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COMMON FIXED POINT THEOREMS FOR SEQUENCE OF MAPPINGS IN GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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Abstract

In 1965, Zadeh [18] initiated the fuzzy sets concept. Thereafter the concept of fuzzy set was generalized as intuitionistic fuzzy set by Atanassov [2] in 1984. In 2004, Park [10] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to George and Veeramani ([5], [6]). In generalized intuitionistic fuzzy metric spaces, we demonstrate a few common fixed point theorems for sequence of mappings.

1. Motivation and Main Results

Definition 1. If the following requirements will meet, a binary operation $* : [0, 1] \times [0, 1]$ is a continuous t-norm:

(a) * has the properties of being both associative and commutative,

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(b) * is continuous,

(c) $u * 1 = u, \forall u \in [0, 1]$,

(d) $u * v \leq i * j$ wherever $u \leq i$ and $v \leq j$ in each case $u, v, i, j \in [0, 1]$.

Examples. $u * v = uv$ and $u * v =$ minimum of $\{u, v\}$.

Definition 2. If the following requirements will meet, a binary operation $* : [0, 1] \times [0, 1]$ is a continuous t-conorm:

(a) $\lozenge$ has the properties of being both associative and commutative,

(b) $\lozenge$ is continuous,

(c) $u \lozenge 0 = u, \forall u \in [0, 1]$,

(d) $u \lozenge v \leq i \lozenge j$ wherever $u \leq i$ and $v \leq j$, for each $u, v, i, j \in [0, 1]$.

Examples. $u \lozenge v =$ minimum of $\{1, u + v\}$ and $u \lozenge v =$ maximum of $\{u, v\}$.

Definition 3. An generalized intuitionistic fuzzy metric space is a 5-tuple $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$ if W is an any non-empty set, * is a continuous t-norm, $\lozenge$ a continuous t-conorm and $\mathcal{M}, \mathcal{N}$ are fuzzy sets on $W^3 \times (0, \infty)$, satisfying the following requirements must be met: for any $w, u, v, a \in W$ and $t, s > 0$.

(a) $\mathcal{M}(w, u, v, t) + \mathcal{N}(w, u, v, t) = 1$,

(b) $\mathcal{M}(w, u, v, t) > 0$,

(c) $\mathcal{M}(w, u, v, t) = 1 \Leftrightarrow w = u = v$,

(d) $\mathcal{M}(w, u, v, t) = \mathcal{M}(p\{w, u, v\}, t)$, here p denotes the permutation function,

(e) $\mathcal{M}(w, u, a, t) * \mathcal{M}(a, u, v, s) \leq \mathcal{M}(w, u, v, t + s)$,

(f) $\mathcal{M}(w, u, v, \cdot) : (0, \infty) \to [0, 1]$ is continuous,

(g) $\mathcal{N}(w, u, v, t) > 0$,

(h) $\mathcal{N}(w, u, v, t) = 0 \Leftrightarrow w = u = v$,

(i) $\mathcal{N}(w, u, v, t) = \mathcal{N}(p\{w, u, v\})$, here p denotes the permutation function,

(j) $\mathcal{N}(w, u, a, t) \lozenge \mathcal{N}(a, u, v, s) \geq \mathcal{N}(w, u, v, t+s)$,

(k) $N(w, u, v, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

Then $(\mathcal{M}, \mathcal{N})$ is generalized intuitionistic fuzzy metric on W.

Example 4. Take $W = R$, $\mathcal{M}$ and $\mathcal{N}$ are fuzzy sets on $W^3 \times (0, \infty)$ defined by $\mathcal{M}(w, u, v, t) = \dfrac{t}{t + |w-u| + |u-v| + |v-w|}$, $\mathcal{N}(w, u, v, t) = \dfrac{|w-u| + |u-v| + |v-w|}{t + |w-u| + |u-v| + |v-w|}$ for each $w, u, v \in W$ and $t > 0$. Then $(\mathcal{M}, \mathcal{N})$ is an generalized intuitionistic fuzzy metric on W.

Definition 5. A generalized intuitionistic fuzzy metric space is $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$. Then

(i) If $\forall t > 0$ and $p > 0$, $\lim_{n\to\infty} \mathcal{M}(w_{n+p}, w_n, w_n, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(w_{n+p}, w_n, w_n, t) = 0$, a sequence $\{w_n\}$ in W is Cauchy.

(ii) If $\forall t > 0$, $\lim_{n\to\infty} \mathcal{M}(w_n, w, w, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(w_n, w, w, t) = 0$, a sequence $\{w_n\}$ in W is converge to a point $w \in W$.

(iii) Every Cauchy sequence in W is convergent $\Leftrightarrow$ an intuitionistic generalized fuzzy metric space $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$ is complete.

Lemma 6. *A generalized intuitionistic fuzzy metric space is defined as* $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$. *With regard to* t, $\forall w, u, v$ *in* W, $\mathcal{M}(w, u, v, t)$ *is non-decreasing and* $\mathcal{N}(w, u, v, t)$ *is non-increasing.*

Lemma 7. *Let a sequence* $\{w_n\}$ *in an generalized intuitionistic fuzzy metric space* $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$, *if* $\exists$ *a constant* $r \in (0, 1)$: $\mathcal{M}(w_n, w_{n+1}, w_{n+1}, rt) \geq \mathcal{M}(w_{n-1}, w_n, w_n, t)$ *and* $\mathcal{N}(w_n, w_{n+1}, w_{n+1}, rt) \leq \mathcal{N}(w_{n-1}, w_n, w_n, t)\ \forall t > 0$. *Then* $\{w_n\}$ *is Cauchy in W.*

Lemma 8. *Let* $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$ *be an generalized intuitionistic fuzzy* *metric space and* $\forall\, w, u, v \in W, t > 0,$ *and if for a number* $r \in (0, 1), \mathcal{M}(w, u, v, rt) \geq \mathcal{M}(w, u, v, t)$ *and* $\mathcal{N}(w, u, v, rt) \leq \mathcal{N}(w, u, v, t)$ *then* $w = u = v.$

Definition 9. If $Tw = w,$ a point $w \in W$ is called a fixed point of the map $T : W \to W.$

Definition 10. If $T_n(w) = w\ \forall\, n,$ a point $w \in W$ is called a common fixed point of sequence of the mappings $T_n : W \to W.$

Theorem 11. *Let* $(W, \mathcal{M}, \mathcal{N}, *, \Diamond)$ *be a complete generalized intuitionistic fuzzy metric space and* $T_n : W \to W$ *be a sequence of maps* $: \forall\, t > 0$ *and* $0 < r < 1$ *satisfying the following conditions* $\mathcal{M}(T_i w, T_j z, T_j z, t) \geq$ *minimum of* $\{\mathcal{M}(w, z, z, t/r), \mathcal{M}(w, w, T_i w, t/r), \mathcal{M}(z, z, T_j z, t/r), \mathcal{M}(w, z, T_i w, t/r), \mathcal{M}(z, T_i w, T_j z, t/r)\}$ $\mathcal{N}(T_i w, T_j z, T_j z, t) \leq$ *maximum of* $\{\mathcal{N}(w, z, z, t/r), \mathcal{N}(w, w, T_i w, t/r), \mathcal{N}(z, z, T_j z, t/r), \mathcal{N}(w, z, T_i w, t/r), \mathcal{N}(z, T_i w, T_j z, t/r)\}$ $\forall\, i \neq j$ *and* $\forall\, w, z \in W.$ *Then* $\{T_n\}$ *has a unique common fixed point.*

12. Proof of Theorem 11. Let $w_0 \in W$ be any arbitrary point.

The sequence $\{w_n\}$ in W defined by $w_{n+1} = T_{n+1} w_n$ for $n = 0, 1, 2, 3, \ldots.$

Now $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) = \mathcal{M}(T_{n+1} w_n, T_{n+2} w_{n+1}, T_{n+2} w_{n+1}, t)$

$\geq$ minimum of $\{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r), \mathcal{M}(w_n, w_n, T_{n+1} w_n, t/r),$
$\mathcal{M}(w_{n+1}, w_{n+1}, T_{n+2} w_{n+1}, t/r), \mathcal{M}(w_n, w_{n+1}, T_{n+1} w_n, t/r),$
$\mathcal{M}(w_{n+1}, T_{n+1} w_n, T_{n+2} w_{n+1}, t/r)\}.$

$=$ minimum of $\{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r), \mathcal{M}(w_n, w_n, w_{n+1}, t/r),$
$\mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r), \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r),$
$\mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r)\}$

$=$ minimum of $\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r), \mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t/r)\}$
$= \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

Therefore, $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

Similarly, $\mathcal{N}(w_{n+1}, w_{n+2}, w_{n+2}, t) \leq \mathcal{N}(w_n, w_{n+1}, w_{n+1}, t/r)$

Thus

$$\begin{aligned}\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) &\geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)\\ &\geq \mathcal{M}(w_{n-1}, w_n, w_n, t/r^2)\\ &\vdots\\ &\geq \mathcal{M}(w_0, w_1, w_1, t/r^{n+1})\end{aligned}$$

We now obtain for any positive integer q and $t > 0$,

$\mathcal{M}(w_n, w_{n+q}, w_{n+q}, t) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/q) * \ldots q$ times $\ldots * \mathcal{M}(w_{n+q-1}, w_{n+q}, w_{n+q}, t/q) \geq \mathcal{M}(w_0, w_1, w_1, t/qr^n) * \ldots q$ times $\ldots * \mathcal{M}(w_0, w_1, w_1, t/qr^{n+q-1})$

Therefore, $\lim_{n\to\infty} \mathcal{M}(w_n, w_{n+q}, w_{n+q}, t) \geq 1^* \ldots q$ times $\ldots * 1 = 1$.

Similarly, $\lim_{n\to\infty} \mathcal{N}(w_n, w_{n+q}, w_{n+q}, t) \leq 0 \lozenge \ldots q$ times $\ldots \lozenge 0 = 0$.

So the sequence $\{w_n\}$ is Cauchy.

Since W is complete, sequence $\{w_n\}$ converges to $w \in W$.

Now $\mathcal{M}(T_m w, w, w, t) = \lim_{n\to\infty} \mathcal{M}(T_m w, w_{n+2}, w_{n+2}, t)$

$= \lim_{n\to\infty} \mathcal{M}(T_m w, T_{n+2} w_{n+1}, T_{n+2} w_{n+1}, t)$

$\geq \lim_{n\to\infty}$ minimum of $\{\mathcal{M}(w, w_{n+1}, w_{n+1}, t/r), \mathcal{M}(w, w, T_m w, t/r),$ $\mathcal{M}(w_{n+1}, w_{n+1}, Tw_{n+2} w_{n+1}, t/r), \mathcal{M}(w, w_{n+1}, T_m w, t/r),$ $\mathcal{M}(w_{n+1}, T_m w, T_{n+2} w_{n+1}, t/r)\}$

$= \lim_{n\to\infty}$ minimum of $\{\mathcal{M}(w, w_{n+1}, w_{n+1}, t/r), \mathcal{M}(w, w, T_m w, t/r),$ $\mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r), \mathcal{M}(w, w_{n+1}, T_m w, t/r), \mathcal{M}(w_{n+1}, T_m w, w_{n+2}, t/r)\}$

$=$ minimum of $\{\mathcal{M}(w, w, w, t/r), \mathcal{M}(w, w, T_m w, t/r), \mathcal{M}(w, w, w, t/r),$ $\mathcal{M}(w, w, T_m w, t/r), \mathcal{M}(w, T_m w, w, t/r)\}$

$=$ minimum of $\{1, \mathcal{M}(w, w, T_m w, t/r\}$

$= \mathcal{M}(w, w, T_m w, t/r)$

$= \mathcal{M}(T_m w, w, w, t/r)$

$\vdots$

$\geq \mathcal{M}(T_m w, w, w, t/r^n)$

$\to 1$ as n tends to ∞.

Hence $\mathcal{M}(T_m w, w, w, t) = 1, \forall t > 0$.

Similarly, $\mathcal{N}(T_m w, w, w, t) = 0, \forall t > 0$.

Hence, $T_m w = w$.

Therefore, $T_n w = w \ \forall n$.

Hence w is a common fixed point of $\{T_n\}$.

Uniqueness: Suppose $w \neq z : T_n z = z \ \forall n$.

Now consider $\mathcal{M}(w, z, z, t) = \mathcal{M}(T_i w, T_j z, T_j z, t)$

$\geq$ minimum of $\{\mathcal{M}(w, z, z, t/r), \mathcal{M}(w, w, T_i w, t/r), \mathcal{M}(z, z, T_j z, t/r), \mathcal{M}(w, z, T_i w, t/r), \mathcal{M}(z, T_i w, T_j z, t/r)\}$

$=$ minimum of $\{\mathcal{M}(w, z, z, t/r), \mathcal{M}(w, w, w, t/r), \mathcal{M}(z, z, z, t/r), \mathcal{M}(w, z, w, t/r), \mathcal{M}(z, w, z, t/r)\}$

$=$ minimum of $\{\mathcal{M}(w, z, z, t/r), 1\}$

$= \mathcal{M}(w, z, z, t/r)$

$\vdots$

$\geq \mathcal{M}(w, z, z, t/r^n)$

$\to 1$ as n tends to ∞

Hence $\mathcal{M}(w, z, z, t) = 1, \forall t > 0$.

Similarly, $\mathcal{N}(w, z, z, t) = 0, \forall t > 0$.

Therefore, $w = z$.

Which is contradiction to $w \neq z$.

Hence $\{T_n\}$ has a unique common fixed point.

Remark 13. The succeeding corollary 14 is obtained by considering $T_i = T_j = T$ in the preceding theorem.

Corollary 14. *Let* $(W, \mathcal{M}, \mathcal{N}, *, \Diamond)$ *be a complete generalized intuitionistic fuzzy metric space and* $T : W \to W$ *be a mapping*: $\mathcal{M}(Tw, Tz, Tz, rt) \geq$ *minimum of* $\{\mathcal{M}(w, z, z, t), \mathcal{M}(w, w, Tw, t), \mathcal{M}(z, z, Tz, t), \mathcal{M}(w, z, Tw, t), \mathcal{M}(z, Tw, Tz, t)\}$ $\mathcal{N}(Tw, Tz, Tz, rt) \leq$ *maximum of* $\{\mathcal{N}(w, z, z, t), \mathcal{N}(w, w, Tw, t), \mathcal{N}(z, z, Tz, t), \mathcal{N}(w, z, Tw, t), \mathcal{N}(z, Tw, Tz, t)\}$ $\forall\, w, z \in W$ *and* $r \in (0, 1)$. *Then T has a unique fixed point.*

Theorem 15. *Let* $(W, \mathcal{M}, \mathcal{N}, *, \Diamond)$ *be a complete generalized intuitionistic fuzzy metric space and* $T_n : W \to W$ *be a sequence of maps* : $\forall\, t > 0$ *and* $0 < r < 1$ *satisfying the following conditions* $5\mathcal{M}(T_i w, T_j z, T_j z, t) \geq \{\mathcal{M}(w, z, z, t/r) + \mathcal{M}(w, w, T_i w, t/r) + \mathcal{M}(z, z, T_j z, t/r) + \mathcal{M}(w, z, T_i w, t/r) + \mathcal{M}(z, T_i w, T_j z, t/r)\}$ $5\mathcal{N}(T_i w, T_j z, T_j z, t) \leq \{\mathcal{N}(w, z, z, t/r) + \mathcal{N}(w, w, T_i w, t/r) + \mathcal{N}(z, z, T_j z, t/r) + \mathcal{N}(w, z, T_i w, t/r) + \mathcal{N}(z, T_i w, T_j z, t/r)\}$ $\forall\, i \neq j$ *and* $\forall\, w, z \in W$. *Then* $\{T_n\}$ *has a unique common fixed point.*

16 Proof of Theorem 15. Let $w_0 \in W$ be any arbitrary point.

The sequence $\{w_n\}$ in W defined by $w_{n+1} = T_{n+1} w_n$ for $n = 0, 1, 2, 3, \ldots$

Now $5\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) = 5\mathcal{M}(T_{n+1} w_n, T_{n+2} w_{n+1}, T_{n+2} w_{n+1}, t)$

$$\geq \{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) + \mathcal{M}(w_n, w_n, T_{n+1} w_n, t/r)$$

$$+\mathcal{M}(w_{n+1}, w_{n+1}, T_{n+2} w_{n+1}, t/r) + \mathcal{M}(w_n, w_{n+1}, T_{n+1} w_n, t/r)$$

$$+\mathcal{M}(w_{n+1}, T_{n+1} w_n, T_{n+2} w_{n+1}, t/r)\}$$

$$= \{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) + \mathcal{M}(w_n, w_n, w_{n+1}, t/r)$$

$$+\mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r) + \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$$

$+ \mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r)\}$

$= 3\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) + 2\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t/r)$

$\geq 3\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) + 2\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t/r)$

Therefore, $3\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) \geq 3\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

That is, $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t/r) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

Similarly, $\mathcal{N}(w_{n+1}, w_{n+2}, w_{n+2}, t) \leq \mathcal{N}(w_n, w_{n+1}, w_{n+1}, t/r)$

Thus $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

$$\geq \mathcal{M}(w_{n-1}, w_n, w_n, t/r^2)$$

$$\vdots$$

$$\geq \mathcal{M}(w_0, w_1, w_1, t/r^{n+1})$$

We now obtain for any positive integer q and $t > 0$,

$\mathcal{M}(w_n, w_{n+q}, w_{n+q}, t) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/\ q) * \ldots q$ times

$\ldots * \mathcal{M}(w_{n+q-1}, w_{n+q}, w_{n+q}, t/q) \geq \mathcal{M}(w_0, w_1, w_1, t/qr^n) * \ldots q$ times

$\ldots * \mathcal{M}(w_0, w_1, w_1, t/qr^{n+q-1})$

Therefore, $\lim_{n\to\infty} \mathcal{M}(w_n, w_{n+q}, w_{n+q}, t) \geq 1 * \ldots q$ times $\ldots * 1 = 1$.

Similarly, $\lim_{n\to\infty} \mathcal{N}(w_n, w_{n+q}, w_{n+q}, t) \leq 0 \lozenge \ldots q$ times $\ldots \lozenge 0 = 0$.

So the sequence $\{w_n\}$ is Cauchy.

Since W is complete, sequence $\{w_n\}$ converges to $w \in W$.

We now show that w is a fixed point of $\{T_n\}$ $\forall\, n$.

$5\mathcal{M}(T_m w, w, w, t) = \lim_{n\to\infty} 5\mathcal{M}(T_m w, w_{n+2}, w_{n+2}, t)$

$= \lim_{n\to\infty} 5\mathcal{M}(T_m w, T_{n+2} w_{n+1}, T_{n+2} w_{n+1}, t)$

$\geq \lim_{n\to\infty} \{\mathcal{M}(w, w_{n+1}, w_{n+1}, t/r) + \mathcal{M}(w, w, T_m w, t/r)$

$+\mathcal{M}(w_{n+1}, w_{n+1}, Tw_{n+2}w_{n+1}, t/r) + \mathcal{M}(w, w_{n+1}, T_m w, t/r)$

$+\mathcal{M}(w_{n+1}, T_m w, T_{n+2}w_{n+1}, t/r)\}$

$= \lim_{n\to\infty}\{\mathcal{M}(w, w_{n+1}, w_{n+1}, t/r) + \mathcal{M}(w, w, T_m w, t/r)$

$+ \mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r) + \mathcal{M}(w, w_{n+1}, T_m w, t/r)$

$+ \mathcal{M}(w_{n+1}, T_m w, w_{n+2}, t/r)\}$

$= \{\mathcal{M}(w, w, w, t/r) + \mathcal{M}(w, w, T_m w, t/r) + \mathcal{M}(w, w, w, t/r)$

$+ \mathcal{M}(w, w, T_m w, t/r) + \mathcal{M}(w, T_m w, w, t/r)\}$

$= \{1 + \mathcal{M}(w, w, T_m w, t/r) + 1 + \mathcal{M}(w, w, T_m w, t/r) + \mathcal{M}(w, w, T_m w, t/r)\}$

$= 2 + 3\mathcal{M}(w, w, T_m w, t/r)$

$\geq 2 + 3\mathcal{M}(T_m w, w, w, t/r).$

Therefore, $2\mathcal{M}(T_m w, w, w, t/r) \geq 2.$

That is, $\mathcal{M}(T_m w, w, w, t/r) \geq 1.$

Hence $\mathcal{M}(T_m w, w, w, t) = 1, \forall t > 0.$

Similarly, $\mathcal{N}(T_m w, w, w, t) = 0, \forall t > 0.$

Hence, $T_m w = w.$

Therefore, $T_n w = w \ \forall n.$

Hence w is a common fixed point of $\{T_n\}$.

Uniqueness: Suppose $w \neq z : T_n z = z \ \forall n.$

Then $5\mathcal{M}(w, z, z, t) = 5\mathcal{M}(T_i w, T_j z, T_j z, t)$

$\geq \{\mathcal{M}(w, z, z, t/r) + \mathcal{M}(w, w, T_i w, t/r) + \mathcal{M}(z, z, T_j z, t/r)$

$+\mathcal{M}(w, z, T_i w, t/r) + \mathcal{M}(z, T_i w, T_j z, t/r)\}$

$= \{\mathcal{M}(w, z, z, t/r) + \mathcal{M}(w, w, w, t/r) + \mathcal{M}(z, z, z, t/r) + \mathcal{M}(w, z, w, t/r)$

$+\mathcal{M}(z, w, z, t/r)\}$

$= \{\mathcal{M}(w, z, z, t/r) + 1 + 1 + \mathcal{M}(w, z, w, t/r) + \mathcal{M}(z, w, z, t/r)\}$

$= 2 + 3\mathcal{M}(w, z, z, t/r)$

$\geq 2 + 3\mathcal{M}(w, z, z, t)$

Therefore, $2\mathcal{M}(w, z, z, t) \geq 2.$

That is, $\mathcal{M}(w, z, z, t) \geq 1.$

Hence $\mathcal{M}(w, z, z, t) = 1, \forall t > 0.$

Similarly, $\mathcal{N}(w, z, z, t) = 0, \forall t > 0.$

Therefore, $w = z.$

Which is contradiction to $w \neq z.$

Hence $\{T_n\}$ has a unique common fixed point.

Remark 17. From the preceding theorem we have, $\mathcal{M}(T_i w, T_j z, T_j z, t)$ $\geq 1/5\{\mathcal{M}(w, z, z, t/r) + \mathcal{M}(w, w, T_i w, t/r) + \mathcal{M}(z, z, T_j z, t/r)$ $+\mathcal{M}(w, z, T_i w, t/r) + \mathcal{M}(z, T_i w, T_j z, t/r)\}$

$\geq$ minimum of $\{\mathcal{M}(w, z, z, t/r), \mathcal{M}(w, w, T_i w, t/r), \mathcal{M}(z, z, T_j z, t/r),$ $\mathcal{M}(w, z, T_i w, t/r), \mathcal{M}(z, T_i w, T_j z, t/r)\}$

Therefore, $\mathcal{M}(T_i w, T_j z, T_j z, t) \geq$ minimum of $\{\mathcal{M}(w, z, z, t/r),$ $\mathcal{M}(w, w, T_i w, t/r), \mathcal{M}(z, z, T_j z, t/r), \mathcal{M}(w, z, T_i w, t/r), \mathcal{M}(z, T_i w, T_j z, t/r)\}.$

Similarly, $\mathcal{N}(T_i w, T_j z, T_j z, t) \leq$ maximum of $\{\mathcal{N}(w, z, z, t/r),$ $\mathcal{N}(w, w, T_i w, t/r), \mathcal{N}(z, z, T_j z, t/r), \mathcal{N}(w, z, T_i w, t/r), \mathcal{N}(z, T_i w, T_j z, t/r)\}.$ Hence we get the succeeding corollary.

Corollary 18. *Let* $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$ *be a complete generalized intuitionistic fuzzy metric space and* $T_n : W \to W$ *be a sequence of maps*: $\forall t > 0$ *and* $0 < r < 1$ *satisfying the conditions* $\mathcal{M}(T_i w, T_j z, T_j z, t) \geq$ *minimum of* $\{\mathcal{M}(w, z, z, t/r), \mathcal{M}(w, w, T_i w, t/r), \mathcal{M}(z, z, T_j z, t/r),$

$\mathcal{M}(w, z, T_i w, t/r), \mathcal{M}(z, T_i w, T_j z, t/r)\}$ $\mathcal{N}(T_i w, T_j z, T_j z, t) \le$ maximum of $\{\mathcal{N}(w, z, z, t/r), \mathcal{N}(w, w, T_i w, t/r), \mathcal{N}(z, z, T_j z, t/r), \mathcal{N}(w, z, T_i w, t/r),$ $\mathcal{N}(z, T_i w, T_j z, t/r)\}$ $\forall i \ne j$ and $\forall w, z \in W$. Then $\{T_n\}$ has a unique common fixed point.

Theorem 19. *Let $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$ be a complete generalized intuitionistic fuzzy metric space with t-norm $a * b =$ minimum of $\{a, b\}$ and t-conorm $a \lozenge b =$ maximum of $\{a, b\}$ $\forall a, b \in [0, 1]$ and $T_n : W \to W$ be a sequence of maps: $\forall t > 0$ and $0 < r < 1$ satisfying the conditions $\mathcal{M}(T_i w, T_j z, T_j z, t) \ge \{\mathcal{M}(w, z, z, t/r) * \mathcal{M}(w, w, T_i w, t/r) * \mathcal{M}(z, z, T_i z, t/r) * \mathcal{M}(w, z, T_i w, t/r) * \mathcal{M}(z, T_i w, T_j z, t/r)\}$ $\mathcal{N}(T_i w, T_j z, T_j z, t) \le \{\mathcal{N}(w, z, z, t/r) \lozenge \mathcal{N}(w, w, T_i w, t/r) \lozenge \mathcal{N}(z, z, T_i z, t/r) \lozenge \mathcal{N}(w, z, T_i w, t/r) \lozenge \mathcal{N}(z, T_i w, T_j z, t/r)\}$ $\forall i \ne j$ and $\forall w, z \in W$. Then $\{T_n\}$ has a unique common fixed point.*

20 Proof of Theorem 19. Let $w_0 \in W$ be any arbitrary point.

The sequence $\{w_n\}$ in W defined by $w_{n+1} = T_{n+1} w_n$ for $n = 0, 1, 2, 3, \ldots$

Now $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) = \mathcal{M}(T_{n+1} w_n, T_{n+2} w_{n+1}, T_{n+2} w_{n+1}, t)$

$\ge \{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) * \mathcal{M}(w_n, w_n, T_{n+1} w_n, t/r)$

$* \mathcal{M}(w_{n+1}, w_{n+2}, T_{n+2} w_{n+1}, t/r) * \mathcal{M}(w_n, w_{n+1}, T_{n+1} w_n, t/r)$

$* \mathcal{M}(w_{n+1}, T_{n+1} w_n, T_{n+2} w_{n+1}, t/r)\} = \{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

$* \mathcal{M}(w_n, w_n, w_{n+1}, t/r) * \mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r)$

$* \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) * \mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r)\}$

$\ge \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) * \mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r)\}$

Therefore, $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) \ge \{\mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r) * \mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t/r)\}$.

Which implies that $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) \ge \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$.

Similarly, $\mathcal{N}(w_{n+1}, w_{n+2}, w_{n+2}, t) \le \mathcal{N}(w_n, w_{n+1}, w_{n+1}, t/r)$.

Thus $\mathcal{M}(w_{n+1}, w_{n+2}, w_{n+2}, t) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/r)$

$$\geq \mathcal{M}(w_{n-1}, w_n, w_n, t/r^2)$$

$$\vdots$$

$$\geq \mathcal{M}(w_0, w_1, w_1, t/r^{n+1})$$

We now obtain for any positive integer q and $t > 0$,

$\mathcal{M}(w_n, w_{n+q}, w_{n+q}, t) \geq \mathcal{M}(w_n, w_{n+1}, w_{n+1}, t/q) * \ldots q$ times

$\ldots * \mathcal{M}(w_{n+q-1}, w_{n+q}, w_{n+q}, t/q) \geq \mathcal{M}(w_0, w_1, w_1, t/qr^n) * \ldots q$ times

$\ldots * \mathcal{M}(w_0, w_1, w_1, t/qr^{n+q-1})$.

Therefore, $\lim_{n\to\infty} \mathcal{M}(w_n, w_{n+q}, w_{n+q}, t) \geq 1 * \ldots q$ times $\ldots * 1 = 1$.

Similarly, $\lim_{n\to\infty} \mathcal{N}(w_n, w_{n+q}, w_{n+q}, t) \leq 0 \lozenge \ldots q$ times $\ldots \lozenge 0 = 0$.

So the sequence $\{w_n\}$ is Cauchy.

Since W is complete, sequence $\{w_n\}$ converges to $w \in W$.

Now $\mathcal{M}(T_m w, w, t) = \lim_{n\to\infty} \mathcal{M}(T_m w, w_{n+2}, w_{n+2}, t)$

$= \lim_{n\to\infty} \mathcal{M}(T_m w, T_{n+2} w_{n+1}, T_{n+2} w_{n+1}, t)$

$\geq \lim_{n\to\infty} \{\mathcal{M}(w, w_{n+1}, w_{n+1}, t/r) * \mathcal{M}(w, w, T_m w, t/r)$

$* \mathcal{M}(w_{n+1}, w_{n+1}, Tw_{n+2} w_{n+1}, t/r) * \mathcal{M}(w, w_{n+1}, T_m w, t/r)$

$* \mathcal{M}(w_{n+1}, T_m w, T_{n+2} w_{n+2}, t/r)\}$

$= \lim_{n\to\infty} \{\mathcal{M}(w, w_{n+1}, w_{n+1}, t/r) * \mathcal{M}(w, w, T_m w, t/r)$

$* \mathcal{M}(w_{n+1}, w_{n+1}, w_{n+2}, t/r) * \mathcal{M}(w, w_{n+1}, T_m w, t/r) * \mathcal{M}(w_{n+1}, T_m w, w_{n+2}, t/r)\}$

$= \{\mathcal{M}(w, w, w, t/r) * \mathcal{M}(w, w, T_m w, t/r) * \mathcal{M}(w, w, w, t/r)$

$* \mathcal{M}(w, w, T_m w, t/r) * \mathcal{M}(w, T_m w, w, t/r)\}$

$= \{1 * \mathcal{M}(w, w, T_m w, t/r) * 1 * \mathcal{M}(w, w, T_m w, t/r) * \mathcal{M}(w, T_m w, w, t/r)\}$

$\geq \mathcal{M}(T_m w, w, t/r)$

$\vdots$

$\geq \mathcal{M}(T_m w, w, t/r^n)$

$\to 1$ as n tends to ∞.

Hence $\mathcal{M}(T_m w, w, w, t) = 1, \forall t > 0$.

Similarly, $\mathcal{N}(T_m w, w, w, t) = 1, \forall t > 0$.

Hence, $T_m w = w$.

Therefore, $T_n w = w \ \forall n$.

Hence w is a common fixed point of $\{T_n\}$.

Uniqueness: Suppose $w \neq z : T_n z = z \ \forall n$.

Then $\mathcal{M}(w, z, z, t) = \mathcal{M}(T_i w, T_j z, T_j z, t)$

$\geq \{\mathcal{M}(w, z, z, t/r) * \mathcal{M}(w, w, T_i w, t/r) * \mathcal{M}(z, z, T_j z, t/r) * \mathcal{M}(w, z, T_i w, t/r)$
$* \mathcal{M}(z, T_i w, T_j z, t/r)\}$

$= \{\mathcal{M}(w, z, z, t/r) * \mathcal{M}(w, w, w, t/r) * \mathcal{M}(z, z, z, t/r) * \mathcal{M}(w, z, w, t/r) *$
$\mathcal{M}(z, w, z, t/r)\}$

$= \{\mathcal{M}(w, z, z, t/r) * 1 * 1 * \mathcal{M}(w, z, w, t/r) * \mathcal{M}(z, w, z, t/r)$

$$\geq \mathcal{M}(w, z, z, t/r)$$

$$\vdots$$

$$\geq \mathcal{M}(w, z, z, t/r^n)$$

$\to 1$ as n tends to ∞.

Hence $\mathcal{M}(w, z, z, t) = 1, \forall t > 0$.

Similarly, $\mathcal{N}(w, z, z, t) = 0, \forall t > 0$.

Therefore, $w = z$.

Which is contradiction to $w \neq z$.

Hence $\{T_n\}$ has a unique common fixed point.

Remark 21. The succeeding corollary 22 is obtained by considering $T_i = T_j = T$ in the preceding theorem.

Corollary 22. *Let* $(W, \mathcal{M}, \mathcal{N}, *, \lozenge)$ *be a complete generalized intuitionistic fuzzy metric space and* $T : W \to W$ *be a mapping*: $\mathcal{M}(Tw, Tz, Tz, rt) \geq \{\mathcal{M}(w, z, z, t) * \mathcal{M}(w, w, Tw, t) * \mathcal{M}(z, z, Tz, t) * \mathcal{M}(w, z, Tw, t) * \mathcal{M}(z, Tw, Tz, t)\}$ $\mathcal{N}(Tw, Tz, Tz, rt) \geq \{\mathcal{N}(w, z, z, t) * \mathcal{N}(w, w, Tw, t) * \mathcal{N}(z, z, Tz, t) * \mathcal{N}(w, z, Tw, t) * \mathcal{N}(z, Tw, Tz, t)\}$ $\forall\, w, z \in Z$ *and* $r \in (0, 1)$. *Then T has a unique fixed point*.

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