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LINGUISTIC INTUITIONISTIC FUZZY VIKOR METHOD FOR MAGDM PROBLEMS WITH SENSITIVITY ANALYSIS

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Abstract

Multiple Attribute Group Decision Making (MAGDM) problems are gaining much importance in the recent days and the application of VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje) method in such problems is unavoidable. The methodology proposed in this article deals with the data sets taking the form of Linguistic Intuitionistic Fuzzy sets (LIFSs) and also deals with the dissension between the data sets which is nothing but the alternatives available in the decision problem. Some novel arithmetic operations are defined for LIFSs and utilised in the VIKOR method. Numerical example with effective illustration for the proposed algorithm is also given. Weight determining methods based on entropy are proposed along with sensitivity analysis for the weighting vectors and comparisons are made for the final ranking of the alternatives. The final ranking of the alternatives are consistent with regard to the sensitivity analysis proposed in this work.

1. Introduction

Multiple Attribute Group Decision Making problems are bottle neck problems in Decision Support Systems (DSS) and plays a major role in Artificial Intelligence and Machine Learning. The VIKOR (Višekriterijumsko Kompromisno Rangiranje) method is one such DSS which operates with the concept of dissension between data sets where attributes with difference of

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opinions are largely involved. The VIKOR method of DSS is mainly intended to select and rank the best alternative out of the available ones in any decision system whenever attributes with difference of opinion are involved. Any decision making problem will concentrate its methodology based on the ranking methods done by measuring the closeness to the positive or negative ideal solution [2,4,5,6,8,11,14]. In recent days, linguistic intuitionistic fuzzy data has gained the attention of researchers to a large extent [3,7,10,14]. After Zadeh [12] introducing the concept of fuzzy sets, researchers have taken the modeling of uncertainty to various levels including linguistic fuzzy and then to different levels of linguistic intuitionistic fuzzy sets. The VIKOR method will be further developed in this research study, coupled with a methodology for solving MAGDM problems as well as weight determining methodologies for an efficient DSS. The study of how the uncertainty in the output of a mathematical model or system might be partitioned and distributed to various sources of uncertainty in its inputs is termed as sensitivity analysis [1, 9, 13]. In this work we have proposed some arithmetic operations for LIFNs and utilised them in computations in the VIKOR method. Six different computations are performed with the available information and where sensitivity analysis is much involved and comparisons in the final ranking are made. The study reveals that our new method is less sensitive to the changes allowed in the weight vectors derived from the entropy method.

2. Road map to the basic concepts of LIFS

2.1. The concept of Linguistic Intuitionistic Fuzzy Numbers (LIFNs)

In LIFN, unlike the other fuzzy data sets, the membership degree and non-membership degree of the linguistic term is the representation. The definition is shown as follows.

Definition 1. Consider the set $S = \{\delta_1, \delta_2, \ldots, \delta_l\}$, where l is the odd number and is a finite and completely ordered discrete term set in the vast majority of real-world scenarios, we take values such as 3,5,7,9, and so on. For illustration, when a set S has been given as follows: $S = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\} =$ {worst, slightly worst, better, slightly good, good}.

Generally, for any linguistic set S, δ_i and δ_j must meet certain requirements:

1. The set S is an ordered set: That is $\delta_i \prec \delta_j$ if and only if $i < j$;

2. The inverting operatorexists and is given as: $inv(\delta_i) = \delta_{\iota - i}$;

3. The utmost operator is defined as follows, $mos(\delta_i, \delta_j) = \delta_i$, if $i \geq j$;

4. The lowest operator is as follows: $low(\delta_i, \delta_j) = \delta_i$, if $i \leq j$;

If the linguistic information has to be preserved, then the set $S = \{\delta_1, \delta_2, \ldots, \delta_l\}$ should be stretched to a continuous linguistic set $\overline{S} = \{\delta_\kappa, \kappa \in R\}$ which is observed to satisfy the above four conditions. The preceding are among the arithmetic operations:

i. $\beta\delta_i = \delta_{\beta \times i}$ ii. $\delta_i \oplus \delta_j = \delta_{i+j}$ iii. $\delta_i / \delta_j = \delta_{i/j}$ iv. $(\delta_i)^n = \delta_{i^n}$ v. $\lambda(\delta_i \oplus \delta_j) = \lambda\delta_i \oplus \lambda\delta_j$ vi. $(\lambda_1 + \lambda_2)\delta_i = \lambda_1\delta_i \oplus \lambda_2\delta_i$.

Definition 2. The Linguistic Intuitionistic Fuzzy Set (LIFS) is defined as follows:

Let $A = \{\langle x[\delta_{\theta(x)}, (\alpha_A(x), \gamma_A(x)]\rangle : x \in X\}$, where $\delta_{\theta(x)} \in \overline{S}$, $\alpha_A(x) : X \to [0, 1]$ and $\gamma_A(x) : X \to [0, 1]$, $\alpha_A(x)$ and $\gamma_A(x)$ satisfying $0 \leq \alpha_A(x) + \gamma_A(x) \leq 1, \forall x \in X$. The numbers $\alpha_A(x)$ is the grade of membership and $\gamma_A(x)$ is the grade of non-membership of the element x to the linguistic index $\delta_{\theta(x)}$. In X, for every LIFS A, indeterminacy of x to the linguistic index $\delta_{\theta(x)}$ is given as $\eta(x) = 1 - \alpha_A(x) - \gamma_A(x), \forall x \in X$, $0 \leq \eta(x) \leq 1, \forall x \in X$.

Definition 3. Let $A = \{\langle x[\delta_{\theta(x)}, (\alpha_A(x), \gamma_A(x))]\rangle : x \in X\}$ be a LIFS. The ternary group is then referred to as a linguistic intuitionistic Fuzzy Number (LIFN), and A can be thought of as a collection of LIFNs.

Definition 4. Let $\widetilde{\sigma}_1 = \langle \delta_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1))\rangle$ and $\widetilde{\sigma}_2 = \langle \delta_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2))\rangle$ be two LIFNs and $\lambda \geq 0$. Then the operations of

LIFNs are defined as follows:

$$\tilde{\sigma}_1 + \tilde{\sigma}_2 = \langle \delta_{\theta(\sigma_1)+\theta(\sigma_2)}, (\alpha(\sigma_1) + \alpha(\sigma_2) - \alpha(\sigma_1)\alpha(\sigma_2), \gamma(\sigma_1)\gamma(\sigma_2))\rangle,$$

$$\tilde{\sigma}_1 \otimes \tilde{\sigma}_2 = \langle \delta_{\theta(\sigma_1)\times\theta(\sigma_2)}, (\alpha(\sigma_1)\alpha(\sigma_2), \gamma(\sigma_1) + \gamma(\sigma_2) - \gamma(\sigma_1)\gamma(\sigma_2))\rangle,$$

$$\lambda\tilde{\sigma}_1 = \langle \delta_{\lambda\times\theta(\sigma_1)}, (1 - (1 - \alpha(\sigma_1))^{\lambda}, (\gamma(\sigma_1))^{\lambda})\rangle, \text{ and}$$

$$\tilde{\sigma}_1^{\lambda} = \langle \delta_{\theta(\sigma_1)^{\lambda}}, (\alpha(\sigma_1))^{\lambda}, 1 - (1 - (\gamma(\sigma_1))^{\lambda})\rangle.$$

We introduce two new LIFN operations in the following section, which will be useful for further computations in this paper.

Definition 5. For any two intuitionistic linguistic numbers $\tilde{\sigma}_1 = \langle \delta_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)\rangle$ and $\tilde{\sigma}_2 = \langle \delta_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)\rangle,$ we have introduced subtraction and division as shown below:

$$\tilde{\sigma}_1 - \tilde{\sigma}_2 = \begin{cases} \left\langle \delta_{|\theta(\sigma_1)-\theta(\sigma_1)|}, \left(\dfrac{\alpha(\sigma_1) - \alpha(\sigma_2)}{1 - \alpha(\sigma_2)}, \dfrac{\gamma(\sigma_1)}{\gamma(\sigma_2)} \right) \right\rangle & \text{if } 0 \le \dfrac{\gamma(\sigma_1)}{\gamma(\sigma_2)} \le \dfrac{1 - \alpha(\sigma_1)}{1 - \alpha(\sigma_2)} \le 1 \\ \langle \delta_{|\theta(\sigma_1)-\theta(\sigma_1)|}, (0, 1)\rangle & \text{otherewise} \end{cases} \tag{1}$$

$$\tilde{\sigma}_1 / \tilde{\sigma}_2 = \begin{cases} \left\langle \delta_{\theta(\sigma_1)/\theta(\sigma_1)|}, \left(\dfrac{\alpha(\sigma_1)}{\alpha(\sigma_2)}, \dfrac{\gamma(\sigma_1) - \gamma(\sigma_2)}{1 - \gamma(\sigma_2)} \right) \right\rangle & \text{if } 0 \le \dfrac{\alpha(\sigma_1)}{\alpha(\sigma_2)} \le \dfrac{1 - \alpha(\sigma_1)}{1 - \alpha(\sigma_2)} \le 1 \\ \langle \delta_{|\theta(\sigma_1)-\theta(\sigma_1)|}, (0, 1)\rangle & \text{otherewise} \end{cases} \tag{2}$$

Next we propose a novel formula for comparing LIFNs which defuzzifies the linguistic characterization and the intuitionistic characterization.

Definition 6. Let $A = \{\langle x[\delta_{\theta(x)}, (\alpha_A(x), \gamma_A(x))]\rangle : x \in X\},$ where $\delta_{\theta(x)} \in \overline{S},$ as in definition 2. Then the Linguistic Median Membership (LMM) function is defined as follows:

$$M_m = \{\theta(x) + (\alpha_A(x) + 1 - \gamma_A(x))\}/2.$$

Where $\theta(x)$ is the linguistic characterization and $\alpha_A(x), \gamma_A(x)$ are the grade of membership and grade of non-membership of the intuitionistic fuzzy characterization respectively.

This LMM is also used to defuzzify the LIFNs into a crisp value.

Theorem 1. *For any two LIFNs* $\tilde{\sigma}_1 = \langle \delta_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1))\rangle$ *and* $\tilde{\sigma}_2 = \langle \delta_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2))\rangle$ *the computational rules are given as follows*:

(i) $\tilde{\sigma}_1 + \tilde{\sigma}_2 = \tilde{\sigma}_2 + \tilde{\sigma}_1$, (ii) $\tilde{\sigma}_1 \otimes \tilde{\sigma}_2 = \tilde{\sigma}_2 \otimes \tilde{\sigma}_1$, (iii) $\lambda(\tilde{\sigma}_1 + \tilde{\sigma}_2) = \lambda\tilde{\sigma}_2 + \lambda\tilde{\sigma}_1$, $\gamma \geq 0$, (iv) $\lambda_1\tilde{\sigma}_1 + \lambda_2\tilde{\sigma}_1 = (\lambda_1 + \lambda_2)\tilde{\sigma}_1, \lambda_1, \lambda_2 \geq 0$, (v) $\tilde{\sigma}_1^{\lambda_1} \otimes \tilde{\sigma}_1^{\lambda_2} = (\tilde{\sigma}_1)^{\lambda_1 + \lambda_2}$, $\lambda_1, \lambda_2 \geq 0$, (vi) $\tilde{\sigma}_1^{\lambda_1} \otimes \tilde{\sigma}_2^{\lambda_1} = (\tilde{\sigma}_1 \otimes \tilde{\sigma}_2)^{\lambda_1}$, $\lambda_1 \geq 0$.

3. Linguistic Intuitionistic Fuzzy VIKOR Method with Entropy

3.1 The General VIKOR method. Most of the real life MAGDM problems are very much complex in nature. To solve many such complex problems, the role of decision makers is to provide a consensus in the ranking process to enhance and ensure a better decision. Opricovicand Tzeng [5,6] supported the concept of compromise solution to alleviate pressure in the decision-making process. The method suggested by them [5,6] is based on finding the feasible solution which is decided by its nature of being close to the ideal solution of the decision problem. The VIKOR method's functioning rule is as follows:

Step 1. Identify the problem's finest attribute q_j^+ and worst attributes q_j^-.

Step 2. Calculate A_i and B_i's values given as follows:

$$A_i = \sum_{j=1}^{M} w_j([q_j^+ - q_{ij}]/[q_j^+ - q_j^-]); B_i = \underset{j}{Max}\{w_j([q_j^+ - q_{ij}]/[q_j^+ - q_j^-])\},$$

$$j = 1, 2, \ldots, M. \tag{3}$$

Step 3. Compute $C_i = v\dfrac{(A_i - A^*)}{(A_i - A^*)} + (1 - v)\dfrac{(B_i - B^*)}{(B_i - B^*)}$, where A^- is the utmost value of A^-, and A^* the lowest value of $A_i; B^-$ is the utmost value of B_i, and B^* is the lowest value of the 'majority of criteria' approach has a weight of v. v has a value of 0 to 1.

Step 4. Three ranking list should be prepared based on A_i, B_i and C_i. The option with the lowest C_i value is picked as the main opportunity.

Step 5. Provide a reasonable compromise for attribute weights, and choose the optimal option, which is the one ranked highest by measure C as given in [5,6].

The theorem given below discusses about the changes in the weights of attributes:

Theorem 2. *If the* p^{th} *attribute's weight is modified to* Δ_p, *the weight of all other attributes changes by.*

$$\Delta_j = (\Delta_p \cdot w_j)/(w_p - 1); j = 1, 2, \ldots, k, j \neq p.$$

3.2 The Entropy Method to determine the weight of each Indicators

The method of finding weights of the attributes by entropy function is given as follows.

Step 1. Calculate $D_{ij} = g_{ij}/\sum_{j=1}^{m} g_{ij}$, r_{ij} is the i^{th} schemes j^{th} indicators value.

Step 2. Estimate the entropy value $ej, e_j = -k\sum_{i=1}^{m} D_{ij} \ln(D_{ij})$, $k = 1/\ln m$, where the number of systems is m.

Step 3. Estimate $w_j = (1 - e_j)/\sum_{j=1}^{n}(1 - e_j)$, the set of aspects is denoted by the letter n, and $0 \leq w_j \leq 1$, $\sum_{j=1}^{n} w_j = 1$.

4. MAGDM Problem–VIKOR Method. Numerical Illustration

A person seeking a better investing strategy wants to put money into an investment firm, and there are four options available. The investment firm consults with experts DM_1, DM_2, and DM_3 to evaluate the alternatives against the attributes R_1, R_2, R_3, and R_4, where R_1 denotes Market condition, R_2 denotes Stability, R_3 denotes Customer Satisfaction, and R_4 denotes the new facilities of the firm. The four options are to be weighed by the decision makers using linguistic intuitionistic fuzzy numbers, whose weighting vector is created using the entropy method. The decision makers have compromised on the weight vector and have used the entropy method to deliver weight information. Table 1 shows the decision matrix along with the LIFNs.

Table 1. The value of indicators.

Alternatives	Market condition R_1	Stability R_2	Customer Satisfaction R_3	New Facilities
S_1	$\langle\delta_5, (0.2, 0.7)\rangle$	$\langle\delta_2, (0.4, 0.6)\rangle$	$\langle\delta_5, (0.5, 0.5)\rangle$	$\langle\delta_3, (0.2, 0.6)\rangle$
S_2	$\langle\delta_4, (0.4, 0.6)\rangle$	$\langle\delta_5, (0.4, 0.5)\rangle$	$\langle\delta_3, (0.1, 0.8)\rangle$	$\langle\delta_4, (0.5, 0.5)\rangle$
S_3	$\langle\delta_3, (0.2, 0.7)\rangle$	$\langle\delta_2, (0.2, 0.7)\rangle$	$\langle\delta_4, (0.3, 0.7)\rangle$	$\langle\delta_5, (0.2, 0.7)\rangle$
S_4	$\langle\delta_6, (0.5, 0.4)\rangle$	$\langle\delta_2, (0.2, 0.8)\rangle$	$\langle\delta_3, (0.2, 0.6)\rangle$	$\langle\delta_3, (0.3, 0.6)\rangle$

Computation 1. VIKOR method with known weights

Assign the known weights $w' = (0.2345, 0.26664, 0.026723, 0.472138)$ to each indicators and use the cost type indicators: $V_{ij} = (\min x_{ij}/x_{ij})$ and the benefit type indicators: $V_{ij} = (x_{ij}/\max x_{ij})$ to derive the following decision-matrix:

$$v = \begin{pmatrix} \langle\delta_{0.833}, (0.4, 0.5)\rangle & \langle\delta_{0.333}, (0, 1)\rangle & \langle\delta_{0.833}, (0, 1)\rangle & \langle\delta_{0.5}, (0, 4, 0.333)\rangle \\ \langle\delta_{0.667}, (0, 1)\rangle & \langle\delta_{0.833}, (0.8, 0.167)\rangle & \langle\delta_{0.5}, (0.2, 0.667)\rangle & \langle\delta_{0.667}, (0, 1)\rangle \\ \langle\delta_{0.5}, (0.4, 0.5)\rangle & \langle\delta_{0.667}, (0.4, 0.5)\rangle & \langle\delta_{0.667}, (0, 1)\rangle & \langle\delta_{0.833}, (0.4, 0.51)\rangle \\ \langle\delta_{1}, (0, 1)\rangle & \langle\delta_{0.333}, (0, 1)\rangle & \langle\delta_{0.5}, (0.4, 0.333)\rangle & \langle\delta_{0.5}, (0.6, 0.333)\rangle \end{pmatrix}$$

Positive-ideal and negative-ideal solutions are, respectively

$$q^* = (\langle\delta_1, (1, 0)\rangle, \langle\delta_{0.833}, (0.8, 0.167)\rangle, \langle\delta_{0.5}, (1, 4, 0.333)\rangle, \langle\delta_{0.5}, (0.6, 0.333)\rangle).$$

$$q^- = (\langle\delta_{0.667}, (1, 0)\rangle, \langle\delta_{0.333}, (0, 1)\rangle, \langle\delta_{0.667}, (0, 1)\rangle, \langle\delta_{0.667}, (0, 1)\rangle).$$

Calculate the A, B, C value for all the alternatives in the decision problem:

$$A_i = \sum_{i=1}^{n} (w_i(q_i^* - q_{ij})/(q_i^* - q_i^-)),\ B_i = \max[w_i(q_i^* - q_{ij})/q_i^-)]$$

$A_1 = \langle \delta_{1.0251}, (1, 0) \rangle,\ A_2 = \langle \delta_{0.7066}, (0, 1) \rangle,\ A_3 = \langle \delta_{1.2928}, (0, 1) \rangle,$

$A_4 = \langle \delta_{0.2666}, (0, 1) \rangle.$

$B_1 = \langle \delta_{0.2666}, (1, 0) \rangle,\ B_2 = \langle \delta_{0.4721}, (0, 1) \rangle,\ B_3 = \langle \delta_{0.7082}, (0, 1) \rangle,$

$B_4 = \langle \delta_{0.2666}, (0, 1) \rangle.$

And then, $C_j = v(A_i - A^*)/(A^- - A^*) + (1 - v)(B_i - B^*)/(B^- - B^*),$

where $A^* = low\, A_j; A = mos\, A_j; B^* = low\, B_j; B = mos\, B_j,$

$A^* = \langle \delta_{0.2666}, (1, 0) \rangle,\ A^- = \langle \delta_{1.2928}, (1, 0) \rangle,\ B^* = \langle \delta_{0.2666}, (1, 0) \rangle,$

$B^- = \langle \delta_{0.7082}, (1, 0) \rangle.$

$C_1 = v(A_1 - A^*)/(A^- - A^*) + (1 - v)(B_i - B^*)/(B^- - B^*),$

$C_1 = \langle \delta_{0.3696}, (1, 0) \rangle, C_2 = \langle \delta_{0.4471}, (1, 0) \rangle, C_3 = \langle \delta_1, (1, 0) \rangle, C_4 = \langle \delta_0, (0, 1) \rangle.$

Table 2. The evaluation value of each Alternative with known weights.

	S_1	S_2	S_3	S_4
A	$\langle \delta_{1.0251}, (1, 0) \rangle$	$\langle \delta_{0.7066}, (1, 0) \rangle$	$\langle \delta_{1.2982}, (1, 0) \rangle$	$\langle \delta_{0.2666}, (1, 0) \rangle$
B	$\langle \delta_{0.2666}, (1, 0) \rangle$	$\langle \delta_{0.4721}, (1, 0) \rangle$	$\langle \delta_{0.7082}, (1, 0) \rangle$	$\langle \delta_{0.2666}, (1, 0) \rangle$
C	$\langle \delta_{0.3696}, (1, 0) \rangle$	$\langle \delta_{0.4471}, (0, 1) \rangle$	$\langle \delta_1, (0, 1) \rangle$	$\langle \delta_0, (0, 1) \rangle$

Table 3. Ranking the alternatives by VIKOR method with known weights.

	S_1	S_2	S_3	S_4
A	3	2	4	1
B	2	3	4	1
C	2	3	4	1

Condition 1. As per $C(A_2) - C(A_1) \geq 1/(m-1)$.

(where $C(A_2)$ is the suboptimal option in C rank table and C's VIKOR evaluation value), we have, $C(S_4) - C(S_1) = 0.1842 = 0.1842 < 1/(4-1)$ (here, $m = 4$) Condition 1 is not met.

Condition 2. S_4 is the best rank by A, B and C.

Condition 2 has been met. Since the Condition 1 is not satisfied, According to the VIKOR method, the ultimate ranking result is $S_4 > S_1 > S_2 > S_3$. Hence, the best supplier is S_4.

Computation 2. VIKOR method with weights from Entropy method

Calculate $D_{ij} = g_{ij}/\sum_{j=1}^{m} g_{ij}$, where g_{ij} is computed from the matrix in Table 1.

$$g_{ij} = \begin{pmatrix} 2.75 & 1.4 & 3 & 1.8 \\ 2.4 & 2.95 & 1.65 & 2.5 \\ 1.75 & 2.25 & 2.3 & 2.75 \\ 3.55 & 1.2 & 1.8 & 1.85 \end{pmatrix}$$

and hence

$$D_{ij} = \begin{pmatrix} 0.3073 & 0.1564 & 0.3352 & 0.2011 \\ 0.2526 & 0.3105 & 0.1737 & 0.2632 \\ 0.1934 & 0.2486 & 0.2541 & 0.3039 \\ 0.4226 & 0.1429 & 0.2143 & 0.2202 \end{pmatrix}$$

Calculate the entropy value $e_j = -k\sum_{i=1}^{m} D_{ij} \ln(D_{ij})$, $k = \frac{1}{\ln(m)}$, where $m = 4$, $k = 1/\ln(4)$, $k = 0.72135$ and hence estimate the weights $w_j = (1 - e_j)/\sum_{j=1}^{n}(1 - e_j)$.

$w_1 = 0.28173$, $w_2 = 0.12775$, $w_3 = 0.07855$, $w_4 = 0.51197$. It can be easily seen that $\sum W_j = 1$.

Hence the weights calculated by entropy method is

$w' = (0.28173, 0.12775, 0.07855, 0.51197)$.

Proceeding Step 1 to Step 4 as in the previous computations with the weights calculated above from Entropy method, then we have:

$$v = \begin{pmatrix} \langle\delta_{0.833}, (0.4, 0.5)\rangle & \langle\delta_{0.333}, (0, 1)\rangle & \langle\delta_{0.833}, (0, 1)\rangle & \langle\delta_{0.5}, (0, 4, 0.333)\rangle \\ \langle\delta_{0.667}, (0, 1)\rangle & \langle\delta_{0.833}, (0.8, 0.167)\rangle & \langle\delta_{0.5}, (0.2, 0.667)\rangle & \langle\delta_{0.667}, (0, 1)\rangle \\ \langle\delta_{0.5}, (0.4, 0.5)\rangle & \langle\delta_{0.667}, (0.4, 0.5)\rangle & \langle\delta_{0.667}, (0, 1)\rangle & \langle\delta_{0.833}, (0.4, 0.51)\rangle \\ \langle\delta_{1}, (0, 1)\rangle & \langle\delta_{0.333}, (0, 1)\rangle & \langle\delta_{0.5}, (0.4, 0.333)\rangle & \langle\delta_{0.5}, (0.6, 0.333)\rangle \end{pmatrix}$$

The positive-ideal and negative-ideal solutions are,

$$q^* = (\langle\delta_1, (1, 0)\rangle, \langle\delta_{0.833}, (0.8, 0.167)\rangle, \langle\delta_{0.5}, (0, 4, 0.333)\rangle, \langle\delta_{0.5}, (0.6, 0.333)\rangle).$$

$$q^- = (\langle\delta_{0.667}, (1, 0)\rangle, \langle\delta_{0.333}, (0, 1)\rangle, \langle\delta_{0.667}, (0, 1)\rangle, \langle\delta_{0.667}, (0, 1)\rangle).$$

Calculate A, B, C value for each alternative:

$A_1 = \langle\delta_{1.4497}, (1, 0)\rangle$, $A_2 = \langle\delta_{0.7937}, (1, 0)\rangle$, $A_3 = \langle\delta_{1.5677}, (1, 0)\rangle$,

$A_4 = \langle\delta_{0.1278}, (1, 0)\rangle$.

$B_1 = \langle\delta_{0.1571}, (1, 0)\rangle$, $B_2 = \langle\delta_{0.5120}, (1, 0)\rangle$, $B_3 = \langle\delta_{0.4226}, (1, 0)\rangle$,

$B_4 = \langle\delta_{0.1278}, (1, 0)\rangle$.

$A^* = \langle\delta_{0.1278}, (1, 0)\rangle$, $A^- = \langle\delta_{1.5677}, (1, 0)\rangle$, $B = \langle\delta_{0.1278}, (1, 0)\rangle$,

$B^- = \langle\delta_{0.5120}, (1, 0)\rangle$.

$C_1 = \langle\delta_{0.4972}, (0, 1)\rangle$, $C_2 = \langle\delta_{0.7312}, (0, 1)\rangle$, $C_3 = \langle\delta_{0.8837}, (0, 1)\rangle$,

$C_4 = \langle\delta_0, (0, 1)\rangle$.

Table 4. The evaluation value of each Alternative with Entropy weights.

	S_1	S_2	S_3	S_4
A	$\langle\delta_{1.4497}, (1, 0)\rangle$	$\langle\delta_{0.7937}, (1, 0)\rangle$	$\langle\delta_{1.5677}, (1, 0)\rangle$	$\langle\delta_{0.1278}, (1, 0)\rangle$
B	$\langle\delta_{0.1571}, (1, 0)\rangle$	$\langle\delta_{0.5120}, (1, 0)\rangle$	$\langle\delta_{0.4226}, (1, 0)\rangle$	$\langle\delta_{0.1278}, (1, 0)\rangle$
C	$\langle\delta_{0.4972}, (1, 0)\rangle$	$\langle\delta_{0.7312}, (0, 1)\rangle$	$\langle\delta_{0.8837}, (0, 1)\rangle$	$\langle\delta_0, (0, 1)\rangle$

Table 5. Ranking the alternatives by VIKOR method with Entropy weights.

	S_1	S_2	S_3	S_4
A	3	2	4	1
B	2	4	3	1
C	2	3	4	1

The ultimate ranking result according to the VIKOR method is as follows: $S_4 > S_1 > S_2 > S_3$. Hence the best alternative is S_4.

5. Sensitivity Analysis for the VIKOR Method

Under sensitivity analysis, the process of recalculating outcomes under various assumptions to identify the effect of a variable can be valuable for a variety of reasons. In this work, the sensitivity analysis is done on the weight vectors derived from entropy method. The weights calculated by entropy method is $w' = (0.28173, 0.12775, 0.07855, 0.51197)$. Now let us analyse the change in the output when changes are allowed in all the vectors of the weights. Let us allow the small change, $\Delta_1 = 0.052$ for the first vector of the weights.

Computation 3. VIKOR method with changed weights starting with w_1 from Entropy method

Let $w_1' = w_1 + \Delta_1 = 0.28172 + 0.052 = 0.33372$. Then the change in the other vectors of the weights is given as:

$$w_j' = ((1 - w_1')/(1 - w_1))w_j = ((1 - 0.33372)/(1 - 0.28172))w_j = 0.92760\, w_j.$$

Hence we obtain the new weight vector as: $w' = (0.33372, 0.11850, 0.07287, 0.47491)$.

Proceeding Step 1 to Step 4 with the new changed weight vector $w' = (0.33372, 0.11850, 0.07287, 0.47491)$ as in previous computations, then we have:

Table 6. The evaluation value of each Alternative with Entropy weights (Sensitivity, 1st weight).

	S_1	S_2	S_3	S_4
A	$\langle \delta_{1.3809}, (1, 0)\rangle$	$\langle \delta_{0.8086}, (1, 0)\rangle$	$\langle \delta_{1.5628}, (1, 0)\rangle$	$\langle \delta_{0.1185}, (1, 0)\rangle$
B	$\langle \delta_{0.1669}, (1, 0)\rangle$	$\langle \delta_{0.4749}, (1, 0)\rangle$	$\langle \delta_{0.5006}, (1, 0)\rangle$	$\langle \delta_{0.1185}, (1, 0)\rangle$
C	$\langle \delta_{0.5003}, (0, 1)\rangle$	$\langle \delta_{0.7053}, (0, 1)\rangle$	$\langle \delta_{1}, (0, 1)\rangle$	$\langle \delta_{0}, (0, 1)\rangle$

Table7. Ranking the alternatives by VIKOR method with Entropy weights (Sensitivity, 1st weight).

	S_1	S_2	S_3	S_4
A	3	2	4	1
B	2	3	4	1
C	2	3	4	1

The ultimate ranking result according to the VIKOR method is as follows: $S_4 > S_1 > S_2 > S_3$. Hence the best alternative is S_4.

Computation 4. VIKOR method with changed weights starting with w_2 from Entropy method

Let $w_2' = w_2 + \Delta_1 = 0.12775 + 0.052 = 0.17975$. Then similar to the computations done in computation-3 and the changed weight vector is as follows: $w' = (0.26493, 0.17975, 0.07387, 0.48145)$.

Proceeding Step 1 to Step 4 with the new changed weighting vector as in previous computations, then we have:

Table 8. The evaluation value of each Alternative with Entropy weights (Sensitivity, 2nd weight).

	S_1	S_2	S_3	S_4
A	$\langle \delta_{1.4229}, (1, 0)\rangle$	$\langle \delta_{0.7464}, (1, 0)\rangle$	$\langle \delta_{0.4941}, (1, 0)\rangle$	$\langle \delta_{0.1798}, (1, 0)\rangle$
B	$\langle \delta_{0.1798}, (1, 0)\rangle$	$\langle \delta_{0.4814}, (1, 0)\rangle$	$\langle \delta_{0.3974}, (1, 0)\rangle$	$\langle \delta_{0.1798}, (1, 0)\rangle$

C	$\langle \delta_{0.4729}, (0, 1) \rangle$	$\langle \delta_{0.7156}, (0, 1) \rangle$	$\langle \delta_{0.8607}, (0, 1) \rangle$	$\langle \delta_{0}, (0, 1) \rangle$

Table 9. The VIKOR method with Entropy weights is used to rank the alternatives (Sensitivity, 2nd weight).

	S_1	S_2	S_3	S_4
A	3	2	4	1
B	2	4	3	1
C	2	3	4	1

The ultimate ranking result according to the VIKOR method is as follows: $S_4 > S_1 > S_2 > S_3$. Hence the best alternative is S_4.

Computation 5. VIKOR method with changed weights starting with w_3 from Entropy method

Let $w_3' = w_3 + \Delta_1 = 0.07856 + 0.052 = 0.13056$.

Then changes in the other vectors of the weights are made similar to the computations done in computation 3 and the changed weight vector is as follows: $w' = (0.26582, 0.12054, 0.13056, 0.48308)$.

Proceeding Step 1 to Step 4 with the new changed weighting vector $w' = (0.26582, 0.12054, 0.13056, 0.48308)$ as in previous computations, then we have:

Table 10. The evaluation value of each Alternative with Entropy weights (Sensitivity, 3rd weight).

	S_1	S_2	S_3	S_4
A	$\langle \delta_{1.4807}, (1, 0) \rangle$	$\langle \delta_{0.7489}, (1, 0) \rangle$	$\langle \delta_{1.5356}, (1, 0) \rangle$	$\langle \delta_{0.1205}, (1, 0) \rangle$
B	$\langle \delta_{0.2611}, (1, 0) \rangle$	$\langle \delta_{0.4831}, (1, 0) \rangle$	$\langle \delta_{0.3987}, (1, 0) \rangle$	$\langle \delta_{0.1205}, (1, 0) \rangle$
C	$\langle \delta_{0.6745}, (0, 1) \rangle$	$\langle \delta_{0.7220}, (0, 1) \rangle$	$\langle \delta_{0.8837}, (0, 1) \rangle$	$\langle \delta_{0}, (0, 1) \rangle$

Table 11. Ranking the alternatives by VIKOR method with Entropy weights (Sensitivity, 3rd weight).

	S_1	S_2	S_3	S_4
A	3	2	4	1
B	2	4	3	1
C	2	3	4	1

The ultimate ranking result according to the VIKOR method is as follows: $S_4 > S_1 > S_2 > S_3$. Hence the best alternative is S_4.

Computation 6. VIKOR method with changed weights starting with w_4 from Entropy method

Let $w'_4 = w_4 + \Delta_1 = 0.51197 + 0.052 = 0.56397$. Then changes in the other vectors of the weights are made similar to the computations done in computation 3 and the changed weight vector is given as: $w' = (0.25171,\ 0.11414,\ 0.07018,\ 0.56397)$.

Proceeding Step 1 to Step 4 with the new changed weighting vector $w' = (0.25171,\ 0.11414,\ 0.07018,\ 0.56397)$ as in previous computations, then we have:

Table 12. The evaluation value of each Alternative with Entropy weights (Sensitivity, 4th weight).

	S_1	S_2	S_3	S_4
A	$\langle \delta_{1.5083}, (1, 0) \rangle$	$\langle \delta_{0.8157}, (1, 0) \rangle$	$\langle \delta_{1.6137}, (1, 0) \rangle$	$\langle \delta_{0.1141}, (1, 0) \rangle$
B	$\langle \delta_{0.1404}, (1, 0) \rangle$	$\langle \delta_{0.5640}, (1, 0) \rangle$	$\langle \delta_{0.3776}, (1, 0) \rangle$	$\langle \delta_{0.1141}, (1, 0) \rangle$
C	$\langle \delta_{0.4940}, (0, 1) \rangle$	$\langle \delta_{0.7339}, (0, 1) \rangle$	$\langle \delta_{0.7928}, (0, 1) \rangle$	$\langle \delta_{0}, (0, 1) \rangle$

Table 13. Ranking the alternatives by VIKOR method with Entropy weights (Sensitivity, 4th weight).

	S_1	S_2	S_3	S_4
A	3	2	4	1
B	2	4	3	1
C	2	3	4	1

The ultimate ranking result according to the VIKOR method is as follows: $S_4 > S_1 > S_2 > S_3$. Hence the best alternative is S_4.

Table14. With different weight vectors, the proposed methods are compared.

PROPOSED VIKOR COMPUTATION METHODS	**RANKING OF ALTERNATIVES**
Computation 1. Using VIKOR method and known weights	$S_4 > S_1 > S_2 > S_3$
Computation 2. Using VIKOR method with ENTROPY	$S_4 > S_1 > S_2 > S_3$
Computation 3. Using VIKOR method with SENSITIVITY analysis starting with 1st weight vector	$S_4 > S_1 > S_2 > S_3$
Computation 4. Using VIKOR method with SENSITIVITY analysis starting with 2nd weight vector	$S_4 > S_1 > S_2 > S_3$
Computation 5. Using VIKOR method with SENSITIVITY analysis starting with 3rd weight vector	$S_4 > S_1 > S_2 > S_3$
Computation 6. Using VIKOR method with SENSITIVITY analysis starting with 4th weight vector	$S_4 > S_1 > S_2 > S_3$

According to the results of the comparison study, the final ranking of the alternatives stays unaltered. It's also worth noting that the sensitivity analysis yields the same ranking of the options. Hence when there is a small change in the weight vectors, the final ranking is unaltered. Hence the proposed Linguistic Intuitionistic Fuzzy VIKOR method proves to be an effective method as well as computationally simple.

7. Conclusion

Since many real world problems under Linguistic Intuitionistic Fuzzy sets play an important role while coming under the sector of decision making problems, the proposed methodology will definitely relieve the biased role of the decision makers involved. Sensitivity analysis is performed on the computed weight vectors through entropy method and it has no effect on the change of the weight vectors involved. New operational rules for LIFNs were proposed namely the subtraction and division operations and then a median membership function for LIFNs. An example of the investor looking for a better investment firm was discussed with VIKOR method for choosing the best alternative among the available ones. The sensitivity analysis of the weighting vectors has no effect on the best alternative's ranking order. In general, the strategy suggested in this research is effective and computationally efficient. Comparison between the different computational methods proposed are also highlighted at the end of the work revealing their consistency.

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