_4\}$.

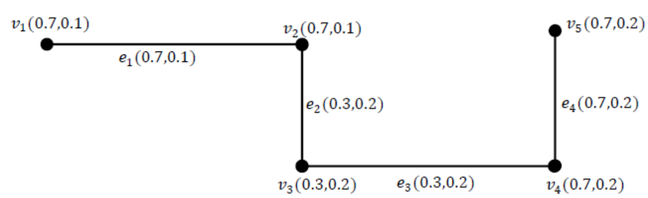

Figure 1. $G = (V, E)$.

In Figure 1, neighbourhood degree of each edge in an IFG $G = \langle V, E \rangle$ is given as follows:

$d_N(e_1) = (0.3, 0.2)$ where $d_{N_\mu}(e_1) = \sum_{e_2 \in N(e_1)} \mu_2(e_j) = \mu_2(e_2) = 0.3$, $d_{N_\nu}(e_1) = \sum_{e_2 \in N(e_1)} \nu_2(e_2) = 0.2$. Similarly $d_N(e_2) = (1, 0.3)$, $d_N(e_3) = (1, 0.4)$, $d_N(e_4) = (0.3, 0.2)$.

In Figure 1, closed neighbourhood degree of each edge in an IFG $G = \langle V, E \rangle$, is given as follows: $d_N[e_1] = (1, 0.3)$ where $d_{N_\mu}[e_1] = [\sum_{e_1 \in N(e_1)} \mu_2(e_1)] + \mu_2(e_1) = \mu_2(e_2) + \mu_2(e_1) = 0.3 + 0.7 = 1$, $d_{N_\nu}[e_1] = [\sum_{e_1 \in N(e_1)} \nu_2(e_1)] + \nu_2(e_1) = \nu_2(e_2) + \nu_2(e_1) = 0.2 + 0.1 = 0.3$. Similarly $d_N[e_2] = (1.3, 0.5)$, $d_N[e_3] = (1.3, 0.6)$, $d_N[e_4] = (1, 0.4)$.

Definition 2.5. A function $f : E \to [0, 1]$ of a strong IFG, $G = \langle V, E \rangle$, is called μ_2-edge independent function of G if $d_{N_\mu}[e_i] = 1$ for every $e_i \in E$, where $\mu_2(e_i) > 0$, $i = 1, 2, \ldots, m$.

Definition 2.6. Let $G = (V, E)$ be a strong IFG where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$.

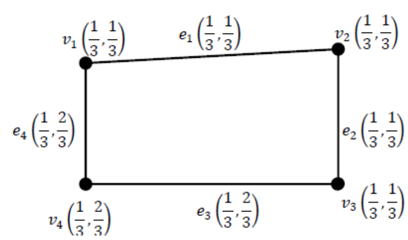

Figure 2. $G = (V, E)$.

In Figure 2, $d_{N_\mu}[e_1] = [\sum_{e_j \in N(e_1)} \mu_2(e_j)] = \mu_2(e_1) = [\mu_2(e_4) + \mu_2(e_2)] + \mu_2(e_1) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. Similarly $d_{N_\mu}[e_1] = 1$ for $i = 2, 3, 4$. Therefore f defined on G is a μ_2-edge independent function.

Definition 2.7. A function $f : E \to [0, 1]$ of a strong IFG, $G = \langle V, E \rangle$ is called ν_2-edge independent function of G if $d_{N_\nu}[e_i] < 1$ for every $e_i \in E$, where $\nu_2(e_i) \neq 1$, $i = 1, 2, \ldots, m$.

Definition 2.8. Let $G = (V, E)$ be a strong IFG where $V = \{v_1, v_2, v_3\}$ and $E = \{e_1, e_2\}$.

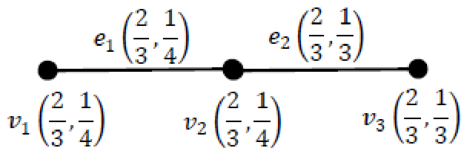

Figure 3. $G = (V, E)$.

In Figure 3, $d_{N_\nu}[e_1] = [\sum_{e_j \in N(e_1)} \nu_2(e_j)] + \upsilon_2(e_1) = [\nu_2(e_2)] + \nu_2(e_1)$ $= \frac{1}{3} + \frac{1}{3} = \frac{7}{12}$. Similarly $d_{N_\mu}[e_2] < 1$. Therefore f defined on G is a ν_2-edge independent function but f is not a μ_2-edge independent function.

Definition 2.9. A function $f : E \to [0, 1]$ of a strong IFG, $G = \langle V, E \rangle$ is said to be an edge independent function if it is both μ_2-edge independent function and a ν_2-edge independent function on G.

Definition 2.10. Let $G = \langle V, E \rangle$ be a strong IFG where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$.

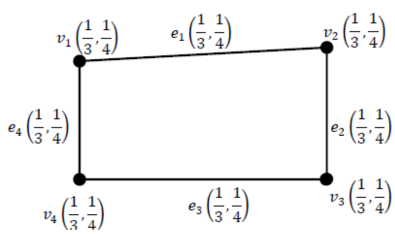

Figure 4. $G = (V, E)$.

Here, the function f on G is μ_2-edge independent function as well as a υ_2-edge independent function. Since $d_{N_\mu}[e_i] = 1$ and $d_{N_\nu}[e_2] < 1$ for every $e_i \in E$. Therefore f is defined on G is an edge independent function of G.

Definition 2.11. Let $G = \langle V, E \rangle$ be an IFG. An edge independent function $f : E \to [0, 1]$ is called a maximal edge independent function of G if there does not exist an edge independent function $f \neq g$, for which $g(e_i) \leq f(e_i)$ for every $e_i \in E$. Equivalently a function f is said to be maximal edge independent function of G if $d_{N_\mu}[e_i] \geq 1$, $d_{N_\nu}[e_i] \neq 1$ for every $e_i \in E$, where $\mu_2(e_i) \geq 0$, $\nu_2(e_i) \neq 1$, $i = 1, 2, \ldots, n$.

Example 2.12. In Figure 1, $d_{N_\mu}[e_i] \geq 1$ and $d_{N_\nu}[e_i] \neq 1$ for every $e_i \in E$, $i = 1, 2, 3, 4$. Therefore f is defined on G is a maximal edge independent function of G.

Definition 2.13. For an edge dominating function f of a strong IFG $G = (V, E)$, the boundary set B'_{if} and positive set P'_{if} are defined by

$$B'_{if} = \{e \in E / d_{N_\mu}[e] = 1, d_{N_\nu}[e] < 1\} \text{ and } P'_{if} = \left\{e \in E / \begin{matrix} 0 \leq \mu_2(e) \leq 1 \\ 0 \leq \upsilon_2(e) < 1 \end{matrix}\right\}.$$

Definition 2.14. Let $G = (V, E)$ be a strong IFG and let $A, B \subseteq E$. A is said to dominate B if each $e \in B - A$ is adjacent to an edge in A. If A dominates B, we write $(A \to B)$.

Example 2.15. In Figure 1, $B'_{if} = \{e_1, e_4\}$ and $P'_{if} = \{e_1, e_2, e_3, e_4\}$. Take $A = \{e_1, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3, e_4\} \subseteq E$. Then $B - A = \{e_1, e_2, e_3, e_4\} - \{e_1, e_4\}$ is adjacent to an edge in A. Therefore B'_{if} dominates P'_{if} denoted by $B'_{if} \to P'_{if}$.

Remark 2.16. An edge dominating function f of a strong IFG G is a minimal edge dominating function f of G if and only if $N[e] \cap B'_{if} \neq \phi$ for all $e \in P'_{if}$.

Theorem 2.17. *A necessary and sufficient condition for an edge dominating function f of a strong IFG G is a minimal edge dominating function of G if* $B'_{if} \to P'_{if}$, *where* B'_{if} *and* P'_{if} *denotes a boundary set and apositive set, respectively.*

Proof. Let $f : E \to [0, 1]$ be a minimal edge dominating function of G

with $e \in P'_{if}$ and suppose to the contrary that B'_{if} does not dominate e, where B'_{if} and P'_{if} denotes a positive set and a boundary set, respectively. Then $N[e] \cap B'_{if} \neq \phi$ which implies that $d_{N_\mu}[e] > 1, d_{N_\upsilon}[e] < 1$ for each $e \in N[e]$. Now define $\varepsilon > 0$ where $\varepsilon \leq \min_{u \in N[e]}(d_{N_\mu}[e] - 1, 1 - d_{N_\nu}[e])$ here $d_{N_\mu}[x]$ and $d_{N_\nu}[x]$ are related to *f* and $g : E \to [0, 1]$ by $g(e) = f(e) - \varepsilon$ and $g(x) = f(x)$ for $x \in E - \{e\}$. For each $u \in N[e]$, $g[u] = (d_{N_\mu}[e] - \varepsilon \geq 1,$ $\varepsilon - d_{N_\upsilon}[e] - \varepsilon < 1)$ and for $x \in E - N[e]$, $g[x] = (d_{N_\mu}[x] \geq 1, d_{N_\nu}[x] < 1)$ here $d_{N_\mu}[x]$ and $d_{N_\nu}[x]$ are related to *f*. Therefore *g* is an edge dominating function. But $g < f$, which is a contradiction of *f*. Hence B'_{if} dominates P'_{if} (i.e., $B'_{if} \to P'_{if}$).

Conversely, assume that $B'_{if} \to P'_{if}$ and let $h : E \to [0, 1]$ is an minimal edge dominating function if there exist a dominating function $h < f$ for which $h(e) < f(e)$ for some $e \in E$. Then $e \in P'_{if}$ and so, by assumption, $u \in B'_{if}$ for some $u \in N[e]$. Now

$h[u] = d_N[u] = (d_{N_\mu}[u], d_{N_\nu}[u])$ where $d_{N_\mu}[u]$ and $d_{N_\nu}[u]$ are related to h. $h[u] = (h(w) + d_N[w]) = (h_{\mu_2}(w) + d_{N_\mu}[w], h_{\nu_2}(w) + d_{N_\nu}[w])$, where $w \in N[u] - \{e\} < (f_{\mu_2}(w) + d_{N_\mu}[w], f_{\nu_2}(w) + d_{N_\nu}[w])$,

where $w \in N[u] - \{e\}$ $h[u] < f[u] = d_N[u] = (d_{N_\mu}[u], d_{N_\upsilon}[u])$ where $d_{N_\mu}[u]$ and $d_{N_\nu}[u]$ are related to *f* and $d_{N_\mu}[u] = 1, d_{N_\nu}[u] < 1$. Therefore *h* is not an edge dominating function which implies that *f* is an minimal edge dominating function.

Definition 2.18. Let f, g be two edge dominating function of an intuitionistic fuzzy graph *G*. Then $h_\lambda = \lambda f + (1 - \lambda)g$ where $0 < \lambda < 1$, called a convex combination of *f* and *g*.

Note 2.19.

(1) Convex combination of two edge dominating functions of a strong IFG *G* is again an edge dominating function of a strong IFG *G*.

(2) A convex combination of two minimal edge dominating functions of a strong IFG need not be a minimal edge dominating function of a strong IFG.

Theorem 2.20. *Let f and g be minimal edge dominating functions of an IFG G, Let* $h_\lambda = \lambda f + (1-\lambda)g$, *where* $0 < \lambda < 1$. *Then* h_λ *is a minimal edge dominating function of G if and only if* $B'_{if} \cap B'_{ig}$ *dominates* $P'_{if} \cup P'_{ig}$.

Proof. We prove that $B'_{h_\lambda} = B'_{if} \cap B'_{ig}$ and $P'_{h_\lambda} = P'_{if} \cup P'_{ig}$. The result is then immediate from Theorem 2.17. If $e \notin P'_{if} \cup P'_{ig}$, then $f(e) = g(e) = h_\lambda(e) = 0$. If $e \in P'_{if}$, then $h_\lambda(e) \geq \lambda f(e) > 0$. Therefore $P'_{h_\lambda} = P'_{if} \cup P'_{ig}$.

Assume $e \in B'_{if} \cap B'_{if}$. In that case $h_\lambda[e] = \lambda f[e] + (1-\lambda) g[e] = (d_{N_\mu}[u] = 1, d_{N_\nu}[u] < 1)$. In the same manner $h_\lambda[e] = (d_{N_\mu}[u] > 1, d_{N_\nu}[u] < 1)$ for $e \notin B'_{if} \cap B'_{ig}$ and hence $B'_{h_\lambda} = B'_{if} \cup B'_{ig}$.

Remark 2.21. If a function $f : E \to [0, 1]$ is an edge independent function of a strong IFG G, then $P'_{if} \subseteq B'_{ig}$.

Theorem 2.22. *Every maximal edge independent function of a strong IFG G is a minimal edge dominating function of G.*

Proof. Let $f : E \to [0, 1]$ be a maximal edge independent function of a strong IFG G. It follows from the definition that $d_{N_\mu}[e] \geq 1, d_{N_\nu}[e] < 1$ for each $e \in E$. Hence f is an edge dominating function of a strong IFG G. Further $P'_{if} \subseteq B'_{ig}$ so that $B'_{ig} \to P'_{if}$. Hence it follows from Theorem 2.17 that f is a minimal edge dominating function of G.

Remark 2.23. Convex combination of two edge independent functions of a strong IFG G need not be an edge independent function of G. Further f and g are both maximal edge independent functions of a strong IFG G and hence a convex combination of two maximal edge independent functions of a strong IFG G need not be an edge independent function of G.

Example 2.24. Let $G = (V, E)$ be an IF path P_3 and a vertex set $V = \{v_1, v_2, v_3\}$, edge set $E = \{v_1v_2 = e_1, v_2v_3 = e_2\}$. Let us take

$(f(e_1), f(e_2)) = \left(\left(\frac{2}{3}, \frac{1}{4}\right), \left(\frac{1}{3}, \frac{1}{4}\right)\right)$ and $(g(e_1), g(e_2)) = ((1, 0), (0, 0.4))$. Here f and g are maximal edge independent functions of a strong IFG G. Now the convex combination of f and g is $h_\lambda = \lambda f + (1-\lambda)g$ where $0 < \lambda < 1$. Take $\lambda = 0.5$ then $(h_{0.5}(e_1), h_{0.5}(e_5)) = ((1.17, 0.17), (0.17, 0.2))$. Then h_λ is an minimal not an edge independent function of a strong IFG G.

Remark 2.25. Let f and g be two edge independent functions of an IFG G. Then $h_\lambda = \lambda f + (1-\lambda)g$ where $0 < \lambda < 1$, is an edge independent function if and only if $P'_{if} \cup P'_{ig} \subseteq B'_{if} \cap B'_{ig}$. Hence either all convex combinations of f and g are edge independent functions of G or none of them is an edge independent function of G.

Theorem 2.26. *Let f and g be two maximal edge independent functions of an IFG G. Then either all convex combinations of f and g are maximal edge independent functions of G or none of them is a maximal edge independent functions of G.*

Proof. Consider $h_\lambda = \lambda f + (1-\lambda)g$ where $0 < \lambda < 1$. Assume that h_{λ_1} is an maximal edge independent functions of an IFG G and take $\lambda \neq \lambda_1$. We prove that h_λ is an maximal edge independent functions of G. Take $e \in E$. Assume $h_\lambda(e) = (\mu_2(e) = 0, 0 < \nu_2(e) < 1)$. In that case $f(e) = g(e) = (\mu_2(e) = 0, 0 < \nu_2(e) < 1)$. Since f and g are maximal edge independent functions of an IFG G, we have $d_{N_\mu}[e] \geq 1, d_{N_\nu}[e] < 1$ are related to f and $d_{N_\mu}[e] \geq 1, d_{N_\nu}[e] < 1$ are related to g. Hence it follows that $d_{N_\mu}[e] \geq 1, d_{N_\nu}[e] < 1$ are related to h_λ.

Now, suppose $h_\lambda(e) = (\mu_2(e) = 0,0 < \nu_2(e) < 1)$. Then either $f(e) = (\mu_2(e) = 0, 0 < \nu_2(e) < 1)$ or $g(e) = (\mu_2(e) = 0,0 < \nu_2(e) < 1)$. Hence $h_{\lambda_1}(e) = (\mu_2(e) = 0, 0 < \nu_2(e) < 1)$. Since h_{λ_1} is an maximal edge independent function of G, $d_{N_\mu}[e] = 1, d_{N_\nu}[e] < 1$ are related to h_{λ_1}, so that $d_{N_\mu}[e] = 1$ is related to f and g are equal and $d_{N_\mu}[e] < 1$ is related to f and g are equal. Hence $d_{N_\mu}[e] = 1$ and $d_{N_\nu}[e] < 1$ are related to h_λ. Thus $P'_{h_\lambda} \subseteq B'_{h_\lambda}$ and

$d_{N_\mu}[e] \geq 1,$ and $d_{N_\nu}[e] < 1$ are related to h_λ, for all $e \in E$. Hence h_λ is an maximal edge independent function of an IFG G.

Lemma 2.26. *Any minimal edge dominating function f of a IFG G with $B'_{if} = E$ is a maximal edge independent function of G.*

Proof. Since $B'_{if} = E$, we have $P'_{h_\lambda} \subseteq B'_{h_\lambda}$ and hence f is an edge independent function of an IFG G. Also since f is an minimal edge dominating function of G, $d_{N_\mu}[e] \geq 1$, $d_{N_\nu}[e] < 1$ for all $e \in E$ and hence f is an maximal edge independent function of G.

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