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PAIRED DOMINATION DECOMPOSITION FOR SOME GRAPHS

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Abstract

Let S be a dominating set of G. A Decomposition $(H_1, H_2, \ldots, H_n)$ of G is said to be paired domination decomposition (pdd) if (i) $E(G) = E(H_1) \cup E(H_2) \cup \ldots \cup E(H_n)$. (ii) Each G_i is connected and in each G_i, $\langle S \rangle$ is a paired dominating set. (iii) $\gamma_{pdd}(G_i) = 2i, 1 \leq i \leq n$. In this paper we find paired domination decomposition for some graphs.

1. Introduction

A set S of vertices of G is a dominating set of G if every vertex in $V(G) - S$ is adjacent to some vertex in S. A minimum dominating set in a graph G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the domination number of G and is denoted by $\gamma(G)$. Paired domination was introduced by T. W. Haynes and P. J. Slater. The concept of Edge domination was introduced by Mitchell and Hedetniemi A set $F \subseteq E$ is an edge dominating set if each edge in E is either

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in F or adjacent to an edge in F. We determined Annihilator domination decomposition for some graphs [1] now we explored it as paired domination decomposition. We provide a brief summary of definitions which are useful for the present investigation.

2. Definitions and Main Results

Definition 2.1. A set $S \subseteq V$ is a paired dominating set [5] if S dominates and the induced subgraph $\langle S \rangle$ has a perfect matching. The paired domination number $\gamma_{pr}(G)$ is the minimum cardinality of a paired dominating set S in G.

Definition 2.2. If U is a nonempty subset of the vertex set $V(G)$ of a graph G, then the subgraph U of G induced by U is the graph having vertex set U and whose edge set consists of those edges of G incident with two elements of U.

Definition 2.3. Two distinct vertices or edges in a graph G are independent if they are not adjacent in G. A set of pair wise independent edges of G is called a matching in G, while a matching of maximum cardinality is a maximum matching in G. If M is a matching in a graph G with the property that every vertex of G is incident with an edge of M, then M is perfect matching in G.

Definition 2.4. The Corona $G_1 \circ G_2$ is the graph obtained by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

Definition 2.5. Let $G = (V, E)$ be a simple connected graph with p vertices and q edges. If $H_1, H_2, \ldots, H_n$ are edge disjoint subgraphs of G with $E(G) = E(H_1) \cup E(H_2) \cup \ldots \cup E(H_n)$, then $(H_1, H_2, \ldots, H_n)$ is said to be a decomposition of G.

Definition 2.6. Let $G = (V, E)$ be a simple graph with p vertices and q edges. Let S be a dominating set of G. A Decomposition $(H_1, H_2, \ldots, H_n)$ of G is said to be paired domination decomposition (pdd) [3] if

(i) $E(G) = E(H_1) \cup E(H_2) \cup \ldots \cup E(H_n)$,

(ii) In each G_i, $\langle S \rangle$ is a paired dominating set.

(iii) $\gamma_{pdd}(G_i) = 2i \; 1 \leq i \leq n$.

Notation 2.7. Paired domination decomposition number of G is denoted by $\gamma_{pdd}(G_i)$.

Definition 2.8. An edge in order to form a single connected graph by connecting disconnected graphs is called a wedge. It is denoted by $\wedge$.

Theorem 2.9. *Path graph* P_{2x^2+5x+4} admits *paired domination decomposition* $(G_1, G_2, \ldots, G_{x+1})$ *such that*

$$\gamma_{pd}(P_{2x^2+5x+4}) = \begin{cases} \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) \; for \; x = 1, 2, 3 \\ \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) \; for \; x > 3 \; and \; k = 2\left\lfloor \frac{x}{4} \right\rfloor. \end{cases}$$

Proof. Let G be a path graph P_{2x^2+5x+4}. Let $v_1, v_2, \ldots, v_{2x^2+5x+4}$ be the vertices of P_{2x^2+5x+4}. Let G_1 be the v_1v_4 path. Then $\gamma_{pdd}(G_1) = 2$. Let G_2 be the v_4v_{11} path. Therefore $\gamma_{pdd}(G_2) = 4$. Let G_3 be the $v_{11}v_{22}$ path. Then $\gamma_{pdd}(G_3) = 6$. G_4 is nothing but the $v_{22}v_{37}$ path. Therefore $\gamma_{pdd}(G_4) = 8$. Let G_i be the path having $4i$ vertices. Then $\gamma_{pdd}(G_i) = 2i$. Let G_{x+1} be the $v_jv_{2x^2+5x+4}$ path where v_j is the last vertex of G_x. Hence G can be decomposed into $(G_1, G_2, \ldots, G_{x+1})$. Also each G_i has paired dominating set. Therefore

$$\sum_{i=1}^{x+1} \gamma_{pdd}(G_i) = 2\left(\frac{(x+1)(x+2)}{2}\right)$$

$$= x^2 + 3x + 2$$

$$\gamma_{pd}(P_{2x^2+5x+4}) = \begin{cases} \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) \text{ for } x = 1, 2, 3 \\ \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) \text{ for } x > 3 \text{ and } k = 2\left\lfloor \frac{x}{4} \right\rfloor. \end{cases}$$

Result 2.10. Path graphs $P_{m-1}, P_{m-2}, P_{m-3}, \ldots, P_{m-(3x+1)}$ where $m = 2x^2 + 5x + 4$ can also admits Paired domination decomposition $(G_1, G_2, \ldots, G_{x+1})$.

Corollary 2.11. *Cycle graph C_{2x^2+5x+3} admits paired domination decomposition $(G_1, G_2, \ldots, G_{x+1})$ such that*

$$\gamma_{pd}(C_{2x^2+5x+3}) = \begin{cases} \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) \text{ for } x = 1, 2 \\ \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) - 2i \text{ for } x = 4i - 1 \text{ to } 4i + 2 \text{ and } i \geq 1. \end{cases}$$

Result 2.12. Cycle graph $C_{m-1}, C_{m-2}, C_{m-3}, \ldots, C_{m-(3x+1)}$ where $m = 2x^2 + 5x + 3$ can also admits Paired domination decomposition $(G_1, G_2, \ldots, G_{x+1})$.

Theorem 2.13. *Wheel graph W_{2x^2+2x} admits paired domination decomposition $(G_1, G_2, \ldots, G_{x+1})$ such that*

$$\gamma_{pd}(W_{2x^2+2x}) = \sum_{i=1}^{x+1} \gamma_{pdd}(G_i) - (x^2 - 3x).$$

Result 2.14. $W_{m+1}, W_{m+2}, \ldots, W_{m-3x}$ where $m = 2x^2 + 2x$ also admits paired domination decomposition $(G_1, G_2, \ldots, G_{x+1})$.

Corollary 2.15. *Helm graph H_{m^2+3m} admits paired domination decomposition $(G_1, G_2, \ldots, G_{m+1})$ such that $\gamma_{pd}(H_{m^2+3m}) = \sum_{i=1}^{x+1} \gamma_{pdd}(G_i)$.*

Theorem 2.16. *Banana tree graph $B(m, n)$; $m = \frac{x^2 + 3x + 2}{2}$ satisfies paired domination decomposition $(G_1, G_2, \ldots, G_{m+1})$ such that $\gamma_{pd}(B(m, n)) = \sum_{i=1}^{x+1} \gamma_{pdd}(G_i)$.*

Theorem 2.17. *The graphs $G_{\frac{n^2 3n+2}{2}} \circ K_{1, n}$ admit paired domination* *decomposition* $(G_1, G_2, \ldots, G_{m+1})$ *such that*

$\gamma_{pd}(G_{\frac{n^2+3n+2}{2}} \circ K_{1,n}) = \sum_{i=1}^{x+1} \gamma_{pdd}(G_i)$ *where* $G_{\frac{n^2 3n+2}{2}}$ *is a cycle or a complete graph on* $\frac{n^2+3n+2}{2}$, $n \geq 1$ *vertices.*

Theorem 2.18. *Wedge graph* $K_{1,n} \wedge P_{2x^2+2x}$ *admits paired domination decomposition* $(G_1, G_2, \ldots, G_{x+1})$ *such that*

$$\gamma_{pd}(K_{1,n} \wedge P_{2x^2+2x}) = \sum_{i=1}^{x+1} \gamma_{pdd}(G_i).$$

Proof. Let u be the apex vertex of the star graph $K_{1,n}$. Let $\{u_1, u_2, \ldots, u_n\}$ be the pendant vertices adjacent to u. Let $v_1, v_2, \ldots, v_{2x^2+2x}$ be the vertices of the path graph P_{2x^2+2x}. These two graphs are connected by an edge uv_1 to obtain wedge graph $K_{1,n} \wedge P_{2x^2+2x}$. Let G_1 be the graph $K_{1,n}$. Therefore $\gamma_{pdd}(G_1) = 2$. Let G_2 be the v_1v_4 path along with the edge uv_1. Then $\gamma_{pdd}(G_2) = 4$. Let G_3 be the v_4v_{12} path. Therefore $\gamma_{pdd}(G_3) = 6$. G_4 is nothing but the $v_{12}v_{24}$ path. Therefore $\gamma_{pdd}(G_4) = 8$. For $x > 1$ let G_i, $i > 2$ be the v_lv_k path having $4i - 3$ vertices. Therefore $\gamma_{pdd}(Gi) = 2i$. For $x > 1$. Let G_{x+1} be the $v_jv_{2x^2+2x}$ path having $4x + 1$ vertices. Then $\gamma_{pdd}(G_{x+1}) = 2(x+1)$. Hence G is decomposed as $(G_1, G_2, \ldots, G_{x+1})$ such that $\gamma_{pd}(K_{1,n} \wedge P_{2x^2+2x}) = \sum_{i=1}^{x+1} \gamma_{pdd}(G_i)$.

Result 2.19. Wedge graph $K_{1,n} \wedge P_{m+1}, K_{1,n} \wedge P_{m+2}, \ldots, K_{1,n} \wedge P_{m+3x}$; $m = 2x^2 + 2x - 1$ also admits paired domination decomposition $(H_1, H_2, \ldots, H_{x+1})$.

Results 2.20.

1. Complete bipartite graph does not admit paired domination decomposition.

2. Star graph does not admit paired domination decomposition, since, $\gamma_{pdd}(G_i) = 2$ for all i.

3. Complete graph does not admit paired domination decomposition.

4. Graphs having isolated vertices does not admit paired domination decomposition.

3. Conclusion

In this paper we have define paired domination decomposition and shown that some graphs concede it.

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