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NUMERICAL MODEL TO STUDY THERMAL CHANGES DUE TO SARCOMA IN TISSUES OF HUMAN LIMBS

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Abstract

The In this paper a model is proposed to study the thermal disturbances caused by sarcoma in the tissues of a human limb. A sarcoma is a type of cancer that develops from bone or muscle. The mechanisms of thermal conduction, blood perfusion and metabolic activity in the normal and abnormal tissues are modelled for a two dimensional steady state case in an elliptical shaped human limb involving sarcoma. The physical conditions due to exposure of the outer surface of the limb to the environment are used to frame the boundary conditions. The numerical results are obtained using finite element method (FEM) and used to analyse the thermal disturbances caused by the location and size of sarcoma in the tissues of a human limb. The thermal information obtained can be useful for detection of the size and location of sarcoma in the tissues of a human limb.

Introduction

The clinical practitioners have been using tissue temperature as an indicator of various clinical and health conditions since ancient times. This tissue temperature depends on thermoregulation of a human body involving processes like blood flow, metabolic activity, and heat transport from body core to the surface and heat loss from body surface to the environment by the

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mechanisms of conduction, convection, radiation and evaporation. Any disturbances in these mechanisms can cause thermal changes in the body. These thermal disturbances may be caused by the environmental changes, physical activity or any diseases leading to disorders in these thermoregulation mechanisms.

Various experimental and theoretical investigations [1-6] are reported to study the relationships of tissue temperature with these mechanisms involved in thermoregulation. Initially theoretical investigations of heat flow were carried out in flat shaped human organs for one dimensional case. Later attempts were made to study the temperature distribution in dermal regions of flat shaped, elliptical cylindrical, circular cylindrical, spherical and ellipsoidal shaped human organs for two and three dimensional cases [9-14]. These studies were carried out under normal environmental and physiological conditions. The research workers also developed models to study the effect of malignant tumors in flat, circular cylindrical, elliptical cylindrical, spherical and ellipsoidal shaped human organs for one, two and three dimensional cases [7, 8, 15-18]. No attempt is reported till date to study the effect of sarcoma on temperature distribution in peripheral and deep tissues of a human limb.

A sarcoma is a type of cancer that develops from certain tissues, like bone or muscle. It is a type of malignant tumor. Bone and soft tissue sarcomas are the main types of sarcoma. Soft tissue sarcomas can develop from soft tissues like fat, muscle, nerves, fibrous tissues, blood vessels, or deep skin tissues. They can be found in any part of the body. Most of them develop in the arms or legs. They can also be found in the trunk, head, neck area, internal organs, and the area in the back of the abdominal cavity (known as the retroperitoneum) [24].

In the present study a finite element model is proposed to study thermal changes in elliptical shaped human limb involving sarcoma in deep tissues for a two dimensional steady state case. The effect of shape and size of sarcoma tumor in the tissues of human limb is analysed at low atmospheric temperatures.

Mathematical Model

The bio-heat equation [22] in elliptical cylindrical shaped human limb is given by [1]:

$$DK\left[\frac{\partial^2 T}{\partial \mu^2}+\frac{\partial^2 T}{\partial \nu^2}\right]+M(T_A-T)+\overline{S}=0, \tag{1}$$

where, $D=\dfrac{1}{d^2(\sinh^2\mu+\sin^2 v)}$, $M=m_b c_b$ and $\overline{S}=S+w.$

Here the effect of blood flow and metabolic heat generation term is given by $M(T_A-T)$ and $\overline{S}$ respectively. $K=$ Thermal conductivity of tissues, $m_b=$ blood mass flow rate, $c_b=$ specific heat of blood, $T=$ tissue temperature, $d=$ eccentricity of tissue layer, μ, v, z are radial, angular and axial coordinates respectively for the elliptical shaped human limbs. T_A = Arterial blood temperature and is taken as equal to body core temperature T_b, as the blood flows in arteries from the body core at core temperature T_b. Here, $w=0$ for normal tissues and $w=\eta S$ for abnormal tissues.

The outer surface of the limb is exposed to the environment and the heat loss from the outer surface to the environment takes place by conduction, convection, radiation and evaporation. Therefore, the boundary condition at the outer surface can be written as [4]

$$-K\frac{\partial T}{\partial \mu}=h(T-T_a)+LE,\ \mu=\mu_3. \tag{2}$$

Where, $h=$ heat transfer coefficient, $T_a=$ atmospheric temperature, $L=$ latent heat, $E=$ rate of sweat evaporation, $-K\dfrac{\partial T}{\partial \mu}=$ heat flux normal to the skin surface.

In this model initial temperature of the bone layer is assumed as core temperature of the limb hence following boundary condition is imposed at the inner surface:

$$T(\mu=\mu_0)=T_b. \tag{3}$$

Where μ_0 is the thickness of bone layer.

The whole region is divided into the four layers namely bone, muscle, fat and skin which are considered to be elliptic with eccentricity d_0, d_1, d_2, d_3 respectively. The whole region is divided into 36 elements with 48 nodal points. The coaxial circular sector elements have been employed to discretize the region as given in the Figure 1. This division of each layer into different number of elements of different sizes has been done in order to match with the geometry and physiological properties of the region

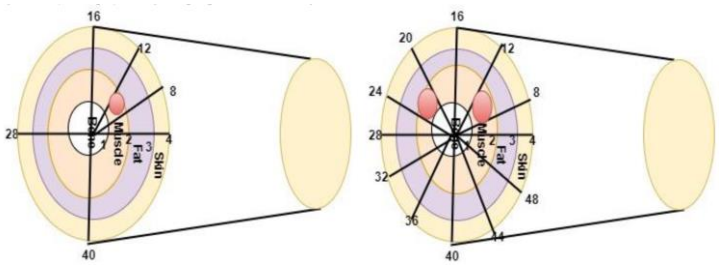

Figure 1. Discretization of human limb in tissue layers.

Here it has been assumed that an elliptical shaped non uniformly perfused tumor named sarcoma is situated in the muscle layer in between $\pi/6$ and $2\pi/6$ in the fourth element. It is also assumed that after some time another sarcoma grows in muscle layer in between $4\pi/6$ and $5\pi/6$ in the thirteenth element. Dividing the region into the number of elements gives us flexibility in assigning the independent values to the physical and physiological parameters in each sub region.

At low atmospheric temperature, the core temperature of the human limb is variable along angular direction. As the warm blood flows in arteries at 37^0C from the core of the trunk to the core of the human limb and the same blood reaching extreme parts of the limb cools down and returns back from extremities of the limb through veins at lower temperature than the body core temperature [19]. Hence the following boundary condition is imposed:

$$T(\mu_0, \upsilon) = \sum_{i=1}^{3} a_i \upsilon^{i-1}. \tag{4}$$

Here a_i's are constant which can be found by following conditions:

$$T(\mu_0, \upsilon) = \alpha, \beta, \gamma \text{ at } \upsilon = 0, \pi, 2\pi \text{ respectively} \tag{5}$$

α, β, γ can be assigned the values based on the temperature at selected points of the core of the human limb.

The partial differential equation (1) for heat flow coupled with boundary conditions (2) to (4), is transformed into the following equivalent discretized variational form [2]:

$$I^{(e)} = \frac{1}{2}\iint\Bigg[D^{(e)}K^{(e)}\left[\left(\frac{\partial T^{(e)}}{\partial \mu}\right)^2 + \left(\frac{\partial T^{(e)}}{\partial v}\right)^2\right] + M^{(e)}(T_b - T^{(e)})^2 - 2\overline{S}^{(e)}T^{(e)}\Bigg]d\mu \cdot dv + \frac{1}{2}(\lambda)^{(e)}\int[h(T^{(e)} - T_a)^2 + 2LET^{(e)}]dv. \tag{6}$$

The following bilinear shape functions are employed in each element

$$T^{(e)} = C_1 + C_2\mu + C_3\nu + C_4\mu v, \tag{7}$$

where, C_1, C_2, C_3 and C_4 are constants which are determined by nodal conditions.

The integrals (6) are evaluated and assembled as follows:

$$I = \sum_{i=1}^{n} I_i(e). \tag{8}$$

The integral I is extremized with respect to each nodal temperature to obtain a system of linear algebraic equations given below:

$$X\overline{T} = Y. \tag{9}$$

Where X is the system of matrix of order 48×48 order and Y is a column vector of order 48×1. Also $\overline{T} = [T_1T_2, \ldots\ldots T_{48}]$. The Gauss elimination method is employed to solve equations (9).

Numerical Results and Discussion

The values of physical and physiological constants given in Table 1 have been used [23] in the present study:

Table 1. The thermal properties of body tissues.

Tissue layers	$\mu_n(m)$	$K_n(W.m-1.m-1)$	ρ(Kg/m3)	c(J.kg–1.K–1)	$m_b(10^{-3})(1/s)$	$S_n(W/m3)$
Bone	0.0153	0.75	1, 357	1, 700	0.0	0.0
Muscle	0.0343	0.42	1, 085	3, 768	2.7	684
Fat	0.0401	0.16	850	2, 300	0.08	58
Skin	0.0418	0.47	1, 085	3, 680	1.26	368
Tumor	0.010	0.558	1, 030	3, 582	6.00	2208

The values of angular positions in degree and radians are as follows:

Table 2. Angular positions in degrees and radians.

Degrees	Radians	Degrees	Radians
0.0	0.0	210.0	3.66519
30.0	0.52360	240.0	4.18879
60.0	1.0472	270.0	4.71239
90.0	1.5708	300.0	5.23599
120.0	2.0944	330.0	5.75959
150.0	2.61799	360.0	6.28319
180.0	3.14159		

Graphs have been plotted among T and v for atmospheric temperature 15^0C. The metabolic activity of tumor is found to vary between 1 to 7 times of that in normal tissues. The blood mass flow rate and metabolic heat generation rate are very low in the tumor core and decreases with increase in the size of the tumor. Therefore as a special case it has been assumed that the rate of blood flow in tumor core is 1/10 of that in normal tissues. Also for particular samples of sarcoma we are assuming that metabolic activity is 3 times and 5 times of that in normal tissues respectively [20].

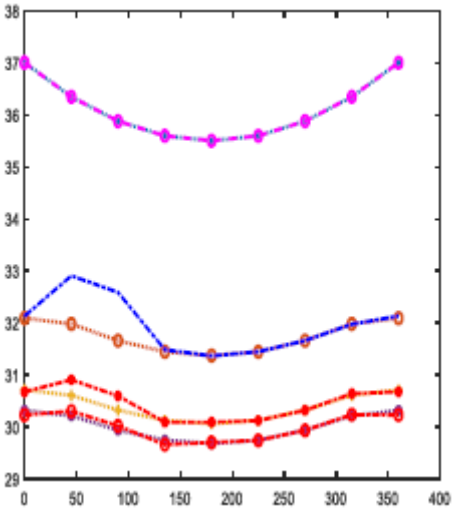

Figure 2. Temperature distribution along angular position for $T_a = 15^0 C$ $E = 0$, $\eta = 5.0$.

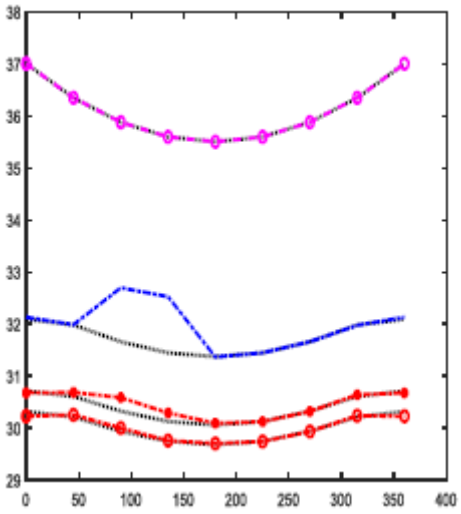

Figure 3. Temperature distribution along angular position for $T_a = 15^0 C$ $E = 0$, $\eta = 3.0$.

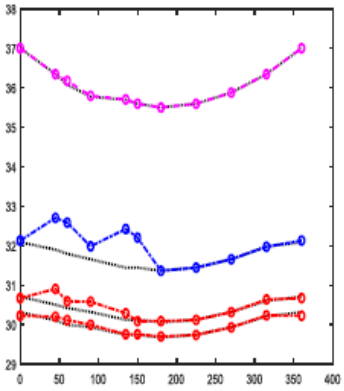

Figure 4. Temperature distribution along angular position for $T_a = 15^0 C$ $E = 0$, $\eta = 5.0$, $\eta = 3.0$.

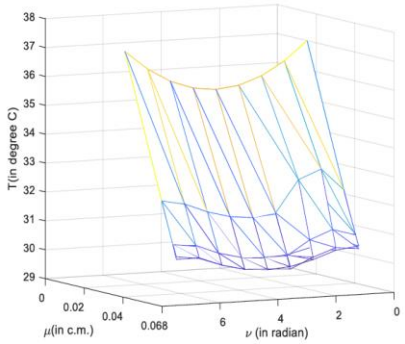

Figure 5. Temperature distribution along μ, v for $T_a = 15^0 C\, E = 0, \eta = 5.0$.

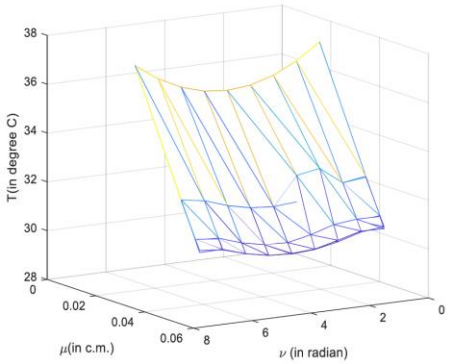

Figure 6. Temperature distribution along μ, v for $T_a = 15^0 C\, E = 0, \eta = 3.0$.

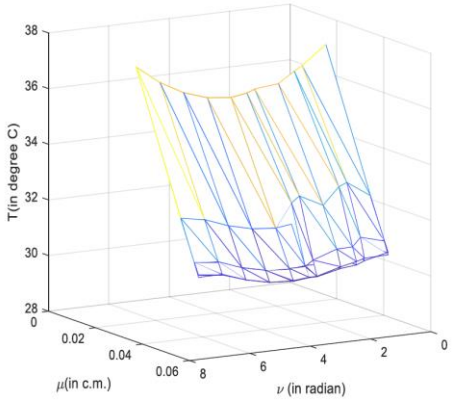

Figure 7. Temperature distribution along μ, υ for $T_a = 15^0 C\, E = 0, \eta = 5.0, \eta = 3.0$.

The figure 2 shows the temperature variation in human limb with sarcoma for $T_a = 15^0 C$, $E = 0$, where sarcoma is situated in between $\pi/6$ and $\pi/3$ in element number 4 with $\eta = 5$. The figure 3 shows the temperature variation in the limb for $T_a = 15^0 C$, $E = 0$, where sarcoma is situated in between $2\pi/3$ and $5\pi/6$ in element number 13 with $\eta = 3$. The figure 4 shows the temperature variation in limb for $T_a = 15^0 C$, $E = 0$, where sarcoma is situated in between $\pi/6$ and $\pi/3$ and $2\pi/3$ and $5\pi/6$ i.e. after some period of time another sarcoma grows on another angular position of the limb. From these figure we observe that the temperature falls down from core to outer skin surface between 37^0C to 30^0C [21]. We observe the change in the slope of the curves at the interfaces of normal tissues and sarcoma region.

Figure 5 shows temperature variations along angular position (ν) radial position (μ) involving sarcoma for $T_a = 15^0 C$, $E = 0$, $\eta = 3$. Figure 6 shows temperature variations along angular position (ν) radial position (μ) involving sarcoma for $T_a = 15^0 C$, $E = 0$, $\eta = 3$. Figure 7 shows temperature variations along μ, υ involving sarcoma for $T_a = 15^0 C$, $E = 0$, $\eta = 5$ and $\eta = 3$. In these figures we observe the change in slope of the curve at the

interface of normal tissues and sarcoma region ($\mu = 0.0343$, $\upsilon = \pi/6$ to $\upsilon = \pi/3$) and ($\mu = 0.0343$, $\upsilon = 2\pi/3$ to $5\pi/6$) and also at the interface of different layers of normal tissues. In figure 5-7 we observe the major thermal disturbances in tumor region. In figure 4 and in figure 7 we observe that the temperature at the interface of normal and sarcoma region first increases from $\pi/6$ to $\pi/3$ then decreases between $\pi/3$ to $2\pi/3$ and again increases from $2\pi/3$ to $5\pi/6$ and further falls down at 2π after $2\pi/3$. This elevation in temperature shows the presence of sarcoma. Also the temperatures in figure 2 and in figure 5 are more as compared to that in figure 3 and figure 6. This is due to different rates of metabolic activity in different sarcoma tumors.

Conclusion

The thermal changes in human limb due to presence of sarcoma have been successfully studied by proposing and employing the finite element model. The present model is quite realistic in terms of shape and size in comparison to the model reported earlier for study of thermal changes in human limb. The proposed model is able to generate quite interesting and useful thermal information in terms of change in the slope of temperature profiles at the interface of the tissues and sarcoma in human limb indicating the position, shape and size of the tumor. The finite element approach has proved to be quite effective in the present study. Such model can be developed for generating thermal information about other types of tumors for clinical applications like detection of position and size of tumors.

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References

[1] M. Agrawal, N. Adlakha and K. R. Pardasani, Three dimensional finite element model to study heat flow in dermal Region of elliptical and tapered shaped human limbs, Applied Mathematics and Computations 217 (2010), 4129-4140.

[2] M. Agrawal, N. Adlakha and K. R. Pardasani, Thermal disturbances in dermal regions of human limbs involving metastasis of tumors, International Mathematical Forum 5 (2010), 1903-1914.

[3] D. B. Gurung, FEM approach to one dimensional unsteady thermal patterns in human dermal parts with quadratic shape function, J Appl Math & Informatics 27 (2009), 301-303.

[4] V. P. Saxena and J. S. Bindra, Steady state thermal patterns in dermal regions of human body with variable blood flow perspiration and self-controlled metabolic heat generation, Indian J Pure Appl Math 15 (1984), 31-42.

[5] V. P. Saxena and D. Arya, Steady state heat distribution in epidermis, dermis and sub dermal tissue, Journal of Theoretical Biology 89 (1981), 423-432.

[6] V. P. Saxena, Temperature Distribution in Human Skin & Sub Dermal Tissue, Journal of Theoretical Biology 102 (1983), 277-286.

[7] V. P. Saxena and K. R. Pardasani, Effect of dermal tumor on thermal patterns in skin with variable blood flow, Bull Mathematical Biology, 53(4), USA (1991), 525-536.

[8] K. R. Pardasani and N. Adlakha, Exact solution to a heat flow problem in peripheral tissue layers with a solid tumor in the dermis, Indian J. Pure Applied Math. 22 (1991), 679-687.

[9] K. R. Pardasani and V. P. Saxena, Exact solution to thermal patterns problem in annular skin layers, Bull Calcutta Math Soc. (1989), 81-108.

[10] K. R. Pardasani and N. Adlakha, Two dimensional steady state thermal patterns in annular tissues of a human or animal body, Indian J. Pure Applied Math. 24 (1993), 721-728.

[11] K. R. Pardasani and N. Adlakha, Coaxial circular sector elements to study two-dimensional heat distribution problem in dermal regions of human limbs, Math. & Comp Modelling 22 (1995), 127-140.

[12] M. A. Khanday and V. P. Saxena, Finite element approach for the study of thermoregulation in human head exposed to cold environment, AIP Conf. Proc. 1146 (2009), 375-385.

[13] M. Agrawa, N. Adlakha and K. R. Pardasani, Semi numerical model to study thermal patterns in peripheral layers of elliptical and tapered shaped human limbs, J. Mech. in Medicine and Biology 10 (2010), 57-72.

[14] A. Makrariya and N. Adlakha, Two-dimensional finite element model of thermal patterns in dermal tissues of extended spherical organs of a human body, International Journal of Biomathematics 6 (2013), 1-15.

[15] A. Makrariya and N. Adlakha, Two dimensional finite element model to study thermal patterns in peripheral regions of extended spherical human organs involving uniformly perfused tumors, International Journal of Biomathematics 8 (2015), 74-104.

[16] A. Makrariya and N. Adlakha, Thermal stress due to tumor in periphery of human breast, Open Science Journal of Bioscience and Bioengineering 2 (2015), 50-59.

[17] A. Makrariya and N. Adlakha, Quantitative study of thermal disturbances due to non-uniformly perfused tumors in peripheral regions of women's breast, Cancer Informatics Volume 16 (2017), 1-13.

[18] M. Agrawal, N. Adlakha and K. R. Pardasani, Finite element model to study thermal effect of uniformly perfused tumor in dermal layers of elliptical shaped human limbs, Int. J. Biomathematics 4 (2011), 241-254.

[19] M. Agrawal, N. Adlakha and K. R. Pardasani, Steady state temperature distribution in dermal regions of an irregular tapered shaped human limb with variable eccentricity, Journal of Thermal Biology 44 (2014), 27-34.

[20] M. Agrawal, N. Adlakha and K. R. Pardasani, Finite element model to study the thermal effect of tumors in dermal regions of irregular tapered shaped human limbs, International Journal of Thermal Sciences 98 (2015), 287-295.

[21] M. Agrawal, N. Adlakha and K. R. Pardasani, Finite element model to study temperature distribution in skin and deep tissues of human limbs, Journal of thermal biology 62 (2016), 98-105.

[22] W. Perl, Heat and matter distribution equation to include clearance by capillary blood flow, Ann. N. Y. Acad. Sci., 108 (1963), 92-105.

[23] D. Fiela, K. J. Lomas and M. Stohrer, A computer model of human thermoregulation for a wide range of environmental conditions, the passive system, Journal of Applied Physiology 87(5) (1999), 1957-1972.

[24] American Cancer Society, What Is a Soft Tissue Sarcoma, https://www.cancer.org/cancer/soft-tissue-sarcoma/about/soft-tissue-sarcoma.html.