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# NEIGHBORHOOD CONNECTED DOMINATION AND ECCENTRIC DOMINATION OF BOOLEAN GRAPH $BG_3(G)$ OF A GRAPH G

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## Abstract

Let $G$ be a simple $(p, q)$ graph with vertex set $V(G)$ and edge set $E(G)$. $BG_3(G)$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to a vertex and an edge incident to it in $G$ or two non-adjacent edges of $G$. In this paper, we study the concept of neighborhood connected domination and eccentric domination in Boolean graph $BG_3(G)$.

## 1. Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph $G$, let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. Let $v \in V$. The open neighborhood $N(v)$ of a vertex $v$ is the set of all vertices adjacent to $v$ in $G$. $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood of $v$. If $S \subseteq V$, then $N(S) = \bigcup v \in sN(v)$ and $N[S] = N(S) \cup S$.

A vertex and an edge are said to cover each other if they are incident. A

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set of vertices which covers all the edges of a graph $G$ is called a point cover for $G$, while a set of edges which covers all the vertices is a line cover. The smallest number of vertices in any point cover for $G$ is called its point covering number or simply covering number and is denoted by $\alpha_0(G)$ or $\alpha_0$. Similarly, $\alpha_1$ is the smallest number of edges in any line cover of $G$ and is called its line cover number. A set of vertices in $G$ is independent if no two of them are adjacent. The largest number of vertices in such a set is called the point independence number of $G$ and is denoted by $\beta_0(G)$ or $\beta_0$. A set of edges in a graph is independent if no two edges in the set are adjacent. By a matching in a graph $G$, we mean an independent set of edges in $G$. The edge independence number $\beta_1(G)$ of a graph $G$ is a maximum cardinality of an independent set of edges. A perfect matching is a matching with every vertex of the graph is incident to exactly one edge of the matching. The graph $G^+$ is obtained from the graph $G$ by attaching a pendant edge to each of the vertices of $G$.

In 2004 [2], Bhanumathi defined the Boolean graph $BG_3(G)$ and studied its properties in [6, 7]. Let $G$ be a simple $(p, q)$ graph with vertex set $V(G)$ and edge set $E(G)$. $BG_3(G)$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices are adjacent if and only if they correspond to a vertex and an edge incident to it in $G$ or two nonadjacent edges of $G$. For simplicity, denote this graph by $BG_3(G)$. The vertices of $BG_3(G)$, which are in $V(G)$ are called point vertices and vertices in $E(G)$ are called line vertices of $BG_3(G)$. $V(BG_3(G)) = V(G) \cup E(G)$ and $E(BG_3(G)) = (E(T(G)) - (E(G) \cup E(L(G))) \cup E(\overline{V(G)}))$. $BG_3(G)$ has $p + q$ vertices, $p$-point vertices and $q$-line vertices, $BG_3(G)$ is a spanning subgraph of $BG_2(G)$. $K_p$ and $L(G)$ are induced subgraphs of $BG_3(G)$.

The concept of domination in graphs was introduced by Ore [9]. A set $D \subseteq V(G)$ is said to be a dominating set of $G$, if every vertex in $V(G) - D$ is adjacent to some vertex in $D$. $D$ is said to be a minimal dominating set if $D - \{u\}$ is not a dominating set for any $u \in D$. The domination number $\gamma(G)$ of $G$ is the minimum cardinality of a dominating set.

A dominating set $D$ of $G$ is called a connected dominating set if the induced subgraph $\langle D \rangle$ is connected. The minimum cardinality of a connected dominating set of $G$ is called the connected dominating number of $G$ and is denoted by $\gamma_c(G)$.

A subset $S$ of $V$ is called a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in $S$. The minimum cardinality of a dominating set is called the domination number of $G$ and is denoted by $\gamma(G)$. Arumugam and Sivagnanam [1] introduced the concept of neighborhood connected domination in graphs. A dominating set $S$ of a connected graph $G$ is called a neighborhood connected dominating set (ncd-set) if the induced subgraph $\langle N(S) \rangle$ of $G$ is connected. The neighborhood connected domination number $\gamma_{nc}(G)$ is the minimum cardinality of a ncd-set. Clearly, $\gamma(G) \leq \gamma_{nc}(G) \leq \gamma_c(G)$.

The distance $d(u, v)$ between two vertices $u$ and $v$ in $G$ is the minimum length of a path joining them if any; otherwise $d(u, v) = \infty$. Let $G$ be a connected graph and $u$ be a vertex of $G$. The eccentricity $e(v)$ of $v$ is the distance to a vertex farthest from $v$. Thus, $e(v) = \max\{d(u, v) : u \in V\}$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity. For any connected graph $G$, $r(G) \leq diam(G) \leq 2r(G)$. The vertex $v$ is a central vertex if $e(v) = r(G)$. The center $C(G)$ is the set of all central vertices. The central sub graph $\langle C(G) \rangle$ of a graph $G$ is the subgraph induced by the center. The vertex $v$ is a peripheral vertex if $e(v) = diam(G)$. The periphery $P(G)$ is the set of all peripheral vertices. For a vertex $v$, each vertex at a distance $e(v)$ from $v$ is an eccentric vertex. Eccentric set of a vertex $v$ is defined as $E(G) = \{u \in V(G) : d(u, v) = e(v)\}$.

Janakiraman, Bhanumathi and Muthammai [5] introduced the concept of eccentric domination number of a graph. A set $D \subseteq V(G)$ is an eccentric dominating set if $D$ is a dominating set of $G$ and for every $v \in V - D$, there exists at least one eccentric point of $v$ in $D$. An eccentric dominating set $D$ is a minimal eccentric dominating set if no proper subset $D' \subseteq D$ is an eccentric dominating set. The eccentric domination number $\gamma_{ed}(G)$ of a graph $G$ equals the minimum cardinality of an eccentric dominating set. $V(G)$ is an eccentric dominating set for any graph $G$. Hence, $\gamma_{ed}(G)$ is an well defined parameter. Obviously, $\gamma(G) \leq \gamma_{ed}(G)$.

In this paper, we study the concept of neighborhood connected domination and eccentric domination in Boolean graph $BG_3(G)$. Let $G$ be a $(p, q)$ Graph.

**Theorem 1.1** [2]. $\gamma(BG_3(G)) = 1$ *if and only if* $G = K_2$.

**Theorem 1.2** [2]. $\gamma(BG_3(G)) = 2$ *if and only if* $G = K_{1,2}, 2K_2, K_1 \cup K_2$ or $K_3$.

**Theorem 1.3** [2]. *Let D be a minimal dominating set for* $V(G)$. *D Dominates* $BG_3(G)$ *if and only if D is a line cover for G.*

**Theorem 1.4** [2]. *Let G be Graph without isolated vertices. Then* $k \leq V(BG_3(G)) \leq \min\{p, q\}$, *where k is the number of pendant vertices of G.*

**Theorem 1.5** [2]. $\gamma(BG_3(G)) \leq p - 1$.

**Theorem 1.6** [2]. *If G has no isolated vertices,* $\gamma(BG_3(G)) = \alpha_1(G)$ *or* $\alpha_1(G) + 1$.

**Theorem 1.7** [2]. *Let G be a graph without isolated vertices.* (i) *When* $G \neq K_{1,n}, K_3, K_4, C_4, P_4, K_4 - e, \gamma_c(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$.

(ii) *When* $G = K_{1,n}, \gamma(BG_3(G)) = n = \alpha_1(G)$ *and* $\gamma_c(BG_3(G)) = n + 1 = \alpha_1(G) + 1$.

(iii) *When* $G = K_3, \gamma(BG_3(G)) = 3, \gamma_c(BG_3(G)) = 4$.

(iv) *When* $G = K_4, C_4, P_4$ or $K_4 - e, \gamma(BG_3(G)) = 3, \gamma_c(BG_3(G)) = 4$.

**Theorem 1.8** [2]. $BG_3(G)$ *has a dominating edge if and only if* $G = K_2$ *or* $2K_2$.

**Theorem 1.9** [2, 8]. *Radius of* $BG_3(G) = 1$ *if and only if* $G = K_2$.

**Theorem 1.10** [2, 8]. *Eccentricity of every point vertices of* $BG_3(G)$ *is two if and only if* $G = K_2$.

**Theorem 1.11** [2, 8]. $BG_3(G)$ *is self-centered with diameter three if and only if* $G = K_3$.

**Theorem 1.12** [2, 8]. *In* $BG_3(G)$, *eccentricity of point vertex is three and eccentricity of line vertex is two if and only if G satisfies the following conditions.*

(i) $r(G) > 1$.

(ii) *For* $u, v \in V(G)$ *either* $uv \in E(G)$ *or there exists non-adjacent edges* $e_u$ *and* $e_v$ *such that* $e_u$ *is incident with u and* $e_v$ *is incident with v.*

**Theorem 1.13** [2, 8]. *Radius of* $BG_3(G)$ *is* 2 *and diameter of* $BG_3(G)$ *is* 4 *if and only if* $r(G) = 1$ *and G has at least two pendant vertices.*

**Theorem 1.14.** [2, 8]. (i) *Radius of* $BG_3(G)$ *is* 2 *and diameter of* $BG_3(G)$ *is* 4 *and eccentricity of every line vertex is* 3 *if and only if* $G = K_{1,n}$.

(ii) *Eccentricity of all point vertices is two and eccentricity of all line vertices is three in* $BG_3(G)$ *is not possible.*

**Theorem 1.15** [2, 8]. $BG_3(G)$ *is self-centered with diameter two if and only if* $G = K_n, n > 3$.

**Theorem 1.16** [2, 8]. $BG_3(G)$ *is bi-eccentric with diameter* 3 *if and only if G satisfies any one of the following*: (i) $r(G) = 1$, $diam(G) = 2$, *G has at most one pendant vertex.* (ii) $diam(G) \geq 2$.

## 2. Neighborhood Connected Domination in Boolean Graph $BG_3(G)$ of A Graph G

In this section we have studied Neighborhood connected domination in Boolean graph $BG_3(G)$ and obtained bounds for $\gamma_{nc}(BG_3(G))$.

Every connected dominating set of a graph is always a neighbourhood connected dominating set. Hence, if $\gamma_{nc}(BG_3(G))$ is the neighbourhood connected domination number of $BG_3(G)$, then $\gamma(BG_3(G)) \leq \gamma_{nc}(BG_3(G)) \leq \gamma_c(BG_3(G))$. Also, if $G$ has $k$ pendant vertices then $k \leq \gamma(BG_3(G))$ by Theorem 1.4 and $\gamma_c(BG_3(G)) \leq \alpha_1(G)+1$ by Theorem 1.6. Hence, $k \leq \gamma_{nc}(BG_3(G)) \leq \alpha_1(G)+1$.

**Theorem 2.1.** *If $G$ is a connected graph with a perfect matching and $p > 4$, then $\gamma_{nc}(BG_3(G)) \leq p/2$.*

**Proof.** Let $G$ be a connected graph. Let $M$ be a perfect matching $f$ $G$. Hence $\beta_1(G) = p/2$. Let $X$ be the set of vertices in $BG_3(G)$ corresponding to the edges in $G$. Let $S$ be the set of vertices in $BG_3(G)$ corresponding to the edges in $M$. Since $G$ is connected and every line vertex in $X - S$ is adjacent to two point vertices in $V(BG_3(G) - X)$, $S$ is a dominating set of $BG_3(G)$. $\langle N(S) \rangle$ is connected, since element in $S$ are adjacent to each other in $BG_3(G)$. Therefore, $S$ is a neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(G)) \leq p/2$.

**Remark 2.1.** When $p = 4$, $S$ is not a dominating set of $BG_3(G)$.

**Theorem 2.2.** *Let $G$ be a connected graph, with $G - v$ has a perfect matching for some $v \in V(G)$ and $\beta_1(G) > 2$. Then $\gamma_{nc}(BG_3(G)) \leq \frac{p+1}{2}$.*

**Proof.** Let $M$ be a perfect matching of the graph $G - v$. Let $S$ be the set of vertices of $BG_3(G)$ corresponding to the edges in $M$. $S_1 = S - \{v\}$ is a dominating set of $BG_3(G)$. In, $BG_3(G)$, $v$ dominates all line vertices which corresponds to edges incident with $v$ in $G$. $v$ is not incident to some edges of $M$ in $G$. Hence there exists $e \in E(G)$ which is incident with $v$ and is not adjacent to $e_1 \in M$. In $BG_3(G)$, $e$ and $e_1$ are adjacent.

Hence, $\langle N(S) \rangle$ is connected. Thus, $\gamma_{nc}(BG_3(G)) \leq \frac{p+1}{2}$.

**Theorem 2.3.** *If $G$ is a graph of radius one and diameter 2, then $\gamma_{nc}(BG_3(G)) \leq p$.*

**Proof.** Consider $u \in V(G)$ with degree $\deg u = p - 1$. In $BG_3(G)$, $N_G(u)$ dominates point vertices and line vertices. Now, consider $S = N_G(u) \cup \{u \text{ or } e\}$, $uv = e \in E(G)$. $S$ is a dominating set of $BG_3(G)$. Also, $\langle N(S) \rangle$ is connected. Thus, $S$ is a ncd-set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(G)) \leq p - 1 + 1 = p$.

This bound is attained for $G = K_{1,n}$.

**Theorem 2.4.** *Let $G \neq K_{1,n}$ be a connected graph with $\alpha_1(G) > 2$, then line cover of G is a $\gamma_{nc}$-set of $BG_3(G)$ and $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$.*

**Proof.** Let $S$ be a line cover of $G$. $S$ covers all the vertices of $G$. Hence, in $BG_3(G)$, $S$ dominates all line vertices and point vertices of $BG_3(G)$. Thus $S$ is a dominating set of $BG_3(G)$. Also $N(S)$ contains all point and line vertices of $BG_3(G)$. This implies that $\langle N(S) \rangle = BG_3(G)$. Hence, $\langle N(S) \rangle$ is connected in $BG_3(G)$. Thus, $S$ is a ncd-set of $BG_3(G)$. Therefore, $\gamma_{nc}(BG_3(G)) \leq \alpha_1(G)$. Also, we know that $\alpha_1(G) \leq \gamma_{nc}(BG_3(G))$. Therefore, $\gamma_{nc}(BG_3(G)) \leq \alpha_1(G)$.

## 3. Neighborhood Connected Domination Number of $BG_3(G)$ For Some Particular Graphs

Here, we find out some minimum neighbourhood connected dominating set of $BG_3(G)$ and $\gamma_{nc}(BG_3(G))$ for some particular classes of graphs.

When $G \neq K_{1,n}$ and $p > 4$, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) \leq \alpha_1(G)$.

**Theorem 3.1.** *For a non-trival path $P_n$ on n vertices with $n - 4$.*

(i) $\gamma_{nc}(BG_3(P_n)) = \lceil n/2 \rceil$, *if n is odd*

(ii) $\gamma_{nc}(BG_3(P_n)) = (n/2)$, *if n is even.*

**Proof.** Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = v_i v_{i+1}, 1 \leq i \leq n - 1$. Let $u_i \in V(BG_3(P_n))$ be the vertex corresponding to $e_i$ in $BG_3(P_n)$.

**Case (i).** $n$ is odd

$S = \{u_1, u_3, u_5, \ldots, u_{n-2}, u_{n-1}\}$. $S$ is a minimum line cover of $G$, and $|S| = \lceil n/2 \rceil$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G) = \lceil n/2 \rceil$.

**Case (i).** $n$ is even

$S = \{u_1, u_3, u_5, \ldots, u_{n-3}, u_{n-1}\}$. $S$ is a minimum neighborhood connected dominating set of $BG_3(P_n)$ and $|S| = (n/2)$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G) = (n/2)$.

**Remark 3.1.** When $G = P_3$. $S = \{v_1, v_3, u_2\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $BG_3(P_n) = 3$. When $G = P_4$. $S = \{v_1, v_3, u_2\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $BG_3(P_n) = 3$.

**Theorem 3.2.** *For a cycle $C_n$ on n vertices with $n > 4$.*

(i) $\gamma_{nc}(BG_3(C_n)) = \lceil n/2 \rceil$, *if n is odd.*

(ii) $\gamma_{nc}(BG_3(C_n)) = (n/2)$, *if n is even.*

**Proof.** Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = v_i v_{i+1}, 1 \leq i \leq n-1$ and $e_n = v_n v_1$. Let $u_i$ be the vertex corresponding to $e_i$ in $BG_3(C_n)$. As in previous theorem, here also $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Hence the theorem follows.

**Remark 3.2.** When $G = C_3$. $S = \{v_1, v_2, v_3\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $BG_3(C_n) = 3$. When $G = C_4$. $S = \{u_1, u_3, u_4\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $BG_3(C_4) = 3$.

**Theorem 3.3.** (i) $\gamma_{nc}(BG_3(K_n)) = (n/2)$ *if n is even and* $n > 4$

(ii) $\gamma_{nc}(BG_3(K_n)) = \lceil n/2 \rceil$, *if n is odd and* $n > 4$

**Proof.** Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of $K_n$ and let $u_{ij}, i < j, i, j = 1, 2, 3, \ldots, n$ be the added vertices corresponding the edges

$e_{ij}$ of $K_n$ to obtain $BG_3(K_n)$. Thus $V(BG_3(K_n)) = \{v_1, v_2, v_3, \ldots, v_n\}$ $\bigcup_{i<j} \{u_{ij}\}$, $i, j = 1, 2, 3, \ldots, n$. The graph $BG_3(K_n)$ has $n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$ vertices.

Let $S = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n-3, n-2}, u_{n-1, n}\}$. $S$ is line cover of $G$. $S$ is a minimal neighborhood connected dominating set of $BG_3(K_n)$. $|S| = (n/2)$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Hence, $\gamma_{nc}(BG_3(K_n)) = (n/2)$ if $n$ is even and $\gamma_{nc}(BG_3(K_n)) = \lceil n/2 \rceil$, if $n$ is odd.

**Remark 3.3.** When $G = K_3$. $S = \{v_1, v_2, v_3\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(K_3)) = 3$. When $G = K_4$. $S = \{v_1, v_2, v_3, v_4\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(K_4)) = 4$. When $G = K_5$. $S = \{u_{12}, u_{34}, u_{45}, v_4\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(K_5)) = 4$.

**Theorem 3.4.** $\gamma_{nc}(BG_3(K_{1,n})) = n + 1$, $n > 4$.

**Proof.** Let $v_1, v_2, v_3, \ldots, v_n, v$ ($v$ is the central vertex of $K_{1,n}$) be the vertices of $K_{1,n}$ and let $e_i = vv_i, 1, 2, \ldots, n$ be the edges of $K_{1,n}$. Let $v_1, v_2, v_3, \ldots, v_n, v, u_1, u_2, u_3, \ldots, u_n$ be the corresponding vertices of $BG_3(K_{1,n})$. Thus $V(BG_3(K_{1,n}))$ has $2n+1$ vertices. Let $S = \{v, u_1, u_2, u_3, \ldots, u_n\}$. $S$ is a neighborhood connected dominating set of $BG_3(K_{1,n})$. $|S| = n+1$. Thus, $\gamma_{nc}(BG_3(K_{1,n})) = n+1$. Also, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G)+1$ and $\alpha_1(K_{1,n}) = n$. Hence, $n \leq \gamma_{nc}(BG_3(K_{1,n})) \leq n+1$. Also, there is no neighbourhood connected dominating set with cardinality $n$. Therefore, $\gamma_{nc}(BG_3(K_{1,n})) = n+1$.

**Theorem 3.5.** $\gamma_{nc}(BG_3(K_{m,n})) = n$ *if* $m \geq n > 2$.

**Proof.** When $G = K_{m,n}$, $V(G) = V_1 \cup V_2$, $|V_1| = m$ and $|V_2| = n$, $E(G) = \{e_{ij}/1 \le i \le m, 1 \le j \le n\}$ where $e_{ij} = u_i v_j$ for all $1 \le i \le m$, $1 \le j \le n$. Thus, $V(BG_3(K_{m,n})) = V_1 \cup V_2 \cup \{e_{ij}/1 \le i \le m, \quad 1 \le j \le n\}$, $S = \{e_{11}, e_{22}, e_{33}, \ldots, e_{nn}, e_{nn+1}, \ldots, e_{nm}\}$ is a line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G) = n$. Therefore, $\gamma_{nc}(BG_3(K_{m,n})) = n$.

**Theorem 3.6.** *Let* $G = W_n$ *with* $n > 4$ *vertices.*

(i) $\gamma_{nc}(BG_3(W_n)) = \lceil n/2 \rceil$, *if n is odd*

(ii) $\gamma_{nc}(BG_3(W_n)) = (n/2) + 1$, *if n is even*

**Proof.** Let $v_1, v_2, \ldots, v_n, v$ ($v$ is the central vertex of $W_n$) be the vertices of $W_n$ and let $e_j = vv_j$, $j = 1, 2, \ldots, n$, $e_{12} = v_1 v_2$, $e_{23} = v_2 v_3, \ldots, e_{n-1,n} = v_{n-1} v_n$, $e_{n1} = v_n v_1$ be the edges of $W_n$. Let $v_1, v_2, \ldots, v_n, v, u_1, u_2, \ldots, u_n$, $u_{12}, u_{23}, \ldots, u_{n-1,n}, u_{n1}$ be the corresponding vertices of $BG_3(W_n)$. Thus $V(BG_3(W_n))$ has $3n + 1$ vertices.

**Case (i).** $n$ is odd

$S = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n-2,n-1}, e_n\}$ is a line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Therefore, $\gamma_{nc}(BG_3(W_n)) = \lceil n/2 \rceil$.

**Case (ii).** $n$ is even

$S = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n-3,n-2}, e_n\}$ is a line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Therefore, $\gamma_{nc}(BG_3(W_n)) = (n/2) + 1$.

**Remark 3.4.** When $G = W_3$, $S = \{u_{12}, u_{23}, u_1\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(W_n)) = 3$. When $G = W_4$, $S = \{v, u_{12}, u_{34}, u_{41}\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{ed}(BG_3(W_n)) = 4$.

**Theorem 3.7.** (i) $\gamma_{nc}(BG_3(F_n)) = \lceil n/2 \rceil$, *if n is odd.*

(ii) $\gamma_{nc}(BG_3(F_n)) = (n/2)+1$, *if n is even*

**Proof.** Let $v_1, v_2, v_3, \ldots, v_n, v$ ($v$ is the central vertex of $F_n$) be the vertices of $F_n$ and let $e_j = vv_j$, $j = 1, 2, \ldots, n$, and $v_iv_j = e_{ij}(i < j = 1, 2, 3, \ldots, n)$ be the edges of $F_n$. Let $v_1, v_2, v_3, \ldots, v_n$, $v, u_1, u_2, \ldots, u_n, u_{12}, u_{23}, \ldots, u_{n-1,n}$ be the corresponding vertices of $BG_3(F_n)$. Thus $V(BG_3(F_n))$ has $3n$ vertices.

**Case (i).** $n$ is odd

$S = \{u_n, u_{12}, u_{34}, u_{56}, \ldots, u_{n-2,n-1}\}$ is a line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Therefore, $\gamma_{nc}(BG_3(F_n)) = \lceil n/2 \rceil$.

**Case (ii).** $n$ is even

$S = \{u_n, u_{12}, u_{34}, u_{56}, \ldots, u_{n-2,n-1}\}$ is a line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Therefore, $\gamma_{nc}(BG_3(F_n)) = (n/2)+1$.

**Remark 3.5.** When $G = F_3$. $S = \{v_2, u_{12}, u_{23}, u_3\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(F_3)) = 4$.

**Theorem 3.8.** $\gamma_{nc}(BG_1(P_n^+)) = n$, $n \geq 3$.

**Proof**. Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $u_i$ be the pendant vertex adjacent to $v_i$, $1 \leq i \leq n$. Let $x_i$ be the vertex of $BG_3(P_n^+)$ corresponding to the edge $u_iv_i$ in $P_n^+$. Then $S = \{x_1, x_2, x_3, \ldots, x_n\}$ is a minimal line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Therefore, $\gamma_{nc}(BG_3(P_2^+)) = n$.

**Remark 3.5.** When $G = P_2^+$. $S = \{x_2, x_2, u_{12}\}$ is a minimum neighborhood connected dominating set of $BG_3(G)$. Hence, $\gamma_{nc}(BG_3(W_3)) = 3$.

**Theorem 3.9.** $\gamma_{nc}(BG_3(C_2^+)) = n$ *if* $n \geq 3$.

**Proof.** Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $u_i$ be the pendant vertex adjacent to $v_i$, $1 \leq i \leq n$. Let $x_i$ be the vertex of $BG_3(C_n^+)$ corresponding to the edge $u_iv_i$ in $C_n^+$. Then $S = \{x_1, x_2, x_3, \ldots, x_n\}$ is a line cover of $G$. Here, $\alpha_1(G) > 2$ and thus by Theorem 2.4, $\gamma_{nc}(BG_3(G)) = \alpha_1(G)$. Therefore, $\gamma_{nc}(BG_3(C_2^+)) = n$.

## 4. Eccentric Domination in Boolean Graph $BG_3(G)$ of A Graph G

In this section, we have studied eccentric domination in Boolean graph $BG_3(G)$ and obtained bounds for $\gamma_{ed}(BG_3(G))$.

If $G$ has no isolated vertices, then $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ by Theorem 1.6 Hence, $\alpha_1(G) \leq \gamma_{ed}(BG_3(G))$.

**Theorem 4.1.** *Let G be a connected graph with greater than four vertices. Then* $\alpha_1(G) \leq \gamma_{ed}(BG_3(G)) \leq p$.

**Proof.** Set of all point vertices form a dominating set of $BG_3(G)$ and if $G \neq K_n$, eccentric vertices are only point vertices. Hence, $V(G)$ is an eccentric dominating set of $BG_3(G)$. Therefore, $\alpha_1(G) \leq \gamma_{ed}(BG_3(G)) \leq p$. If $G = K_n$, set of all point vertices form a dominating set of $BG_3(G)$ and line vertices have both line and point vertices as eccentric vertices. Hence, again $V(G)$ is an eccentric dominating set of $BG_3(G)$. Therefore, $\gamma_{ed}(BG_3(G)) \leq p$.

Also, we know that $\alpha_1(G) \leq \gamma_{ed}(BG_3(G)) \leq p \leq \gamma_{ed}(BG_3(G))$. Hence, $\alpha_1(G)\gamma_{ed}(BG_3(G)) \leq p$.

The bounds are sharp, since $\gamma_{ed}(BG_3(G)) = 3$ if $G = P_5$ and $\gamma_{ed}(BG_3(G)) = p$ if $G = K_{1,n}$.

**Theorem 4.2.** *Let* $G \neq K_n$ *be a connected graph with* $p > 2$. *Let S be the set of all line vertices of* $BG_3(G)$. *Then S cannot be an eccentric dominating set of* $BG_3(G)$.

**Proof.** Let $S = E(G)$. S contains all line vertices of $G$. Hence, S dominates all point and line vertices of $BG_3(G)$. Also, $V(BG_3(G) - E(G))$ has their eccentric vertices in $V(G)$ only. Therefore, $S$ cannot be an eccentric dominating set of $BG_3(G)$.

**Theorem 4.3.** *Let S be the set of all pendant vertices of G. Then S cannot be an eccentric dominating set of* $BG_3(G)$.

**Proof.** Let $S$ be the set of all pendant vertices of $G$. In $BG_3(G)$, $S$ dominates incident edges only. Therefore, $S$ cannot be an eccentric dominating set of $BG_3(G)$.

**Theorem 4.4.** *Let S be a minimal line cover or vertex cover of G, where* $G \neq K_n$. *Then S cannot be an eccentric dominating set of* $BG_3(G)$.

**Proof. Case (i).** Let $S$ be a line cover of $G$. In $BG_3(G)$, $S$ dominates all point vertices and line vertices. Every point vertices of $V(BG_3(G))$ has their eccentric vertices in $V(G)$ only. Therefore, $S$ cannot be an eccentric dominating set of $BG_3(G)$.

**Case (ii).** Let $S$ be a minimal point cover of $G$. In $BG_3(G)$, $S$ dominates all line vertices only. Therefore, $S$ cannot be an eccentric dominating set of $BG_3(G)$.

## 5. Eccentric Domination Number of $BG_3(G)$ for Some Particular Graphs

Here, we find out some minimum eccentric dominating set of $BG_3(G)$ and the exact value of $\gamma_{ed}(BG_3(G))$ for some particular classes of graphs.

**Theorem 5.1.** *For a non-trival path* $P_n$ *on n vertices*

(i) $\gamma_{ed}(BG_3(P_n)) = \lceil n/2 \rceil + 1$, *if n is odd.*

(ii) $\gamma_{ed}(BG_3(P_n)) = (n/2) + 1$, *if n is even.*

**Proof.** Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = v_i v_{i+1}, 1 \leq i \leq n-1$. Let

$u_i \in V(BG_1(P_n))$ be the vertex corresponding to $e_i$ in $BG_3(P_n)$ Then $\{v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_{n-1} \in V(BG_3(P_n)$. Thus $|V(BG_3(P_n))| = 2n - 1$.

**Case (i).** $n$ is odd

Let $S = \{v_1, v_n, u_2, u_4, u_6, \ldots, v_{n-1}\}$. is a minimal eccentric dominating set of $BG_3(P_n)$. Vertices $v_1, v_n$ are eccentric vertices of elements of $V - S$. $|S| = \lceil n/2 \rceil + 1$. Therefore, $\gamma_{ed}(BG_3(P_3) \leq \lceil n/2 \rceil + 1$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ and $\alpha_1(P_n) = \lceil n/2 \rceil$ Hence, $\lceil n/2 \rceil \leq \gamma_{ed}(BG_3(P_n)) \leq \lceil n/2 \rceil + 1$. Also, there is no eccentric dominating set with cardinality $\lceil n/2 \rceil$. Therefore, $\gamma_{ed}(BG_3(P_n)) \leq \lceil n/2 \rceil + 1$.

**Case (ii).** $n$ is even

Let $S = \{v_1, v_n, u_2, u_4, u_6, \ldots, u_{n-2}\}$. $S$ is a minimal eccentric dominating set of $BG_3(P_n) \cdot v_1, v_n$ are eccentric vertices of elements of $V - S$. $|S| = \lceil n/2 \rceil + 1$. Therefore, $\gamma_{ed}(BG_3(P_3) \leq (n/2) + 1$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ and $\alpha_1(P_n) = (n/2)$. Hence, $(n/2) \leq \gamma_{ed}(BG_3(P_n)) \leq (n/2) + 1$. Also, there is no eccentric dominating set with cardinality $(n/2)$. Therefore, $\gamma_{ed}(BG_3(P_n)) = (n/2) + 1$.

**Theorem 5.2.** (i) $\gamma_{ed}(BG_3(C_n)) = \lceil n/2 \rceil + 1$, *if* $n \geq 5$.

**Proof.** Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$. and $e_n = v_i v_{i+1}, 1 \leq i \leq -1$ and $e_n = v_n v_1$. Let $u_i$ be the vertex corresponding to $e_i$ in $BG_3(C_n)$. Then $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n \in V(BG_3(C_n))$. Thus $|V(BG_3(C_n))| = 2n$. Let $S = \{v_1, v_4, u_2, u_5, u_7, \ldots, u_{n-1}\}$. $S$ is an eccentric dominating set of $BG_3(C_n)$. $|S| = \lceil n/2 \rceil + 1$. Therefore, $\gamma_{ed}(BG_3(C_n)) \leq \lceil n/2 \rceil + 1$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ and $\alpha_1(C_n) = \lceil n/2 \rceil$. Hence, $\lceil n/2 \rceil \leq \gamma_{ed}(BG_3(C_n)) \leq \lceil n/2 \rceil + 1$. Also, there is no eccentric dominating set with cardinality $\lceil n/2 \rceil$. Therefore, $\gamma_{ed}(BG_3(C_n)) \leq \lceil n/2 \rceil + 1$.

**Remark 5.1.** When $G = C_3$. $S = \{v_1, v_2, v_3\}$. $v_1$ is a eccentric vertex of $u_2$, $v_2$ is a eccentric vertex of $u_3$, $v_3$ is a eccentric vertex of $u_1$. Therefore $S$ is a minimum eccentric dominating set of $BG_3(G)$. Hence, $\gamma_{ed}(BG_3(C_3)) = 3$. When $G = C_4$. $S = \{v_1, v_2, v_3, v_4\}$. Therefore, $S$ is a minimum eccentric dominating set of $BG_3(G)$. Hence, $\gamma_{ed}(BG_3(C_4)) = 4$.

**Theorem 5.3.** (i) $\gamma_{ed}(BG_3(K_3)) = (n/2)$, *if n is even and* $n > 4$

(ii) $\gamma_{ed}(BG_3(K_n)) = \lceil n/2 \rceil$, *if n is odd*, $n > 4$.

**Proof.** Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of $K_n$ and let $u_{ij}, i < j$, $i, j = 1, 2, 3, \ldots, n$ be the added vertices corresponding the edges $e_{ij}$ of $K_n$ to obtain $BG_3(K_n)$. Thus $V(BG_3(K_n)) = \{v_1, v_2, v_3, \ldots, v_n\} \quad \cup_{i<j}\{u_{ij}\}, i$, $j = 1, 2, 3, \ldots, n$. The graph $BG_3(K_n)$ has $n + \frac{n(n-1)}{2} = \frac{n(n-1)}{2}$ vertices. Eccentricity of every point vertex and line vertex of $BG_3(K_n)$ is two. Therefore it is a self-centered graph.

**Case (i).** $n$ is even

Let $S = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n-1,n}\}$. $S$ is a minimal eccentric dominating set of $BG_3(K_n)$. $|S| = (n/2)$. Therefore, $\gamma_{ed}(BG_3(K_n)) \leq (n/2)$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ and $\alpha_1(K_n) = (n/2)$. Hence, $(n/2) = \gamma(BG_3(K_n)) \leq \gamma_{ed}(BG_3(K_n))$. This implies that $\gamma_{ed}(BG_3(K_n)) = (n/2)$. Therefore, $\gamma_{ed}(BG_3(K_n)) \leq \lceil n/2 \rceil$.

**Case (ii).** $n$ is odd

Let $S = \{u_{12}, u_{34}, u_{56}, \ldots, u_{n,1}\}$. $S$ is a minimal eccentric dominating set of $BG_3(K_n)$. $|S| = \lceil n/2 \rceil$. Therefore, $\gamma_{ed}(BG_3(K_n)) \leq \lceil n/2 \rceil$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ and $\alpha_1(K_n) = \lceil n/2 \rceil$. Also, $\lceil n/2 \rceil = \gamma(BG_3(K_n)) \leq \gamma_{ed}(BG_3(K_n))$. This implies that $\gamma_{ed}(BG_3(K_n)) = \lceil n/2 \rceil$. Therefore, $\gamma_{ed}(BG_3(K_n)) = \lceil n/2 \rceil$.

**Remark 5.2.** When $G = K_3 \cdot S = \{v_1, v_2, v_3\}$ is a minimum eccentric dominating set of $BG_3(G)$. Hence, $\gamma_{ed}(BG_3(K_n)) = 3$ When $G = K_4$. $S = \{v_1, v_2, v_3, v_4\}$ is a minimum eccentric dominating set of $BG_3(G)$. Hence, $\gamma_{ed}(BG_3(K_n)) = 4$.

**Theorem 5.4.** $\gamma_{ed}(BG_3(K_{1,n})) = n,\ n \geq 3.$

**Proof.** Let $v_1, v_2, v_3, \ldots, v_n, v$ ($v$ is the central vertex of $K_{1,n}$) be the vertices of $K_{1,n}$ and let $e_i = vv_i$, $i = 1, 2, \ldots, n$ be the edges of $K_{1,n}$. Let $v_1, v_2, v_3, \ldots, v_n, v, u_1, u_2, u_3, \ldots, u_n$, be the corresponding vertices of $BG_3(K_{1,n})$. Thus $(BG_3(K_{1,n}))$ has $2n+1$ vertices. $V' = V(BG_3(K_{1,n}))$. Let $S = \{v_1, v_n, u_2, u_3, u_4, \ldots, u_{n-1}\}$, where $v_1, v_n$ are pendant vertices of $G$. $v_1, v_n$ are eccentric vertices of elements of $V' - S$. Eccentricity of every point vertices are 2 or 3 and eccentricity of every line vertices are 2 in $BG_3(K_{1,n})$. Radius $r(BG_3(K_{1,n})) = 2$ and diameter $d.(BG_3(K_{1,n})) = 3$ $S$ is an eccentric dominating set of $BG_3(K_{1,n})$ and $|S| = n$. Thus, $\gamma_{ed}(BG_3(K_{1,n})) \leq n$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G)+1$ and $\alpha_1(K_{1,n}) = n$. Also, $\gamma(BG_3(G)) \leq \gamma_{ed}(BG_3(G))$. This implies that $\gamma(BG_3(K_{1,n}) \geq n$. Hence, $\gamma(BG_3(K_{1,n}) = n$.

**Remark 5.3.** When $G = K_{1,2}$. Let $S = \{v, v_1, v_2\}$ is a minimum eccentric dominating set of $BG_3(G)$. Hence, $\gamma_{ed}(BG_3(K_{1,2})) = 3$.

**Theorem 5.5.** $\gamma_{ed}(BG_3(K_{m,n})) = n + 2,\ n \geq m > 2.$

**Proof.** When $G = K_{m,n} \cdot V(G) = V_1 \cup V_2 \cdot |V_1| = m$ and $|V_2| = n$. $E(G) = \{e_{ij}/1 \leq i \leq m, 1 \leq j \leq j \leq n\}$ where $e_{ij} = u_iv_j$ for all $1 \leq i \leq m$, $1 \leq j \leq n$. Thus $V(BG_1(K_{m,n})) = (V_1 \cup V_2) \cup \{e_{ij}/1 \leq i \leq m, 1 \leq j \leq n\}$. Let $S = \{v_1, u_1, e_{11}, e_{22}, e_{33}, \ldots, e_{mm}\} \cup \{e_{mm+1}, \ldots, e_{mn}\}$, $v_1 \in V_1$ and $u_1 \in V_2$. S is an eccentric dominating set of $BG_3(K_{m,n}) \cdot |S| = n+2$. Thus, $\gamma_{ed}(BG_3(K_{m,n})) \leq n + 2$. Also, $(BG_3(G) = \alpha_1(G)$ or $\alpha_1(G)+1$ and $\alpha_1(K_{m,n}) = n$. We know that $\gamma(BG_3(K_{m,n})) \leq \gamma_{ed}\gamma(BG_3(K_{m,n}))$. Eccentricity of every point vertices are 3 and eccentricity of every line vertices are 2 in $BG_3(K_{m,n})$. $r(BG_3(K_{m,n}))$. $r(BG_3(K_{m,n})) = 2$ and $d(BG_3(K_{m,n})) = 3$. Every point vertices of $V(BG_3(K_{m,n}))$ has their eccentric vertices in $V_1$ or $V_2$ only.

Also, there is no eccentric dominating set with cardinality $n$ and $n+1$. Therefore, $\gamma_{ed}(BG_3(K_{m,n})) = n+2$.

**Theorem 5.6.** (i) $\gamma_{ed}(BG_3(W_n)) = (n/2)+2$, *if n is even*

(ii) $\gamma_{ed}(BG_3(W_n)) = \lceil n/2 \rceil + 1$, $\gamma$ *if n is odd*, $n \geq 4$.

**Proof.** Let $v_1, v_2, \ldots, v_n, v$ ($v$ is the central vertex of $W_n$) be the vertices of $W_n$ and let $e_j = vv_j$, $j = 1, 2, \ldots, n$, $e_{12} = v_1v_2$, $e_{23} = v_2v_3, \ldots, v_3, \ldots$, $e_{n-1}, n = v_n$, $e_{n1} = v_nv_1$ be the edges of $W_n$. Let $v_1, v_2, \ldots, v_n, v, u_1$, $u_2, \ldots, u_n, u_{12}, u_{23}, u_{n-1}, n, u_{n1}$ be the corresponding vertices of $BG_3(W_n)$. Thus $V(BG_3(W_n))$ has $3n+1$ vertices, where $v_1, v_4$ are eccentric vertices of all point vertices in $BG_3(W_n)$. Eccentricity of every point vertex and line vertex of $BG_3(W_n)$ is three or two.

**Case (i).** $n$ is even

Let $S = \{v_1, v_4, u_{23}, u_{56}, \ldots, u_{n-1}, n, u_n\} \cdot S$ is a minimal eccentric dominating set of $BG_3(W_n) \cdot |S| = (n/2)+2$. Therefore, $\gamma_{ed}(BG_1(W_n)) \leq (n/2)+2$. Also, we know that $\gamma_{ed}(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G)+1$ and $\alpha_1(W_n) = (n/2)+1$. Hence, $(n/2)+1 \in \gamma_{ed}(BG_3(W_n) \leq (n/2)+2$ Also, there is no eccentric dominating set with cardinality $(n/2)+1$. Therefore, $\gamma_{ed}(BG_3(W_n) = (n/2)+2$.

**Case (ii):** $n$ is odd

Let $S = \{v_1, v_4, u_{23}, u_{56}, \ldots, u_{n-2,n-1}, u_n\}$. S is a minimal eccentric dominating set of $BG_3(W_n)$. $|S| = \lceil n/2 \rceil + 1$. Therefore, $\gamma_{ed}(BG_1(W_n)) \leq \lceil n/2 \rceil + 1$ Also, we know that $\gamma_{ed}(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G)+1$ and $\alpha_1(W_n) = \lceil n/2 \rceil$. Hence, $\lceil n/2 \rceil \leq \gamma_{ed}(BG_3(W_n)) \leq \lceil n/2 \rceil + 1$. Also, there is no eccentric dominating set with cardinality $\lceil n/2 \rceil$. Therefore, $\gamma_{ed}(BG_3(W_n) = \lceil n/2 \rceil + 1$.

**Theorem 5.7.** (i) $\gamma_{ed}(BG_3(F_n) = (n/2)+2$, *if n is even*

(ii) $\gamma_{ed}(BG_3(F_n) = \lceil n/2 \rceil + 1$, *if n is odd*, $n \geq 4$.

**Proof.** Let $v_1, v_2, v_3, \ldots, v_n, v$ ($v$ is the central vertex of $F_n$) be the vertices of $F_n$ and let $e_j = vv_j$, $j = 1, 2, \ldots, n$, and $v_iv_j = e_{ij}(j = i+1, i = 1, 2, 3, \ldots, n)$ be the edges of $F_n$. Let $v_1, v_2, \ldots, v_n$, $v, u_1, u_2, \ldots, u_n, u_n, u_{12}, u_{23}, \ldots, u_{n-1,n}$ be the corresponding vertices of $BG_3(F_n)$. Thus $V(BG_3(F_n))$ has $3n$ vertices.

**Case (i).** $n$ is even

$S = \{v_1, v_4, u_{23}, u_{45}, \ldots, u_{n-2,n-3}, u_{n-1}\}$ is the eccentric dominating set of $BG_3(F_n)$. $S$ is a minimal eccentric dominating set of $BG_3(F_n)$. $|S| = \lceil n/2 \rceil + 1$. Therefore, $\gamma_{ed}(BG_3(F_n)) \leq (n/2) + 2$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$. and $\alpha_1(F_n) = (n/2)$. Hence, $(n/2) \leq \gamma_{ed}(BG_3(F_n)) \leq (n/2) + 2$. Also, there is no eccentric dominating set with cardinality $(n/2)$. Therefore, $\gamma_{ed}(BG_3(F_n)) = (n/2) + 1$.

**Case (i).** $n$ is odd

$S = \{v_1, v_4, u_{23}, u_{45}, \ldots, u_{n-2,n-3}, u_{n-1}\}$ is the eccentric dominating set of BG3(Fn). $S$ is a minimal eccentric dominating set of $BG_3(F_n)$. $|S| = \lceil n/2 \rceil + 1$. Therefore, $\gamma_{ed}(BG_3(F_n)) \leq \lceil n/2 \rceil + 1$. Also, we know that $\gamma(BG_3(G)) = \alpha_1(G)$ or $\alpha_1(G) + 1$ and $\alpha_1(F_n) = \lceil n/2 \rceil$. Hence, $\lceil n/2 \rceil \leq \gamma_{ed}(BG_3(F_n)) \leq \lceil n/2 \rceil + 1$. Also, there is no eccentric dominating set with cardinality $\lceil n/2 \rceil$. Therefore, $\gamma_{ed}(BG_3(F_n)) = \lceil n/2 \rceil + 1$.

**Theorem 5.8 (i).** (i) $\gamma_{ed}(BG_3(F_n^+)) = \left\lceil \frac{3n-1}{2} \right\rceil$, *if n is even*

(ii) $\gamma_{ed}(BG_3(P_n^+)) = \left( \frac{3n-1}{2} \right)$, *if n is odd.*

**Proof.** Let $G = P_n^+$ be a graph obtained from $P_n$ by attaching exactly one pendant edge at each vertex of $P_n$. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, e_{34}, \ldots, e_{n-1,n}$ be the edges in $P_n$ where $e_{i,i+1} = v_iv_{i+1}$, $i = 1$, $2, 3, \ldots, n-1$. Let $u_i$ be the pendant vertex attached to $v_i$ in $P_n^+$, $i = 1$, $2, 3, \ldots, n$ and let $e_i = v_iu_i \in E(G)$, $i = 1, 2, 3, \ldots, n$. Then $v_1, v_2, v_3, \ldots,$

$v_n, u_1, u_2, u_3, \ldots, u_n, e_{11}, e_{22}, e_{33}, \ldots, e_{nn}, e_{12}, e_{22}, e_{23}, e_{34}, \ldots, e_{n-1,n}$ $\in V(BG_3(P_n^+))$ Thus $|V(BG_3(P_n^+))| = 4n-1$. $u_2, u_4, \ldots, u_{n-1}$ are peripheral vertices of $BG_3(P_n^+)$.

**Case (i).** $n$ is even

Let $S = \{e_{11}, u_2, e_{33}, e_{12}, u_4, e_{34}, \ldots, e_{n-1,n-1}, u_n, e_{n-1,n}\}$. $S$ is an eccentric dominating set of $BG_3(P_n^+)$. $|S| = \left\lceil \frac{3n-1}{2} \right\rceil$. Thus, $\gamma_{ed}(BG_3(P_n^+)) \leq \left\lceil \frac{3n-1}{2} \right\rceil$. Let $S$ be an eccentric dominating set of $BG_3(P_n^+)$. Vertices $u_1, u_2, u_3, \ldots, u_n$ are pendant vertices in $BG_3(P_n^+)$. So either $u_i$ or $e_i \in S$. Now, consider the line vertex $e_i \in S$. Eccentric vertices of $e_i e_{i+1}$ in $BG_3(P_n^+)$ are $u_i$ or $u_{i-1}$ only. So, $S$ must contain at least $n + \left\lceil \frac{n-1}{2} \right\rceil$ vertices. Hence, $n + \left\lceil \frac{n-1}{2} \right\rceil \leq \gamma_{ed}(BG_3(P_n^+))$. Therefore, $\gamma_{ed}(BG_3(P_n^+)) = \left\lceil \frac{3n-1}{2} \right\rceil$.

**Case (ii).** $n$ is odd

Let $S = \{e_{11}, u_2, e_{12}, e_{33}, u_4, e_{34}, \ldots, e_{nn}, u_{n-1}, e_{n-2,n,1}\}$. $S$ is an eccentric dominating set of $BG_3(P_n^+)$. $|S| = \left( \frac{3n-1}{2} \right)$. Thus, $\gamma_{ed}(BG_3(P_n^+)) \leq \left( \frac{3n-1}{2} \right)$. Let $S$ be an eccentric dominating set of $BG_3(P_n^+)$. Vertices $u_1, u_2, u_3, u_n$ are pendant vertices in $BG_3(P_n^+)$. So either $u_i$ or $e_i \in S$. Now, consider the line vertex $e_i e_{i+1}$. Eccentric vertices of $e_i e_{i+1}$ in $BG_3(P_n^+)$ are $u_i$ or $u_{i-1}$ only. So, $S$ must contain at least $n + \left( \frac{n-1}{2} \right)$ vertices. Hence, $n + \left( \frac{n-1}{2} \right) \leq \gamma_{ed}(BG_3(P_n^+))$. Therefore, $\gamma_{ed}(BG_3(P_n^+)) \leq \left( \frac{3n-1}{2} \right)$.

**Theorem 5.9.** (i) $\gamma_{ed}(BG_3(C_n^+)) \leq \left\lceil \frac{3n}{2} \right\rceil$ *if n is odd.*

(ii) $\gamma_{ed}(BG_3(C_n^+)) = (3n/2)$ *if n is even.*

**Proof.** Let $G = C_n^+$ be a graph obtained from $C_n$ by attaching exactly one pendant edge at each vertex of $C_n$. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices and $e_{12}, e_{23}, e_{34}, \ldots, e_{n1}$ be the edges in $C_n$, where $e_{i,i+1} = v_i v_{i+1}$, $1 \le i \le n-1$ and $e_{n1} = v_n v_1$. Let $u_i$ be the pendant vertex attached to $v_i$ in $C_n^+$, $i = 1, 2, \ldots, n$, where $e_i = u_i v_i$, $1 \le i \le n$. Then $v_1, v_2, v_3, \ldots, v_n, u_1, u_2, u_3, \ldots, u_n, e_1, e_2, e_3, e_{12}, e_{23}, e_{34}, \ldots, e_{n1} \in V(BG_3(C_n^+))$.

Thus $|V(BG_3(C_n^+))| = 4n$. has all $u_j$'s and $v_j$'s, as eccentric vertices in $V - S$.

**Case (i).** $n$ is odd

Let $S = \{u_1, e_2, e_{12}, u_3, u_4, e_{34}, \ldots, u_n, e_{n1}, e_{n-1,n-1}\}$. $S$ is an eccentric dominating set of $BG_3(C_n^+)$. $|S| = \left\lceil \frac{3n}{2} \right\rceil$. Therefore, $\gamma_{ed}(BG_3(C_n^+) \le \left\lceil \frac{3n}{2} \right\rceil$. Let $S$ be an eccentric dominating set of $BG_3(C_n^+)$. Vertices $u_1, u_2, u_3, \ldots, u_n, \ldots, u_n$ are pendant vertices in $BG_3(C_n^+)$. So either $u_i$ or $e_i \in S$. Now, consider the line vertex $e_i e_{i+1}$. Eccentric vertices of $e_i e_{i+1}$ in $BG_3(C_n^+)$ are $u_i$ or $u_{i-1}$ only. So, $S$ must contain at least $n + \left\lceil \frac{n}{2} \right\rceil$ vertices. Hence, $n + \left\lceil \frac{n}{2} \right\rceil \le \gamma_{ed}(BG_3(C_n^+))$. Therefore, $\gamma_{ed}(BG_3(C_n^+)) = \left\lceil \frac{3n}{2} \right\rceil$.

**Case (ii).** $n$ is even

Let $S = \{u_1, e_2, e_{12}, u_3, e_4, e_{34}, \ldots, u_{n-1}, e_{n-1,n}, e_n\}$. $S$ is an eccentric dominating set of $BG_3(C_n^+)$. $|S| = (3n/2)$. Therefore, $\gamma_{ed}(BG_3(C_n^+)) \le (3n/2)$. Let $S$ be an eccentric dominating set of $BG_3(C_n^+)$. Vertices $u_1, u_2, u_3, \ldots, u_n$ are pendant vertices in $BG_3(C_n^+)$. So either $u_i$ or $e_i \in S$. Now, consider the line vertex $e_i e_{i+1}$. Eccentric vertices of $e_i e_{i+1}$ in $BG_3(C_n^+)$ are $u_i$ or $u_{i-1}$ only. So, $S$ must contain at least $n + (n/2)$ vertices. Hence, $n + (n/2) \le \gamma_{ed}(BG_3(C_n^+))$. Therefore, $\gamma_{ed}(BG_3(P_n^+)) \le (3n/2)$.

## Conclusion

In this paper we computed the exact value of the neighborhood connected domination number and eccentric domination number for Boolean graph of path, cycle, complete graph, complete bipartite graph and some special graphs. Also we found some upper bounds for neighborhood connected domination number and eccentric domination number for Boolean graph $BG_3(G)$ of a graph.

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