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A NEW TYPE OF DOMINATING FUZZY GRAPHS

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Abstract

A fuzzy graph $G:(\sigma, \mu)$ with underlying graph $G^*:(\sigma^*, \mu^*)$ be given. Let G^* be (V, E) and S is the collection of all minimal dominating set of G. The dominating fuzzy graph of G is denoted by $D(G):(\sigma_D, \mu_D)$ with vertex set the disjoint union of VUS. In this paper, characterizations are given for fuzzy graphs whose dominating fuzzy graph is connected and complete. Some other properties of dominating fuzzy graphs are also obtained.

1. Introduction

The study of domination set was initiated by Ore [11] and Berge [1]. The domination number and the independent domination number were introduced by Cockayne and Hedetniemi [2]. Rosenfeld [12] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Somasundaram and Somasundaram [13] discussed domination in fuzzy graphs. They defined domination using effective edges in fuzzy graphs. Nagoor Gani and Chandrasekaran [8, 9] discussed domination in fuzzy graph using strong arcs. Nagoor Gani and Vadivel [10] discussed domination, independent domination and irredundance in fuzzy graphs using strong arcs. V. R. Kulli et al. [6,7] introduced various type of dominating graphs which are graph valued functions in the field of domination theory. B. Basavanagoud and S. M. Hosamani [4] introduced a new class of intersection graphs in the field of

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domination theory. In this paper we introduce a new type of fuzzy graph in the field of fuzzy domination theory.

2. Preliminaries

Definition 2.1. A fuzzy graph $G : (\sigma, \mu)$ is a pair of function $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ where $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for $u, v \in V$.

The underlying crisp graph of $G = (\sigma, \mu)$ is denoted by $G^* = (V, E)$ where $V = \{u \in V : \sigma(u) > 0\}$ and $E = \{(u, v) \in V \times V : \mu(u, v) > 0\}$.

Definition 2.2. The order p and size q of the fuzzy graph $G = (\sigma, \mu)$ are defined by $p = \sum_{v \in V} \sigma(v)$ and $q = \sum_{(u,v) \in E} \mu(u, v)$.

Definition 2.3. A path ρ in a fuzzy graph is a sequence of distinct nodes $x_0, x_1, x_2, \ldots, x_n$ such that $\mu(x_{i-1}, x_i) > 0, 1 \leq i \leq n$; here $n \geq 0$ is called the length of the path.

Definition 2.4. An arc (u, v) in a fuzzy graph $G = (\sigma, \mu)$ is said to be strong if $\mu^{\infty}(u, v) = \mu(u, v)$ and the node v is said to be strong neighbor of u. If $\mu(u, v) = 0$ for every $v \in V$ then u is called isolated node.

Definition 2.5. Two nodes of a fuzzy graph are said to be fuzzy independent if there is no strong arc between them. A subset S of V is said to be a fuzzy independent set of G if any two nodes of S are fuzzy independent.

Definition 2.6. The strength of connectedness between two nodes u, v in a fuzzy graph G is $\mu^{\infty}(u, v) = \sup\{\mu^k(u, v); k = 1, 2, 3, \ldots\}$ where $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \mu(u, u_2) \wedge \ldots \wedge \mu(u_{k-1}, v)\}$.

Definition 2.7. Let $G = (\sigma, \mu)$ be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every $v \in V - D$ there exists $u \in D$ such that (u, v) is a strong arc. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set.

3. Dominating Fuzzy Graphs

Definition 3.1. A fuzzy graph $G = (\sigma, \mu)$ with underlying crisp graph $G^* : (\sigma^*, \mu^*)$ be given. Let G^* be (V, E), S is the collection of all minimal dominating set of G. The dominating fuzzy graph of G is denoted by $D(G) : (\sigma_D, \mu_D)$ with node set the disjoint union of $V \cup S$, where

$\sigma_D(u) = \sigma(u)$ if $u \in \sigma^*$

$= \mu^{\infty}(u, v)$ if $u, v \in \mu^*$

$= 0$ otherwise.

$\mu_D(v_i, v_j) = 0$ if $v_i, v_j \in \sigma^*$

$\mu_D(v_i, e_j) = \mu(e_j)$ if $v_i \in \sigma^*, e_j \in \mu^*$

$= 0$ otherwise.

As σ_D is defined only through the values of σ and μ, $\sigma_D : V \cup E \to [0, 1]$ is a well- defined fuzzy subset on $V \cup E$. Also μ_D is a fuzzy relation on σ_D and $\mu_D(u, v) \leq \sigma_D(u) \Lambda \sigma_D(v) \forall u, v$ in $V \cup E$.

Example 3.2.

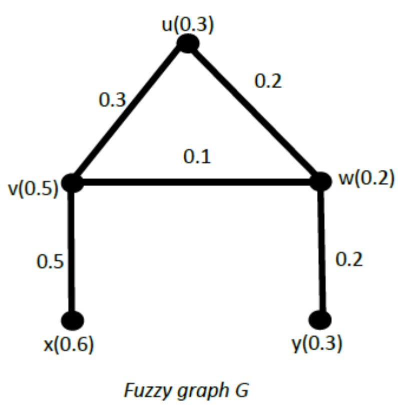

Figure 3.1

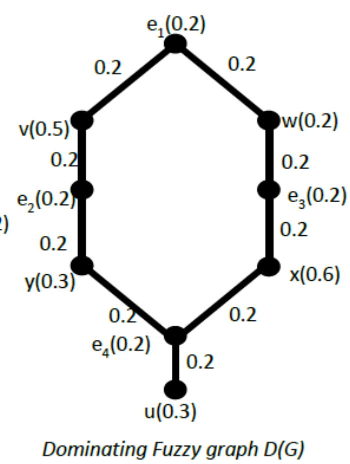

Figure 3.2

Properties of a dominating fuzzy graph

Theorem 3.3. *Order* $[D(G)] = \text{Order}[G] + \sum_{(u)\in S} \mu^{\infty}(u)$.

Proof of Theorem 3.3.

$$\text{Order } [D(G)] = \sum_{u\in V\cup S} \sigma_D(u)$$

$$= \sum_{u\in V} \sigma_D(u) + \sum_{u\in S} \sigma_D(u)$$

$$= \sum_{u\in\sigma^*} \sigma(u) + \sum_{u\in\mu^*} \mu^{\infty}(u) \text{ [By definition 3.1]}$$

$=$ Order $[G]+$ Sum of the weights of strongest arc for all minimal dominating set of G.

Hence, Order $[D(G)] = \text{Order}[G] + \sum_{(u)\in S} \mu^{\infty}(u)$.

Theorem 3.4. *Size* $D[G] = |S| \sum_{(u)\in S} \mu^{\infty}(u)$.

Proof of Theorem 3.4.

$$\text{Size } D[G] = \sum_{u, v\in V\cup S} \mu_D(u, v)$$

$$= \sum_{u, v\in V} \mu_D(u, v) + \sum_{u, v\in S} \mu_D(u, v) + \sum_{u\in V, v\in S} \mu_D(u, v)$$

$$= 0 + 0 + \sum_{v_i\in\sigma^*, e_j\in\mu^*} \mu(e_j) \text{ [By Definition 3.1]}$$

= Sum of the weights of the strongest arc connecting the nodes in S for all minimal dominating set of G, as each arc in E is incident with exactly number of nodes in the corresponding minimal dominating set.

Hence, Size $D[G] = |S| \sum_{(u)\in S} \mu^{\infty}(u)$.

Theorem 3.5. *A Dominating Fuzzy Graph* $D(G)$ *is a strong fuzzy graph*.

Proof of Theorem 3.5.

Consider an edge (u, v) is $D(G)$.

Then $\mu_D(u, v) = 0$ if $u, v \in \sigma^*$

$\mu_D(u, v) = \mu(e_j)$ if $u \in \sigma^*$, $v = e_j \in \mu^*$

where $\mu(e_j) = \sigma_D(u)$

$= \mu^{\infty}(u)$ if $u \in \mu^*$.

Since $\mu^{\infty}(u)$ is a strength of connectedness between the nodes of minimal dominating set.

Therefore, every arc of $D(G)$ is a strong arc.

Hence $D(G)$ is a strong fuzzy graph.

Theorem 3.6. *$D(G)$ is not a complete fuzzy graph even if G is a complete fuzzy graph.*

Proof of Theorem 3.6. Given G is a complete fuzzy graph, then every pair of vertices are adjacent in G^*.

But by the definition of $D(G)$, $\mu_D(v_i, v_j) = 0$ if $v_i, v_j \in \sigma^*$ i.e.) No two nodes in $V(G)$ are neighbours in $D(G)$. So $D(G)$ is not a complete fuzzy graph.

Theorem 3.7. *If G is a complete fuzzy graph, then* $|E(D(G))| = 0$.

Proof of Theorem 3.7. Given G is a complete fuzzy graph.

Then every node of G is a minimal dominating set.

By the definition of $D(G)$, every edge in the dominating fuzzy graph is a strength of connectedness between the nodes of minimal dominating set.

Since $\mu^{\infty}(u) = 0$, then the graph will be a null graph.

Hence $|E(D(G))| = 0$.

Theorem 3.8. *For any fuzzy graph G, the dominating fuzzy graph $D(G)$ is a fuzzy bipartite graph.*

Proof of Theorem 3.8. The dominating fuzzy graph is a disjoint union of $V\ U\ S$, where S is the collection of all minimal dominating set of G.

Then the node set V can be partitioned into two nonempty sets V and S such that V and S are fuzzy independent sets.

Every arc in $D(G)$ is a strong arc since $D(G)$ is a strong fuzzy graph.

Thus every strong arc of $D(G)$ has one end in V and the other end in S.

Hence $D(G)$ is a fuzzy bipartite graph.

Theorem 3.9. *For any dominating fuzzy graph* $\gamma(D(G)) = p$ *if and only if* $G = K_p$.

Proof of Theorem 3.9. If G is a complete fuzzy graph with p vertices, then every vertices of G is a minimal dominating set.

We have p minimal dominating set for a given fuzzy graph G.

By the definition of dominating fuzzy graph $D(G)$, will be a null graph with p vertices.

Hence the cardinality of minimal dominating set of the dominating fuzzy graph is equal to p.

Theorem 3.10. *For a dominating fuzzy graph* $D(G), \delta(D(G)) \leq \gamma(G) \leq \Gamma(G) \leq \Delta(D(G))$.

Proof of Theorem 3.10. Here we consider only a minimal dominating sets of G. Let $\gamma(G)$ be a minimum cardinality of the minimal dominating set and $\Gamma(G)$ be a maximum cardinality of the minimal dominating set.

The degree of vertices of $D(G)$ is depending upon the minimal dominating set of G.

If $u \in S$, then $d(D(G)) =$ number of vertices which are in the minimal dominating set.

If $u \in V$, then $d(D(G)) =$ a vertex is exactly the number of times occurs in the minimal dominating set.

Hence the maximum and minimum degree of the dominating fuzzy graph lies in this interval $\delta(D(G)) \leq \gamma(G) \leq \Gamma(G) \leq \Delta(D(G))$.

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