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RETAILER'S OPTIMAL INVENTORY STRATEGY FOR NEW PRODUCT AND BUY BACK STRATEGY FOR USED PRODUCT

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Abstract

To sustain finite resources and encourage green production not only manufacturers and government, consumer also is concerned about effects of manufacturing on environment. As a result, consumers have shown their interest to buy used or refurbished product. In this article we considered a case that, retailer sells new product as well as collects and sell the used product to the customers. An optimal pricing and ordering strategy for new product and optimal buy back strategy for used product is formulated and discussed in detail. The rate of demand is assumed to be nonlinear function of price and time for new product and linear function of price and time for buy back used product. The objective is to maximize total profit per time unit for retailer with respect to optimal price and ordering quantity for new product and optimal buy back quantity for used product. The model is illustrated with numerical examples and sensitivity analysis is performed for key parameters.

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1. Introduction

Extensive use of technology helps all the industrial sectors to reach the consumer from any corner of the world. This leads to boost in demand of any product. Thus, the manufacturing sectors are producing more to meet the demands and industries are competing with each other to capture the market. And so, industries are using more natural resources for excess production which generates more trash. Now, the key issue is to improve strategies for ecological support to preserve our limited natural resources and reduce the generation of trash as specified government regulations. Customers are concerned about environmental issues and they prefer to purchase product from manufacturer having green image. That is why many manufacturers have started collecting used products which are discarded by the customers. After refurbishing or recycling those products, manufacturer sells the product to a new customer at lower price. The process of recycling or refurbishing product is not a novel idea. It has been a common practice for the products like, aeronautics, metal, glass, ornaments, paper etc. in the last two three decades. Recently, it has been observed for many other products like plastic bags, water bottles, mobile phones, marker pens, etc. Thierry et al defined four synonyms of reuse as direct reuse, repair, recycling and remanufacturing. Reuse in deterministic model is introduced by Schrady [18] with a constant rate of demand. Two cost components fixed cost and holding cost were considered in Schrady's [18] model. Mabini et al. [12] extended Schrady's model for multi-items having same repair facility. One may refer [9], Richter [16], Kannan et al. [10], Govindan et al. [5], Chen et al. [1], etc. and it's cited references for more about recycling or reusable item. Other motivating work in this area are batteries recycling, Daniel et al. [4], electronic waste recycling, Nagurney and Toyasaki [14], glass recycling, Gonzlez-Torre and Adenso-Daz [7], paper recycling, Pati et al. [15], etc.

In recent time, consumers are price sensitive and purchase the product after looking to the price only.

Selling price of an item is prime factor to influence demand; studies on optimal pricing policies have received considerable attention these days. Most commonly, it is observed that the demand of an item is inversely proportionate to its retail selling price. Therefore, price varying demand pattern needs to be highlighted and considered. Whitin [25] was the first to discuss inventory system with price dependent demand. Mondal, Bhunia and Maiti [11] also discussed price dependent demand in their study. Thereafter, many inventory models are formulated with price dependent demand like, Mukhopadhyay, Mukherjee and Chaudhary [13], Soni [19], Shah et al. [3], Jaggi et al. [8], Shah et al. [21], Shah et al. [22], Shukla et al. [23], Sundarajan et al. [17], Pareek et al. [22], Shaikh and Cardenas-Barron [23], Giri and Masanta [24] and many more. Cárdenas-Barrón and Sana [2] developed a concavity by Eigen values of hessian matrix.

In this article, with assumption that retailer sells the new item to the customers as well as collects the used item to resell it, the optimal pricing and ordering policy for new item and optimal ordering policy for used item is formulated in order to maximize retailer's total profit per time unit. The demand of an item is considered to be price and time responsive. All the assumptions and notations required to formulate problem mathematically are given in section 2. Mathematical formulation is discussed in section 3. To demonstrate the methodology numerical example is given in sections 4 and 5 and sensitivity analysis is carried out to discuss strategic implications in section 6. We summarize the article in section 7.

2. Assumptions and Notations

2.1 Assumptions

1. The inventory system deals with single item.

2. The replenishment is instantaneous and planning horizon is infinite.

3. The holding cost is considered to be constant for new as well as used product and $h > h_u$.

4. The rate of demand for new product is taken as $R_n(p, t) = ap^{-b}e^{-\varepsilon t}$, $0 \le t \le T$ where, $a > 0$ denotes the scale demand, $0 < b < 1$ denotes the price elasticity and $0 < \varepsilon < 1$.

5. The rate of demand for used buy back product is taken as a

$R_u(p, t) = \alpha(1 - \beta t) - p(1 - p_0)$, $\tau \le t \le T$ where, $a > 0$ denotes the scale demand and $0 < \beta < 1$.

6. The Lead time is negligible or zero and shortages are not allowed.

2.2 Notation

Table 1. List of notations used.

A	Retailer ordering cost (in ₹/order)
C	Purchase cost (Constant) (in ₹/unit)
h	Inventory holding cost (in ₹/unit) for new product
h_u	Inventory holding cost (in ₹/unit) for used buy back product
Q	The replenishment quantity for new product
Q_u	The quantity of used buy back product
T	The length of ordering cycle (a decision variable) (years)
P	Selling Price (in ₹/unit) (a decision variable)
τ	The point of time when collection and sell of used buy back products starts (years) $0 \leq \tau < T$.
$R_n(p, t)$	Demand rate for new product at $0 \leq t < T$. (units)
$R_u(p, t)$	Demand rate for used product at $\tau \leq t < T$. (units)
$I(t)$	Inventory level at time $0 \leq t \leq T$. for new product (units)
$I_u(t)$	Inventory level at time $\tau \leq t \leq T$. for used buy back product (units)
p_0	Rate of discount on selling price for

	used buy back product
d	Rate of depreciation on purchase cost for used buy back product

3. Mathematical Formulation

The inventory level of the new product at time t over the period $[0, T]$ can be represented by the following differential equation,

$$\frac{dI(t)}{dt} = -R_n(p, t),\ 0 \le t \le T \tag{1}$$

At time $t = T$, the inventory level reaches zero i.e. $I(T) = 0$,

The solution of the differential equation (1) is given by

$$I(t) = \frac{ap^{-b}}{\varepsilon}(e^{-\varepsilon t} - e^{-\varepsilon T}),\ 0 \le t \le T \tag{2}$$

But $I(0) = Q$ gives that

$$Q = \frac{ap^{-b}}{\varepsilon}(1 - e^{-\varepsilon T}) \tag{3}$$

Now, for the used product during the period $[\tau, T]$, the inventory level is affected by the return rate of the used product so the governing differential equation for inventory level $I_u(t)$ at any time t,

$$\frac{dI_u(t)}{dt} = -R_u(p, t),\ \tau \le t \le T \tag{4}$$

But used product inventory level also reached zero at time $t = T$ i.e. $I_u(T) = 0$,

The solution of the differential equation (4) is given by

$$I_u(t) = \alpha\left((T - t) - \frac{\beta}{2}(T^2 - t^2)\right) - p(1 - p_0)(T - t) \tag{5}$$

Thus, the quantity of used product given by

$$Q_u = p(1-p_0)(T-t) - \alpha\left((T-\tau) - \frac{\beta}{2}(T^2 - \tau^2)\right) \tag{6}$$

Now to calculate total profit for new product, we calculate all the components as below:

Sales revenue from new product

$$SR_n = \frac{p}{T}\left(\int_0^T ap^{-b}e^{-\varepsilon t}dt\right) \tag{7}$$

Purchase cost for new product

$$PC_n = \frac{CQ}{T} \tag{8}$$

Holding cost for new product

$$HC_n = \frac{1}{T}\int_0^T [h \cdot I(t)]dt \tag{9}$$

Ordering cost

$$OC_n = \frac{A}{T} \tag{10}$$

Thereupon, the total profit for new product during the cycle is from (7) to (10)

$$\pi_n(p, T) = SR_n - OC_n - HC_n - PC_n \tag{11}$$

Now, to calculate total profit from used buy back product, we calculate all the components as below:

Sales revenue from used product

$$SR_u = p\,\frac{p(1-p_0)}{T}\left(\int_\tau^T (\alpha(1-\beta t) - p(1-p_0))dt\right) \tag{12}$$

Purchase cost for used product

$$PC_u = \frac{C(1-d)Q_u}{T-\tau} \tag{13}$$

Holding cost for used product

$$HC_u = \frac{1}{T}\int_{\tau}^{T} [h_u \cdot I_u(t)]dt \tag{14}$$

Thereupon, the total profit for used product during the cycle is

$$\pi_u(p, T) = SR_u - HC_u - PC_u \tag{15}$$

Hence, the total profit from both products is given by from (11) and (15)

$$\pi(p, T) = \pi_n(p, T) + \pi_u(p, T) \tag{16}$$

$$= \frac{ap^{(1-b)}(1-e^{-\varepsilon T})}{T\varepsilon} - \frac{A}{T} - \frac{ha(e^{\varepsilon T} - \varepsilon T - 1)e^{-\varepsilon T}p^{-b}}{\varepsilon^2 T} - \frac{Cap^{-b}(1-e^{-\varepsilon T})}{T}$$

$$+ \frac{1}{T}\left(p(1-p_0)(\alpha - p(1-p_0))(T-\tau) - \frac{1}{2}(p(1-p_0)\alpha\beta(T^2-t^2)\right)$$

$$- \frac{h_u}{T}\left(\frac{1}{6}\alpha\beta(T^2-t^2) + \frac{1}{2}(\alpha - p(1-p_0))(T^3-t^2)\right.$$

$$\left. + (T-\tau)\alpha(T - \frac{1}{2}\beta T^2) - p(1-p_0)T\right)$$

$$- \frac{C(1-d)}{T-\tau}\left(p(1-p_0)(T-\tau) - \alpha(T - \tau - \frac{1}{2}\beta(T^2-t^2)\right)$$

The total profit is a function is a function of selling price p and the replenishment cycle time T. The objective is to find the optimal selling price and the replenishment cycle time such that the retailer's total profit is maximized.

4. Solution Procedure

To obtain the optimal selling price that corresponds to maximizing the total profit, for given T, we first check necessary and sufficient conditions.

The necessary condition for finding the optimal selling price p^* for fix value of T is given as follows:

$$\frac{\partial \pi(p, T)}{\partial p} = -\frac{ap^{1-b}(1-b)}{Tp\varepsilon}[e^{-T\varepsilon} - 1] - \frac{Cap^{-b}b}{Tp\varepsilon}[e^{-T\varepsilon} - 1]$$

$$+\frac{habp^{-b}}{Tp\varepsilon^{2}}[e^{-T\varepsilon}\varepsilon-\varepsilon T-1]$$

$$+\frac{(1-p_0)}{T}\left[-\frac{1}{2}\alpha\beta(T^2-\tau^2)+(\alpha-p(1-p_0))(T-\tau)-p(1-p_0)(T-\tau)\right] \quad (17)$$

$$-C(1-d)(1-p_0)-\frac{h_u(1-p_0)}{2T}[(T^2-\tau^2)-2(T-\tau)]=0$$

Theorem 4.1. *For a given value of T, we have*

I. *The Equation* (17) *has a unique solution.*

II. *The solution in* (i) *satisfies the second-order conditions for the maximum.*

Proof. See Appendix A1.

Now, to obtain the optimal cycle time that correspond to maximising the total profit, for given fix selling price, we first check necessary and sufficient conditions.

The necessary condition for finding the optimal cycle time T^* for fix value of p is given as follows:

$$\frac{\partial\pi(p,T)}{\partial T}=\frac{ab^{1-b}}{T^2\varepsilon}[e^{-\varepsilon T}-1]+\frac{ab^{-b}e^{-\varepsilon T}}{T}$$

$$+hap^{-b}\left[\frac{(-\varepsilon T+e^{\varepsilon T}-1)}{T^2\varepsilon^2}-\frac{(-\varepsilon+\varepsilon e^{\varepsilon T})}{T\varepsilon^2}+\frac{(-\varepsilon T+e^{\varepsilon T}-1)}{T\varepsilon}\right]$$

$$-Cap^{-b}\left[\frac{e^{-\varepsilon T}}{T}+\frac{(e^{-\varepsilon T}-1)}{\varepsilon T^2}\right]+\frac{A}{T^2}+\left(\frac{-p(1-p_0)\alpha\beta T+p(1-p_0)(\alpha-p(1-p_0))}{T}\right)$$

$$-\left(\frac{-\frac{1}{2}p(1-p_0)\alpha\beta(T^2-\tau^2)+p(1-p_0)(\alpha-p(1-p_0))(T-\tau)}{T^2}\right)$$

$$+h_u\left[\left(\frac{-\frac{1}{6}\alpha\beta(T^3-\tau^3)+\frac{1}{2}(-\alpha+p(1-p_0))(T^2-\tau^2)+\alpha((T-\frac{1}{2}\beta T^2)-p(1-p_0))(T-\tau)}{T^2}\right)\right.$$

$$-\left(\frac{-\frac{1}{2}\alpha\beta T^2+(\alpha-p(1-p_0))T+\alpha((-T\beta+1)-p(1-p_0))(T-\tau)+\alpha((T-\frac{1}{2}\beta T^2)-p(1-p_0))T}{T^2}\right)\Bigg]\Bigg]$$

$$+\frac{C(1-d)}{(T-\tau)^2}[p(1-p_0)(T-\tau)-\alpha(T-\tau\frac{1}{2}\beta(T^2-\tau^2)))$$

$$-(p(1-p_0)(T-\tau)-\alpha(-T\beta+1)(T-\tau)))]=0 \tag{18}$$

Theorem 4.2. *For a given value of p, we have*

i. *The Equation* (18) *has a unique solution*.

ii. *The solution in* (i) *satisfies the second-order conditions for the maximum*.

Proof. See Appendix A2.

5. Numerical Example

The proposed models are illustrated below by considering the following example.

The following numerical values of the parameter in proper unit were considered as input for numerical, graphical and sensitivity analysis of the model, $a = 255$, $b = 0.4$, $\alpha = 100$, $\beta = 0.3$, $C = 55$, $A = 100$, $h = 0.5$, $h_u = 0.2$, $d = 0.15$, $\tau = \frac{30}{365}$, $p_0 = 0.5$, $\varepsilon = 0.90$ Using mathematical software like, MATLAB or Mathematica or Maple 18 software, the optimal values of decision variables are obtained as $p^* = 103.8575$₹ and $T^* = 0.3698$ Year.

The optimum quantity of new product and used buy back product is $Q^* = 12.52$ units and $Q^* = 12.52$ units respectively. The maximum profit of retailer is 4978.10₹.

The concavity of the profit function is developed by the well-known Hessian matrix.

Consider Hessian Matrix

$$H(p, T) = \begin{pmatrix} \frac{\partial^2 \pi(p, T)}{\partial p^2} & \frac{\partial^2 \pi(p, T)}{\partial p \partial T} \\ \frac{\partial^2 \pi(p, T)}{\partial T \partial p} & \frac{\partial^2 \pi(p, T)}{\partial T^2} \end{pmatrix} (M)$$

$$H(p^*, T^*) = \begin{pmatrix} -0.5639599422 & -20.64316302 \\ -20.64316302 & -11439.53819 \end{pmatrix}$$

As per Cárdenas-Barron and Sana [2], if the Eigen values of the Hessian matrix at the solution (p^*, T^*) are all negative then the profit function $\pi(p^*, T^*)$ is maximum at the solution. Here, eigenvalues of the above Hessian matrix are $\lambda_1 = -11439.5, \lambda_2 = -0.5267$. Therefore, the profit function $\pi(p^*, T^*)$ is maximum.

From above Hessian Matrix, define that $\Delta_{11} \frac{\partial^2 \pi(p, T)}{\partial p^2}$, $\Delta_{22} = \frac{\partial^2 \pi(p, T)}{\partial T^2}$ and $\Delta_{12} \frac{\partial^2 \pi(p, T)}{\partial p \partial T}$ for optimal value of p^* and T^*, it is clear that $\Delta_{11} = -0.5639599422 < 0, \Delta_{22} = -11439.53819 < 0$ and $\Delta_{11}\Delta_{22} - (\Delta_{12})^2 = 6025.3 > 0$ then the optimal value of p^* and T^* satisfies the equations (17) and (18) and value of p^* and T^* is unique and maximize $\pi(p, T)$.

The concavity of profit function is also shown in Figure 1, Figure 2 and Figure 3 as below:

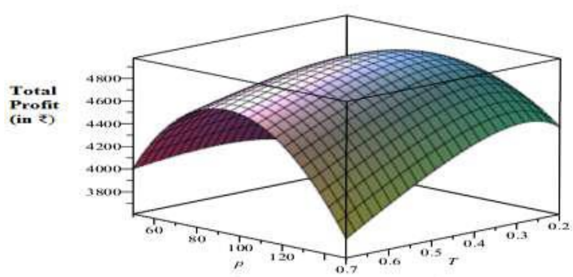
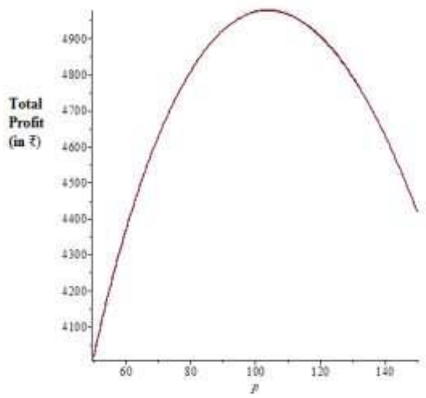


Figure 1. Concavity of total profit function.



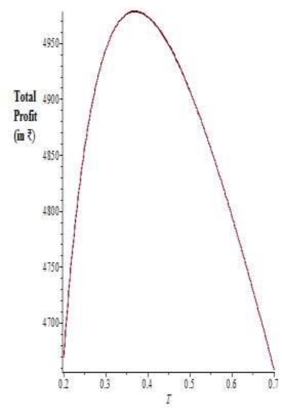


Figure 2. Total Profit Vs Cycle Time. **Figure 3.** Total Profit Vs Selling Price.

6. Sensitivity Analysis

The sensitivity analysis is performed with change in key parameter values in relative steps of –20%, –10%, 10%, 20%, taking one parameter at a time and the remaining values of the parameters are unchanged.

Table 2. Sensitivity with respect to key parameters.

Inventory Parameter	Change % Values	Value	T^*	p^*	Q^*	Q_u^*	Profit (in ₹)	Eigen Value
	-20	204	0.4021	92.83	11.24	14.82	4680.76	(-14678.4, -0.5538)
	-10	229.5	0.3859	98.37	11.94	13.30	4821.11	(-10111.7, -0.5350)
a	0	255	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	280.5	0.3538	109.31	13.00	10.54	5151.26	(-12907.3, -0.5285)
	20	306	0.3381	114.77	13.38	9.29	5340.26	(-14668.4, -0.5057)
	-20	0.32	0.2858	135.26	13.36	5.47	5948.68	(-22437.21, -0.4908)

	-10	0.36	0.3335	116.98	13.23	8.86	5358.99	(-15207.62, -0.5330)
b	0	0.4	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	0.44	0.3982	93.83	11.57	14.50	4726.21	(-9212.12, -0.5882)
	20	0.48	0.4204	85.91	10.53	16.74	4557.53	(-7795.78, -0.6077)
	-20	80	0.3665	91.63	13.07	8.19	3399.35	(-8805.77, -0.6122)
	-10	90	0.3677	97.63	12.77	10.02	4177.53	(-10115.86, -0.5860)
α	0	100	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	110	0.3726	110.30	12.30	13.75	5802.03	(-12780.10, -0.5152)
	20	120	0.3758	116.95	12.10	15.64	6650.24	(-14140.53, -0.5093)
	-20	0.24	0.3997	103.79	13.37	13.44	5100.56	(-9013.07, -0.5702)
	-10	0.27	0.3839	103.83	12.92	12.61	5038.20	(-10205.44, -0.5670)
β	0	0.3	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	0.33	0.3572	103.87	12.16	11.22	4920.01	(-12714.14, -0.5109)
	20	0.36	0.3458	103.88	11.83	10.64	4863.74	(-14028.15, -0.5001)
	-20	44	0.3567	110.44	11.85	10.49	4975.18	(-12552.03, -0.5242)
	-10	49.5	0.3633	107.05	12.19	11.19	4976.90	(-11977.32, -0.5233)
C	0	55	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)

	10	60.5	0.3762	100.84	12.85	12.55	4987.38	(-10934.44, -0.5275)
	20	66	0.3824	97.99	13.18	13.22	5001.41	(-10458.61, -0.5295)
	-20	80	0.3559	104.37	12.10	11.29	5033.22	(-11983.69,-0.5589)
	-10	90	0.3629	104.11	12.31	11.58	5005.39	(-11704.71, -0.5315)
A	0	100	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	110	0.3766	103.61	12.73	12.16	4951.30	(-11188.80, -0.5100)
	20	120	0.3833	103.37	12.93	12.45	4924.98	(-10950.72, -0.5010)
	-20	0.4	0.3699	103.85	12.53	11.88	4978.69	(-11429.29, -0.5266)
	-10	0.45	0.3699	103.85	12.52	1188	4978.39	(-11434.41, -0.5267)
h	0	0.5	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	0.55	0.3697	103.86	12.52	11.87	4977.80	(-11444.7,-0.5267)
	20	0.6	0.3697	103.87	12.52	11.87	4977.50	(-11450.10, -0.5267)
	-20	0.16	0.3699	103.85	12.52	11.88	4978.27	(-11433.5,-0.5267)
	-10	0.18	0.3698	103.85	12.52	11.88	4978.19	(-11436.5, -0.5267)
h_u	0	0.2	0.3698	103.86	12.52	11.88	4978.10	(-11439.5, -0.5267)
	10	0.22	0.3698	103.86	12.52	11.88	4978.01	(-11441.3,-0.5267)
	20	0.24	0.3697	103.86	12.52	11.88	4977.92	(-11444.3, -0.5267)
	-20	0.12	0.3703	102.38	12.61	12.11	5046.83	(-11414.60,

								-0.5329)
	-10	0.135	0.3701	103.12	12.56	11.99	5012.31	(-11426.70, -0.5297)
d	0	0.15	0.3698	103.86	12.52	11.88	4978.10	(-11439.50, -0.5267)
	10	0.165	0.3695	104.60	12.48	11.76	4944.18	(-11453.40, -0.5255)
	20	0.18	0.3693	105.35	12.44	11.64	4910.57	(-11469.35, -0.5243)
	-20	0.0658	0.3428	104.58	11.71	11.52	5099.33	(-12511.00, -0.5442)
	-10	0.0740	0.3565	104.22	12.12	11.71	5037.54	(-11945.78, -0.5352)
τ	0	0.0822	0.3698	103.86	12.52	11.88	4978.10	(-11439.50, -0.5267)
	10	0.0904	0.3827	103.50	12.91	12.03	4920.78	(-10939.30, -0.5132)
	20	0.0986	0.3952	103.16	13.27	12.16	4865.39	(-10309.27, -0.5002)
	-20	0.4	0.3973	83.50	14.51	13.46	4478.27	(-9313.19, -0.7823)
	-10	0.45	0.3839	92.61	13.53	12.69	4711.78	(-10274.10, -0.6480)
p_0	0	0.5	0.3698	103.86	12.52	11.88	4978.10	(-11439.50, -0.5267)
	10	0.55	0.3548	118.08	11.48	10.99	5286.80	(-12398.87, -0.4102)
	20	0.6	0.3385	136.59	10.41	10.01	5651.96	(-13540.70, -0.3202)
	-20	0.72	0.3779	105.14	13.11	11.99	5032.15	(-10810.00, -0.5341)
	-10	0.81	0.3737	104.50	12.80	11.93	5004.57	(-11138.50, -0.5304)
ε	0	0.9	0.3698	103.86	12.52	11.88	4978.10	(-11439.50, -0.5267)

	10	0.99	0.3663	103.23	12.26	11.84	4952.66	(-11710.70, -0.5127)
	20	1.08	0.3632	102.61	12.02	11.81	4928.19	(-1201.15, -0.5012)

7. Observations

To observe the sensitivity of the inventory parameters on the optimal solution, the data provided in the numerical example are considered.

1. Here observed that Eigen values of Hessian Matrix (M) at corresponding value of p^* and T^* all are negative, means that profit is maximize at (p^*, T^*)

2. Optimal selling price increase when system parameters a, α, d and p_0 are increased but if parameters b, C, ε increased then selling price will be decrease. However, selling price almost unchanged for changes in holding cost parameters, τ and β, Admittedly, selling price is highly positive sensitive to a, α, d and p_0 and strongly negative sensitive to b, C, ε.

3. When the value of the parameters a, α, C, τ and p_0 are increase, the optimal total profit will be increase. However, for increasing in parameters b, β, A, d and ε then the total profit will be decrease. Other parameters not more effect on total profit.

4. It is noted that replenishment cycle time is positive related to system parameters b, α, C, A, τ and negative related to a, β, d, ε. However, not much effect in replenishment cycle time for changes in holding cost parameters.

5. Replenishment of optimal order quantity of new product will be increase if the parameters a, α, C, τ are increased but optimal order quantity of new product will be decreased if the parameters b, β and ε are increased.

Appendices

Appendix: 1

Proof of Theorem 4.1. Taking second-order derivative in Equation (17) with respect to p and simplifying terms is given as follows:

$$\frac{\partial^2 \pi(p, T)}{\partial p^2} = \frac{abp^{-b}(e^{-\varepsilon T} - 1)(1-b)}{Tp^2\varepsilon} - \frac{habp^{-b}(1 - e^{-\varepsilon T} - \varepsilon T e^{-\varepsilon T})(b+1)}{Tp^2\varepsilon^2}$$

$$+ \frac{Cabp^{-b}(e^{-\varepsilon T} - 1)(b+1)}{Tp^2\varepsilon} - \frac{2(1-p_0)^2(T-\tau)}{T}$$

Since $e^{-\varepsilon T} - 1 < 0$, $1 - e^{-\varepsilon T} - \varepsilon T e^{-\varepsilon T} > 0$ and $T > \tau$

Hence $\frac{\partial^2 \pi(p, T)}{\partial p^2} < 0$

Hence, the Equation (17) has a unique solution and this satisfies the second-order condition for the maximum.

Appendix: 2

Proof of Theorem 4.2. Taking second-order derivative in Equation (18) with respect to T and simplifying terms is given as follows:

$$\frac{\partial^2 \pi(p, T)}{\partial T^2} = -\frac{2ap^{1-b}}{T^3\varepsilon}\left[\left(1 + T\varepsilon + \frac{T^2\varepsilon^2}{2}\right)e^{-\varepsilon T} - 1\right]$$

$$-\frac{2ap^{-b}}{T}\left[(1 - e^{-\varepsilon T})\left(\frac{C}{\varepsilon T^2}\frac{h}{T\varepsilon} - h\right) - C\left(\frac{1}{T} + \frac{\varepsilon}{2}\right)e^{-\varepsilon T}\right]$$

$$- hap^{-b}(1 - (\varepsilon T - 1)e^{-\varepsilon T})\left[\frac{1}{T} + \frac{2}{T^3\varepsilon^2} + \frac{2}{T^2\varepsilon}\right]$$

$$-\frac{p(1-p_0)}{T^3}[2(\alpha - p(1-p_0))\tau - \alpha\beta(T^2 + \tau^2)]$$

$$-\frac{C(1-d)}{(T-\tau)^2}[0] + \frac{2h_u}{T^2}\left[\left(\frac{\alpha\beta T p(1-p_0)}{2h_u}\right) - (\alpha\beta T(T, \tau+1) + (T, \tau)(p(1-p_0) - \alpha))\right]$$

$$-\frac{2h_u}{T^3}\left(\frac{\alpha\beta}{6}(T^3-\tau^2)+\frac{1}{2}(p(1-p_0)-\alpha)(T^2-\tau^2)+\alpha\left((T\frac{1}{2}\beta T^2)-p(1-p_0)T\right)(T-\tau)\right)$$

$$-\frac{2A}{T^3}$$

For notation convenience, let's take,

$$\Psi_1 = (1+T\varepsilon+\varepsilon+\frac{T^2\varepsilon^2}{2})\varepsilon^{-\varepsilon T-1}$$

$$\Psi_2 = (1-e^{-\varepsilon T})\left(\frac{C}{\varepsilon T^2}-\frac{h}{T\varepsilon}-h\right)-C\left(\frac{1}{T}-\frac{\varepsilon}{2}\right)e^{-\varepsilon T},$$

$$\Psi_3 = 1(\varepsilon T-1)e^{-\varepsilon T},$$

$$\Psi_4 = 2(\alpha-p(1-p_0))\tau-\alpha\beta(T^2+\tau^2),$$

$$\Psi_5 = \left(\frac{\alpha\beta Tp(1-p_0)}{2h_u}\right)-(\alpha\beta T(T-\tau+1)+(T+\tau)(p(1-p_0)-\alpha)),$$

$$\Psi_6 = \frac{\alpha\beta}{6}(T^3-\tau^3)+\frac{1}{2}(p(1-p_0)-\alpha)(T^2-\tau^2)$$
$$+\left(\alpha(T-\frac{1}{2}\beta T^2)-p(1-p_0)T\right)(T-\tau)$$

For the conditions $T \geq 0$, $T > \tau$, $p > C$ and $a > 0$, $0 < b < 1$, $0 < \varepsilon < 1$, $\alpha > 0$, $0 < \beta < 1$.

We have,

$$\Psi_1 > 0 \text{ for } (1+T\varepsilon+\frac{T^2\varepsilon^2}{2})e^{-\varepsilon T} > 1,$$

$$\Psi_2 > 0 \text{ for } (1-e^{-\varepsilon T})\left(\frac{C}{\varepsilon T^2}-\frac{h}{T\varepsilon}-h\right) > C\left(\frac{1}{T}+\frac{\varepsilon}{2}\right)e^{-\varepsilon T},$$

$$\Psi_3 > 0 \text{ for } 1-(\varepsilon T-1)e^{-\varepsilon T} > 0,$$

$$\Psi_4 > 0 \text{ and } 2(\alpha-p(1-p_0))\tau > \alpha\beta(T^2+\tau^2),$$

$$\Psi_5 > 0 \quad \text{for} \quad \left(\frac{\alpha\beta T p(1-p_0)}{2h_u}\right) > (\alpha\beta T(T-\tau+1)+(T+\tau)(p(1-p_0)-\alpha))$$

and

$\Psi_6 > 0$ for

$$\frac{\alpha\beta}{6}(T^3-\tau^3)+\frac{1}{2}(p(1-p_0)-\alpha)(T^2-\tau^2)+\left(\alpha(T-\frac{1}{2}\beta T^2)-p(1-p_0)T\right)(T-\tau)>0.$$

6. Conclusion

In this article, we presented retailer's ordering, pricing and buy back strategy for the items having time and price responsive demand. We assumed that the retailer sells the new product and collects used product to sell it. Mathematical model is formulated to maximize total profit of the retailer by acquiring optimum value of selling price and length of ordering cycle. The optimal selling price, replenishment time, ordering quantity of new product, and optimal buy back quantity of used product are determined using classical optimization. The numerical examples have been solved to validate the proposed model, finally, we made the sensitivity analysis of the parameters on the optimal solutions to derive optimal strategy. The possible extension of this model is to be consider product have a deteriorating nature, advertisement dependent demand, stock dependent demand, trade credit, etc.

References

[1] C. K. Chen, T. C. Weng and C. C. Lo, Optimal replenishment quantity for new products and return rate of used products for a retailer, Applied Mathematical Modelling 40(23-24) (2016), 9754-9766

[2] L. E. Cárdenas-Barrón and S. S. Sana, Multi-item EOQ inventory model in a two-layer supply chain while demand varies with a promotional effort, Appl. Math Model 39(21) (2015) 6725-6737.

[3] B. K. Dey, B. Sarkar, M. Sarkar and S. Pareek, An integrated inventory model involving discrete setup cost reduction, variable safety factor, selling price dependent demand, and investment, RAIRO - Operations Research 53(1) (2019), 39-57.

[4] S. E. Daniel, C. P. Pappis and T. G. Voutsinas, Applying life cycle inventory to reverse supply chains: a case study of lead recovery from batteries, Resources, Conservation and Recycling 37(4) (2003), 251-281.

[5] K. Govindan, H. Soleimani and D. Kannan, Reverse logistics and closed-loop supply

chain: A comprehensive review to explore the future, European Journal of Operational Research 240(3) (2015), 603-626.

[6] B. C. Giri and M. Masanta, Developing a closed-loop supply chain model with price and quality dependent demand and learning in production in a stochastic environment, International Journal of Systems Science: Operations & Logistics 7(2) (2020), 147-163.

[7] P. L. González-Torre and B. Adenso-Díaz, Reverse logistics practices in the glass sector in Spain and Belgium, International Business Review 15(5) (2006), 527-546.

[8] C. K. Jaggi, S. Tiwari and S. K. Goel, Credit financing in economic ordering policies for noninstantaneous deteriorating items with price dependent demand and two storage facilities. Annals of Operations Research 248(1-2) (2016), 253-280.

[9] S. G. Koh, H. Hwang, K. I. Sohn and C. S. Ko, An optimal ordering and recovery policy for reusable items, Computers & Industrial Engineering 43(1-2) (2002), 59-73.

[10] G. Kannan, P. Sasikumar and K. Devika A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling, Applied Mathematical Modelling 34(3) (2010), 655-670.

[11] B. Mondal, A. K. Bhunia and M. Maiti, An inventory system of ameliorating items for price dependent demand rate, Computers & Industrial Engineering 45(3) (2003), 443-456.

[12] M. C. Mabini, L. M. Pintelon and L. F. Gelders, EOQ type formulations for controlling repairable inventories, International Journal of Production Economics 28(1) (1992), 21-33.

[13] S. Mukhopadhyay, R. N. Mukherjee and K. S. Chaudhuri, Joint pricing and ordering policy for a deteriorating inventory, Computers & Industrial Engineering 47(4) (2004) 339-349.

[14] A. Nagurney and F. Toyasaki, Reverse supply chain management and electronic waste recycling: a multitiered network equilibrium framework for e-cycling, Transportation Research Part E: Logistics and Transportation Review 41(1) (2005), 1-28.

[15] R. Pati, P. Vrat and P. Kumar, A goal programming model for paper recycling system, Omega 36(3) (2008), 405-417.

[16] K. Richter and I. Dobos, Production-inventory control in an EOQ-type reverse logistics system, Supply Chain Management and Reverse Logistics (2004), 139-160.

[17] R. Sundararajan, M. Palanivel and R. Uthayakumar, An EOQ model of non-instantaneous deteriorating items with price, time-dependent demand and backlogging, Journal of Control and Decision, (2019), DOI: 10.1080/23307706.2019.1656560.

[18] D. A. Schrady, A deterministic inventory model for reparable items, Naval Research Logistics Quarterly 14(3) (1967), 391-398.

[19] H. N. Soni, Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment, International Journal of Production Economics 146(1) (2013), 259-268.

[20] N. H. Shah U. Chaudhari and M. Y. Jani, Optimal Policies for Time-Varying Deteriorating Item with Preservation Technology Under Selling Price and Trade Credit

Dependent Quadratic Demand in a Supply Chain, International Journal of Applied and Computational Mathematics 3(2) (2016), 363-379.

[21] N. H. Shah and C. R. Vaghela, Retailer's replenishment and pricing decisions for noninstantaneous deterioration and price-dependent demand, International Journal of Industrial Engineering & Production Research 28(2) (2017), 101-111.

[22] N. H Shah and C. R. Vaghela, Retailer's optimal pricing and replenishment policy for new product and optimal take-back quantity of used product, Yugoslav Journal of Operations Research 28(3) (2018), 345-353.

[23] M. S. Suthar and K. T. Shukla, Pricing and ordering policy for deteriorating inventory system with price sensitive ramp type demand, pre and post deterioration discounts, Ravista Investigacion Operacional 40(2) (2019), 267-274.

[24] A. A. Shaikh and L. E. Cárdenas-Barrón, An EOQ model for non instantaneous deteriorating product s with advertisement and price sensitive demand under order quantity dependent trade credit, Ravista Investigacion Operacional 41(2) (2020), 168-187.

[25] T. M. Whitin, Inventory Control and Price Theory, Management Science 2(1) (1955), 61-68.