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ON SELECTING BEST NEW BETTER THAN USED OF SPECIFIED AGE DISTRIBUTION

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Abstract

The present study considers the problem of selecting the 'Best' population among the several populations which belong to new better than used of age t_0 (NBU-t_0) class of life distributions. The selection procedure is based on a measure of departure from exponentiality towards NBU-t_0 due to Hollander, Park and Proschan [12]. Hollander, Park and Proschan [12] proposed a test procedure based on U-statistics to test exponentiality against NBU-t_0 which is an estimate of measure proposed. The selection procedure is based on large sample properties of the statistic and the performance of the selection procedure proposed here is evaluated in terms of probability of correct selection (PS).

1. Introduction

The principle of ranking and selection of the 'best population' among the several populations is well attended problem in the literature. A best population which can be modeled by some parametric family is selected with reference to a parameter of the family. Lehmann's [8] studied this problem using another approach in which a population is selected as the best based on a procedure using ranks. Barlow and Proschan's [2] approach is based on

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partial orderings of probability distributions. Further, a selection procedure based on means for selecting from class of increasing failure rate (IFR) distributions is given by Patel [10]. An extensive review of ranking and selection problems can be found in Gupta and Panchapakesan [6]. Pandit and Math [9] developed a selection procedure to select a population among several increasing failure rate average (IFRA) populations based on U-statistic. Recently, a selection procedure to select best new better than used (NBU) is developed by Inginashetty and Pandit [7]. Recently, Pandit and Joshi (2017) studied the problem of selecting best decreasing mean residual life (DMRL) population. Here, we consider selection problem of selecting the best population possessing new better than used of specified age $t_0(\text{NBU-}t_0)$ property among several populations possessing $\text{NBU-}t_0$.

Here, we consider selection problems for the $\text{NBU-}t_0$ classes of life distributions. Definition of $\text{NBU-}t_0$ is given below.

Definition. A life distribution of F is new better than used of specified age $t_0(\text{NBU-}t_0)$ if

$$\overline{F}(x + t_0) \leq \overline{F}(x)\overline{F}(t_0) \text{ for all } x \geq 0.$$

Hollander, Park and Proschan [12] gave $\gamma(F) = \int_0^\infty \{\overline{F}(x)\overline{F}(t_0) - \overline{F}(x + t_0)\} dF(x)$ as a measures the degree of $\text{NBU-}t_0$ ness and used it to develop a test for testing exponentially against $\text{NBU-}t_0$ class of distributions. When F is exponential, $\gamma(F) = 0$ and if F is $\text{NBU-}t_0$ $\gamma(F) \geq 0$.

It is easy seen that, if $\gamma(F) \geq \gamma(G)$, then a life distribution F is said to possess more $\text{NBU-}t_0$-ness property than that of another life distribution G.

Here, the criterion for selecting the best distribution possessing more $\text{NBU-}t_0$-ness property among $k\,\text{NBU-}t_0$ populations is based on the value of $\gamma(F)$, assuming that the underlying distribution F is continuous.

The selection procedure here is based on the U-statistics T_n, an unbiased estimate of $\gamma(F)$ which is defined as below:

Let $h_1(X_1, X_2) = \frac{1}{2}\{\psi(X_1, X_2 + t_0) + \psi(X_2, X_1 + t_0)\}$ and $h_2(X_1) = \frac{1}{2}\psi(X_1, t_0)$ be the kernels of degree 2 and 1 corresponding to $T_1(F)$ and $T_2(F)$ respectively where $\psi(a, b) = \begin{cases} 1 \text{ if } a > b \\ 0 \text{ if } a \le b \end{cases}$.

Let

$$T = \frac{1}{2n}\sum_{i=1}^{n} \psi(X_i, t_0) - \frac{1}{n(n-1)}\Sigma' \psi(X_{\alpha_1}, X_{\alpha_2} + t_0).$$

Where Σ' is the sum taken over all $n(n-1)$ sets of two integers (α_1, α_2) such that $1 \le \alpha_i \le n (i = 1, 2)$ and $\alpha_1 \ne \alpha_2$.

In this paper, the asymptotic normality of $T_n(F)$ is used for selecting the best NBU-t_0 distribution among k NBU-t_0 distributions.

2. Selection Procedure

Let $\pi_1, \pi_2, \ldots, \pi_k$ denote the k population with unknown NBU-t_0 distribution function $F_1, F_2, \ldots, F_k$ respectively and hence the function form of F_i and the NBU-t_0 ness measure $\gamma(F_i)$ of F_i as defined earlier are unknown, but it is assumed that F_i are continuous. For the sake of convenience, we denote $\gamma(F_i)$ by γ_i. The goal is to select the population which has largest $\gamma_{[k]}$, where $\gamma_{[1]} \le \gamma_{[2]} \le \ldots \le \gamma_{[k]}$ denote the ordered NBU-t_0-ness measure for k distributions. Selecting largest $\gamma_{[k]}$ is to select the most NBU-t_0 distribution.

Let $\gamma = (\gamma_{[1]}, \gamma_{[2]}, \ldots, \gamma_{[k]})$ and $\underline{\Omega} = \left\{\underline{\gamma} : \frac{1}{m+0} \le \gamma_{[1]} \le \gamma_{[2]} \le \ldots \le \gamma_{[k]} \le 1\right\}$ be the parameter space which is partitioned into a preference zone $\Omega(\delta^*)$ and an indifference zone $\Omega - \Omega(\delta^*)$, where $\Omega(\delta^*)$ is defined by $\Omega(\delta^*) = \{\underline{\gamma} : \gamma_{[2]} - \underline{\gamma}_{[1]} \ge \delta^*\}$.

The quantity $\frac{1}{m+1} \leq \delta^*$ and $\frac{1}{k} < p^* < 1$ are pre-assigned by the experiment and selection procedure R is required to satisfy the condition

$$P(CS \mid R) \geq P^*, \text{ for all } \underline{\gamma} \in \Omega(\delta^*). \tag{A}$$

Selection of any population with $\gamma_{[k]}$ is regarded as the correct selection [CS] and condition (A) is referred to as P^* condition.

The selection procedure here is based on the U-Statistics $T_n(F)$ and utilize the large sample properties of $T_n(F)$.

Let $\xi_1^{(1)} = E\{h_1(X_1, X_2)h_1(X_1, X_3)\} - \{T_1(F)\}^2$

$\xi_2^{(1)} = E\{h_1^2(X_1, X_2)\} - \{T_1(F)\}^2$

$\xi_1^{(2)} = E\{h_2^2(X_1)\} - \{T_2(F)\}^2$ and

$\xi^{(1,2)} = E\{h_1(X_1, X_2) - T_1(F)\}\{h_2(X_1) - T_2(F)\}$. Then

$$Var(T) = \frac{1}{\binom{n}{2}} \sum_{k=1}^{2} \binom{2}{k}\binom{n-2}{2-k} \xi_k^{(1)} + \frac{1}{n}\xi_1^{(2)} - \frac{4}{n}\xi^{(1,2)}$$

and $\sigma^2 = \lim_{n \to \infty} Var(T) = 4\xi_1^{(1)} + \xi_1^{(2)} - 4\xi_1^{(1,2)}$

where $\xi_1^{(1)} = \int\int \overline{F}(x_2 + t_0)\overline{F}(x_3 + t_0)dF(x_2)dF(x_3)\int \overline{F}(x + t_0)dF(x)$

$$\xi_1^{(1)} = \overline{F}(t_0)(1 - \overline{F}(t_0))$$

$$\xi^{(1,2)} = \int \overline{F}(x + t_0)dF(x) - \{\int \overline{F}(x + t_0)dF(x)\}\{\overline{F}(t_0)\}.$$

The asymptotic distribution of $\sqrt{n}(T_n(F) - \gamma(F))$ is normal with mean zero and variance $\sigma^2(F) = 4\xi_1^{(1)} + \xi_1^{(2)} - 4\xi_1^{(1,2)}$. Since, the asymptotic distributions of each $\sqrt{n}(T_n(F))$ is normal with mean $\gamma(F)$ and variance

$4\xi_1^{(1)} + \xi_1^{(2)} - 4\xi_1^{(1,\,2)}$, the problem of selecting the more NBU-t_0 population can be treated as selection of the largest mean of the normal population (see Dudewicz [4] and Dudewicz and Dalal [5]).

A strongly consistent estimator of $\sigma^2(F)$ is obtained by replacing F by its empirical distribution function and denoted as $\hat{\sigma}^2(F)$.

Now a two stage selection procedure R is proposed to select the population possessing more NBU-t_0-ness property among k NBU-t_0 populations based on large samples.

Let $t = t(k,\ p^*) > 0$ be the unique solution of the equation

$$\int_{-\infty}^{\infty} \Phi^{k-1}(z + t) d\Phi(z) = p^*,$$

where $\Phi(\cdot)$ is the distribution function of standard normal random variable.

Procedure *R*:

The selection procedure to select the best NBU-t_0 distribution involves two stages that is as explained below:

Take an initials sample of size no from the population π_{i1} $i = 1, 2, \ldots, k$ and compute $T(F_i)$ and $\hat{\sigma}^2_{n_0}(F_i)$, define $n_i = \max(2n_0, [1/c])$ where $[x]$ denote the smallest integer which is greater than or equal to x and $\frac{1}{c} = \hat{\sigma}^2_{n_0}(F_i)\left(\frac{1}{\delta^*}\right)^2$.

The second stage sample size from π_i denote by n_i is determined as follows:

$$n' = \begin{cases} 0 & \text{if } \left[\frac{1}{c}\right] \leq n_0 \\ n_i - n_0 & \text{if } \left[\frac{1}{c}\right] > n_0. \end{cases}$$

Compute $T_{n_i}(F_i)$ based on n_i, the additional sample taken from π_i and define $U_i = a_i T_{n_0}(F_i) + (1 - a_i) T_{n_i}(F_i)$, where $0 < a_i < 1$ is determined to satisfy $\hat{\sigma}^2_{n_0}(F_i)\left[\frac{a_i^2}{n_0} + \frac{(1-a_i)^2}{n_i}\right] = \left(\frac{\delta^*}{t}\right)^2$.

Here, $\hat{\sigma}^2_{n_0}(F_i)\left[\frac{a_i^2}{n_0} + \frac{(1-a_i)^2}{n_i}\right]$ is strongly consistent estimator for variance of U_i.

Note. If $n_i = 0$, i.e., no second-stage sample is taken from the i^{th} population, we take $a(1 - a_i)T_{n_i}(F) = 0$ and $(1 - a_i^2)n_i = 0$.

Lemma. *There exists* a_i *satisfying*

$$\hat{\sigma}^2_{n_0}(F_i)\left[\frac{a_i^2}{n_0} + \frac{(1-a_i)^2}{n_i}\right] = \left(\frac{\delta^*}{t}\right)^2.$$

Proof. Define

$$a_i = \begin{cases} 1 & \text{if } \frac{1}{c} \le n_0 \\ \frac{1}{2}\left(1 + \sqrt{(2n_0 c - 1)}\right) & \text{if } n_0 < \frac{1}{c} \le 2n_0. \\ \frac{n_0}{n_1} & \text{if } \frac{1}{c} > 2n_0 \end{cases}$$

Then, it is straight forward to show that a_i defined above satisfies $\hat{\sigma}^2_{n_0}(F_i)\left[\frac{a_i^2}{n_0} + \frac{(1-a_i)^2}{n_i}\right] = \left(\frac{\delta^*}{t}\right)^2$ as $\hat{\sigma}^2_{n_0}(F_i)\left[\frac{t}{\delta^*}\right]^2 = \frac{1}{c}$.

It is to be noted that a_i can also chosen to be $(1 - (\sqrt{2n_0 c - 1}))/2$ and $(1 + (\sqrt{2n_0 c - 1}))/2$. In such a case, the initial sample size n_0 is equal to the additional sample size n_i and the coefficients a_i and $(1 - a_i)$ become interchangeable.

Theorem. *For any* p^*, $p^* \in \left(\frac{1}{k}, 1\right)$ *there exists* n_0 *large enough such that* $\inf_{\Omega(\delta^*)} p(CS \mid R) \cong p^*$.

Proof. For any i, we can write

$$P\left(\frac{U_i-\gamma_i}{(\delta^*/t} \leq y\right) = P\left[\frac{U_i-\gamma_i}{\left(\hat{\sigma}^2_{n_0}(F_i)\left[\frac{a_i^2}{n_0}+\frac{(1-a_i)^2}{n_i}\right]\right)} \leq y\right]$$

$$= E_{\hat{\sigma}^2_{n_0}(F_i)} P\left[\frac{U_i-\gamma_i}{\sqrt{\left(\hat{\sigma}^2_{n_0}(F_i)\left[\frac{a_i{}^2}{n_0}+\frac{(1-a_i)^2}{n_i'}\right]\right)}} \leq y\frac{\sqrt{\hat{\sigma}^2_{n_0}(F_i)}}{\sqrt{\hat{\sigma}^2_{n_0}(F_i)}} \;\middle|\hat{\sigma}^2_{n_0}(F_i)\right]$$

$$\cong E_{\hat{\sigma}^2_{n_0}(F_i)} \Phi\left[y\frac{\sqrt{\hat{\sigma}^2(F_i)}}{\sqrt{\sigma^2(F_i)}} \mid \hat{\sigma}^2_{n_0}(F_i)\right] \cong \Phi(y) \tag{B}$$

as $\left[\frac{\sqrt{\hat{\sigma}^2_{n_0}(F_i)}}{\sqrt{\sigma^2_{n_0}}}\right] \to 1.$

Let $U_{(i)}$ denote the statistic corresponding to the population having parameter $\gamma_{[i]}$, $i = 1, 2, \ldots, k$.

Then, we have

$$P(CS \mid R) = P[U_{(i)} \geq U_{(1)}, i = 1, 2, \ldots, k-1]$$

$$= P\left[\frac{U_{(i)} - \gamma_{[i]}}{\delta^*/t} > \frac{U_{(1)} - \gamma_{(1)}}{\delta^*/t} - \frac{\gamma_{[i]} - \gamma_{[1]}}{\delta^*/t}, i = 1, 2, 3, \ldots, k-1\right]$$

$$\cong \int_{-\infty}^{\infty} \underset{i=1}{\overset{k-1}{\pi}} \left[1 - \Phi\left(z - \frac{\gamma_{[i]} - \gamma_{[1]}}{\delta^*/t}\right)\right] d\Phi(z) \tag{B'}$$

$$\geq \int_0^{\infty} \Phi^{k-1}(z-t) d\Phi(z)$$

$$= \int_0^{\infty} \Phi^{k-1}(z-t) d\Phi(z) \tag{C}$$

$$= p^*. \tag{D}$$

Since $U'_{(i)}s$ are independent.

The inequality (C) is true since the right hand side of (B') is minimized, when $\gamma_{[1]} = \gamma_{[2]} = \ldots = \gamma_{[k-1]} = \gamma_{[k]}$ and (D) follows from

$$\int_{-\infty}^{\infty} \Phi^{k-1}(z-t)d\Phi(z) = p^*.$$

To select the most NBU-t_0 distribution, we select the distribution which yields $U_{[k]}$. In this case the preference zone is defined for fixed δ^* as $\{\gamma : \gamma_{[2]} - \gamma_{[1]} \geq \delta^*\}$, $\delta^* > 0$. The probability of correct selection is given by $P\lfloor U_{(i)} \geq U_{[1]};\ i = 1, 2, \ldots, k-1 \rfloor$.

This probability of Correct Selection is minimized when $\gamma_{[1]} = \gamma_{[2]} = \ldots = \gamma_{[k-1]} = \gamma_{[k]}$ and minimum value is the assigned P^* used compute t using $\int_{-\infty}^{\infty} \Phi^{k-1}(z+t)d\Phi(z) = p^*$.

3. Simulation Study

The performance of the selection procedure R is evaluated using simulation for linear failure rate (LFR), Makeham and NBU-t_0 distributions. The distributions functions of the life distributions considered for simulation are given below:

1. Linear failure rate (LFR) distribution:

$$F(x) = 1 - \exp\left\{-\left(x + \frac{\theta x^2}{2}\right)\right\},\ x \geq 0,\ \theta \geq 0.$$

2. Makeham Distribution:

$$F(x) = 1 - \exp\{-(x + \theta(x + \exp\{-x\} - 1))\},\ x \geq 0,\ \theta \geq 0.$$

3. NBU-t_0 Distribution:

$$F(x) = 1 - \exp\left\{-x - \frac{\theta}{2t_0}x^2\right\} I(0 \leq x \leq t_0).$$

In table 1, the number of populations, values of parameters and initial sample size considered for simulation are presented. A simulation study is conducted to estimate $P(CS)$ and expected sample size. Tables 2 gives values of $\gamma(F)$ for different values of θ. The selection procedure is repeated 1000 times and the proportion of correct selection are computed. The estimate of $P(CS)$ is the proportion of correct selection in 1000 repetitions and the estimate of expected sample size is the average of the total sample sizes needed to reach the decision. The estimates of probability of correct decision $P(CS)$ and expected sample size for LFR, Makeham and NBU-t_0 distributions are tabulated in tables 3, 4 and 5 respectively. The value of t corresponding $k = 3$ and $k = 4$ are read from Bechhofer [3] and the values of t are 1.6524 and 1.8932 respectively when $P^* = 0.80$.

Table 1. Values of k, θ and n_0 considered for simulation.

Life Distributions	K	Θ	Initial sample size n_0
LFR	3	1,2,3,	15, 30, 40, 50
	4	1,2,3,4	15, 30, 40, 50
Makeham	3	1,2,3	15, 30, 40, 50
	4	1,2,3,4	15, 30, 40, 50
NBU-t_0	3	2,3,4	15, 30, 40, 50
	4	2,3,4,5	15, 30, 40, 50

Table 2. Total Expected Sample Size Estimates and $P(CS)$ among k LFR distributions with $P^* = 0.8$.

LFR	$t_0 = 0.2$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	42		39		41		58	
$\delta^* = 0.0005$	2	50	0.82	31	0.89	56	0.90	64	0.91
	3	39		54		42		51	
$k = 4$	1	41		46		50		75	
$\delta^* = 0.0005$	2	66	0.85	87	0.90	63	0.93	84	0.90
	3	54		36		69		59	
	4	42		43		52		51	

LFR	$t_0 = 0.5$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	42		32		43		83	
$\delta^* = 0.0005$	2	49		65		56		87	0.93
	3	38	0.85	60	0.91	71	0.92	91	
$k = 4$	1	56		38		32		56	
$\delta^* = 0.0005$	2	51	0.87	45	0.90	65	0.95	73	0.94
	3	32		42		74		85	
	4	21		81		92		65	

LFR	$t_0 = 1.0$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$ $\delta^* = 0.0005$	1	41	0.87	31	0.91	35	0.93	51	0.94
	2	64		43		49		72	
	3	89		54		41		32	
$k = 4$ $\delta^* = 0.0005$	1	56	0.87	81	0.93	47	0.97	71	0.96
	2	79		83		62		80	
	3	91		65		65		65	
	4	93		42		50		50	

LFR	$t_0 = 1.6$								
		$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$ $\delta^* = 0.0005$	1	42	0.87	43	0.93	41	0.91	81	0.95
	2	43		61		74		78	
	3	31		56		69		86	
$k = 4$ $\delta^* = 0.0005$	1	65	0.89	31	0.95	30	0.97	51	0.96
	2	50		42		61		70	
	3	31		41		70		83	
	4	22		74		90		45	

LFR	$t_0 = 2.0$								
		$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	39		82		54		64	
$\delta^* = 0.0005$	2	56		35		86		101	0.97
	3	31	0.84	64	0.90	51	0.93	64	
$k = 4$	1	43		67		87		56	
$\delta^* = 0.0005$	2	60	0.87	43	0.92	54	0.96	80	0.98
	3	53		81		60		52	
	4	41		40		43		65	

Table 3. Total Expected Sample Size Estimates and $P(CS)$ among k Makeham distributions with $P^* = 0.8$.

Makeham	$t_0 = 0.2$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	45		89		90		65	
$\delta^* = 0.0007$	2	57		56		54		78	0.92
	3	62	0.84	60	0.86	73	0.87	90	
$k = 4$	1	56		49		90		84	
$\delta^* = 0.0007$	2	78	0.82	91	0.88	65	0.90	77	0.91
	3	43		42		83		69	
	4	58		53		79		91	

Makeham	$t_0 = 0.5$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	35		90		43		84	
$\delta^* = 0.0007$	2	43		86		90		98	0.93
	3	56	0.85	45	0.91	73	0.92	90	
$k = 4$	1	89		58		54		106	
$\delta^* = 0.0007$	2	43	0.87	87	0.90	89	0.95	54	0.96
	3	62		90		65		76	
	4	45		85		79		85	

Makeham	$t_0 = 1.0$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	34		56		89		65	
$\delta^* = 0.0007$	2	65		71		71		77	0.96
	3	92	0.86	84	0.87	73	0.92	92	
$k = 4$	1	55		74		65		88	
$\delta^* = 0.0007$	2	86	0.88	92	0.90	87	0.95	102	0.97
	3	43		65		101		91	
	4	82		55		75		65	

Makeham	$t_0 = 1.6$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	56		85		65		67	
$\delta^* = 0.0007$	2	48		78		91		74	0.97
	3	83	0.88	54	0.90	43	0.94	96	
$k = 4$	1	92		55		90		88	
$\delta^* = 0.0007$	2	44	0.89	97	0.92	54	0.95	51	0.98
	3	76		91		83		79	
	4	87		74		91		105	

Makeham	$t_0 = 2.0$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	56		78		49		54	
$\delta^* = 0.0007$	2	79		59		67		91	0.98
	3	32	0.89	98	0.90	91	0.95	76	
$k = 4$	1	89		86		52		65	0.98
$\delta^* = 0.0007$	2	94	0.90	92	0.94	63	0.97	87	
	3	65		79		82		61	
	4	52		54		95		109	

Table 4. Total Expected Sample Size Estimates and $P(CS)$ among k NBU-t_0 distributions with $P^* = 0.8$.

NBU-t_0	$t_0 = 0.2$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$ $\delta^* = 0.0005$	1	69	0.86	70	0.87	47	0.89	58	0.90
	2	70		62		80		92	
	3	50		39		92		86	
$k = 4$ $\delta^* = 0.0005$	1	98	0.87	72	0.88	95	0.90	65	0.91
	2	49		86		85		90	
	3	73		54		42		76	
	4	65		90		76		93	

NBU-t_0	$t_0 = 0.5$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$ $\delta^* = 0.0005$	1	67	0.87	78	0.88	65	0.89	87	0.89
	2	89		90		80		95	
	3	54		45		71		65	
$k = 4$ $\delta^* = 0.0005$	1	90	0.87	97	0.89	101	0.90	71	0.92
	2	87		65		62		86	
	3	65		96		90		83	
	4	72		50		72		73	

NBU-t_0		$t_0 = 1.0$							
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	45		40		86		91	
$\delta^* = 0.0005$	2	82		96		80		80	0.91
	3	50	0.88	72	0.89	91	0.90	74	
$k = 4$	1	60		52		83		76	
$\delta^* = 0.0005$	2	49	0.89	90	0.90	104	0.92	60	0.92
	3	76		91		74		92	
	4	80		80		65		74	

NBU-t_0		$t_0 = 1.6$							
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	78		45		56		85	
$\delta^* = 0.0005$	2	65		76		94		91	0.98
	3	49	0.89	59	0.90	53	0.96	83	
$k = 4$	1	80		32		80		89	
$\delta^* = 0.0005$	2	62	0.90	50	0.95	91	0.97	95	0.98
	3	75		96		73		93	
	4	79		57		90		98	

NBU-t_0	$t_0 = 2.0$								
	θ	$n_0 = 15$		$n_0 = 30$		$n_0 = 40$		$n_0 = 50$	
		EN	P(CS)	EN	P(CS)	EN	P(CS)	EN	P(CS)
$k = 3$	1	86		96		75		83	
$\delta^* = 0.0005$	2	72		52		93		95	0.98
	3	65	0.90	75	0.92	50	0.97	55	
$k = 4$	1	89		82		69		94	
$\delta^* = 0.0005$	2	65	0.89	90	0.92	89	0.98	101	0.99
	3	86		63		75		88	
	4	82		92		80		86	

Applications

A significant application of the procedure R is that, it can be used to select the "best" distribution according to its NBU-t_0-ness property to a number of interesting situations, for which no other selection procedures are available. It can be used to select among k given life distributions, with the same functional form but different shape parameters, the distribution with the largest mean life or the distribution with the largest mean residual life at a fixed time $t_0 > 0$. The procedure can be used to select the best of k NBU-t_0 distributions all of which have the same mean.

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