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LEVEL OPERATORS ON BIPOLAR INTUITIONISTIC FUZZY α-IDEAL AND BIPOLAR INTUITIONISTIC ANTI FUZZY α-IDEAL OF A BP-ALGEBRA

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Abstract

The concept of a bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzy α-ideal are a new algebraic structure of BP-algebra and to use level operators. The purpose of this study is to implement the fuzzy set theory and ideal theory of a BP-algebra. The relation between the operation of level operators on bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzy α-ideal are established.

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1. Introduction

The concept of fuzzy sets was initiated by L. A. Zadeh [14] then it has become a vigorous area of research in engineering, medical science, graph theory. S. S. Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was introduced by K. J. Lee [6] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval $[0, 1]$ to $[-1, 1]$. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree $(0, 1]$ indicates that elements somewhat satisfy the property and the negative membership degree $[-1, 0)$ indicates that elements somewhat satisfy the implicit counter property. The author W. R. Zhang [15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1998. K. Chakrabarthy and Biswas R. Nanda [3] investigated note on union and intersection of intuitionistic fuzzy sets. A. Rajeshkumar [13] was analyzed fuzzy groups and level subgroups. M. Palanivelrajan, K. Gunasekaran and S. Nandakumar [12] introduced the level operators on intuitionistic fuzzy primary ideal and semiprimary ideal. K. Gunasekaran, S. Nandakumar and S. Sivakaminathan [16] introduced the definition of bipolar intuitionistic fuzzy α-ideal of a BP-algebra.

2. Preliminaries

Definition 1. Let A and B be any two bipolar intuitionistic fuzzy set $A = (\mu^P_{\alpha_A}, \mu^N_{\alpha_A}, \nu^P_{\alpha_A}, \nu^N_{\alpha_A})$ and $B = (\mu^P_{\alpha_B}, \mu^N_{\alpha_B}, \nu^P_{\alpha_B}, \nu^N_{\alpha_B})$ in X, we define

(i) $A \cap B = \{(x, \min(\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)), \max(\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)),$

$$\max(\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)), \min(\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x))/x \in X\}$$

(ii) $A \cup B = \{(x, \max(\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)), \min(\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)),$

$$\min(\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)), \max(\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x))/x \in X\}.$$

(iii) $\overline{A} = \{(x, \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x), \mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x))/x \in X\}.$

Definition 2. A bipolar intuitionistic fuzzy set $A = \{(x, \mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x))/x \in X\}$, of BP-algebra X is called a bipolar intuitionistic fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \geq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \leq \mu^N_{\alpha_A}(x)$

(ii) $\mu^P_{\alpha_A}(y * z) \geq \min\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\}$

(iii) $\mu^N_{\alpha_A}(y * z) \leq \max\{\mu^N_{\alpha_A}(x * z), \mu^N_{\alpha_A}(x * y)\}$

(iv) $\nu^P_{\alpha_A}(0) \leq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \geq \nu^N_{\alpha_A}(x)$

(v) $\nu^P_{\alpha_A}(y * z) \leq \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}$

(vi) $\nu^N_{\alpha_A}(y * z) \geq \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}$, for all $x, y, z \in X$.

Definition 3. A bipolar intuitionistic fuzzy set $A = \{(x, \mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x))/x \in X\}$, of BP-algebra X is called a bipolar intuitionistic anti fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \leq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \geq \mu^N_{\alpha_A}(x)$

(ii) $\mu^P_{\alpha_A}(y * z) \leq \max\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\}$

(iii) $\mu^N_{\alpha_A}(y * z) \geq \min\{\mu^N_{\alpha_A}(x * z), \mu^N_{\alpha_A}(x * y)\}$

(iv) $\nu^P_{\alpha_A}(0) \geq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \leq \nu^N_{\alpha_A}(x)$

(v) $\nu^P_{\alpha_A}(y * z) \geq \min\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}$

(vi) $\nu^N_{\alpha_A}(y * z) \leq \max\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}$, for all $x, y, z \in X$.

Definition 4. Let A is a bipolar intuitionistic fuzzy set of X, then the level operator $!$ is defined by $!A = \left\{\left(x, \max\left(\frac{1}{2}, \mu^{P}_{\alpha_A}(x)\right), \min\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(x)\right),\right.\right.$ $\left.\left.\min\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(x)\right), \max\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(x)\right)\right)/x \in X\right\} = L_1.$

Definition 5. Let A is a bipolar intuitionistic fuzzy set of X, then the level operator $?$ is defined by $?A = \left\{\left(x, \min\left(\frac{1}{2}, \mu^{P}_{\alpha_A}(x)\right), \max\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(x)\right),\right.\right.$ $\left.\left.\max\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(x)\right), \min\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(x)\right)\right)/x \in X\right\} = L_2.$

3. Level Operators on Bipolar Intuitionistic Fuzzy α-Ideal

Theorem 1. *If A is a bipolar intuitionistic fuzzy α-ideal of X, then $!A$ is a bipolar intuitionistic fuzzy α-ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider $0, x, y, z \in A$.

(i) Now $\mu^{P}_{\alpha_{!A}}(0) = \max\left(\frac{1}{2}, \mu^{P}_{\alpha_A}(0)\right)$

$$\geq \max\left(\frac{1}{2}, \mu^{P}_{\alpha_A}(x)\right)$$

$$= \mu^{P}_{\alpha_{!A}}(x)$$

Therefore $\mu^{P}_{\alpha_{!A}}(0) \geq \mu^{P}_{\alpha_{!A}}(x)$

Now $\mu^{N}_{\alpha_{!A}}(0) = \min\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(0)\right)$

$$\leq \min\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(x)\right)$$

$$= \mu^{N}_{\alpha_{!A}}(x)$$

Therefore $\mu^{N}_{\alpha_{!A}}(0) \leq \mu^{N}_{\alpha_{!A}}(x)$

(ii) Now $\mu^{P}_{\alpha!A}(y*z) = \max\left(\frac{1}{2},\, \mu^{P}_{\alpha_A}(y*z)\right)$

$$\geq \max\left(\frac{1}{2},\, \min\{\mu^{P}_{\alpha_A}(x*z),\, \mu^{P}_{\alpha_A}(x*y)\}\right)$$

$$= \min\left\{\max\left(\frac{1}{2},\, \mu^{P}_{\alpha_A}(x*z)\right),\, \max\left(\frac{1}{2},\, \mu^{P}_{\alpha_A}(x*y)\right)\right\}$$

$$= \min\{\mu^{P}_{\alpha!A}(x*z),\, \mu^{P}_{\alpha!A}(x*y)\}$$

Therefore $\mu^{P}_{\alpha!A}(y*z) \geq \min\{\mu^{P}_{\alpha!A}(x*z),\, \mu^{P}_{\alpha!A}(x*y)\}$

(iii) Now $\mu^{N}_{\alpha!A}(y*z) = \min\left(\frac{-1}{2},\, \mu^{N}_{\alpha_A}(y*z)\right)$

$$\leq \min\left(\frac{-1}{2},\, \max\{\mu^{N}_{\alpha_A}(x*z),\, \mu^{N}_{\alpha_A}(x*y)\}\right)$$

$$= \max\left\{\min\left(\frac{-1}{2},\, \mu^{N}_{\alpha_A}(x*z)\right),\, \min\left(\frac{-1}{2},\, \mu^{N}_{\alpha_A}(x*y)\right)\right\}$$

$$= \max\{\mu^{N}_{\alpha!A}(x*z),\, \mu^{N}_{\alpha!A}(x*y)\}$$

Therefore $\mu^{N}_{\alpha!A}(y*z) \leq \max\{\mu^{N}_{\alpha!A}(x*z),\, \mu^{N}_{\alpha!A}(x*y)\}$

(iv) Now $\nu^{P}_{\alpha!A}(0) = \min\left(\frac{1}{2},\, \nu^{P}_{\alpha_A}(0)\right)$

$$\leq \min\left(\frac{1}{2},\, \nu^{P}_{\alpha_A}(x)\right)$$

$$= \nu^{P}_{\alpha!A}(x)$$

Therefore $\nu^{P}_{\alpha!A}(0) \leq \nu^{P}_{\alpha!A}(x)$

Now $\nu^{N}_{\alpha!A}(0) = \max\left(\frac{-1}{2},\, \nu^{N}_{\alpha_A}(0)\right)$

$$\geq \max\left(\frac{-1}{2},\, \nu^{N}_{\alpha_A}(x)\right)$$

$$= \nu^{N}_{\alpha_{!A}}(x)$$

Therefore $\nu^{N}_{\alpha_{!A}}(0) \geq \nu^{N}_{\alpha_{!A}}(x)$

(v) Now $\nu^{P}_{\alpha_{!A}}(y * z) = \min\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(y * z)\right)$

$$\leq \min\left(\frac{1}{2}, \max\{\nu^{P}_{\alpha_A}(x * z), \nu^{P}_{\alpha_A}(x * y)\}\right)$$

$$= \max\left\{\min\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(x * z)\right), \min\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(x * y)\right)\right\}$$

$$= \max\{\nu^{P}_{\alpha_{!A}}(x * z), \nu^{P}_{\alpha_{!A}}(x * y)\}$$

Therefore $\nu^{P}_{\alpha_{!A}}(y * z) \leq \max\{\nu^{P}_{\alpha_{!A}}(x * z), \nu^{P}_{\alpha_{!A}}(x * y)\}$

(vi) Now $\nu^{N}_{\alpha_{!A}}(y * z) = \max\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(y * z)\right)$

$$\geq \max\left(\frac{-1}{2}, \min\{\nu^{N}_{\alpha_A}(x * z), \nu^{N}_{\alpha_A}(x * y)\}\right)$$

$$= \min\left\{\max\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(x * z)\right), \max\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(x * y)\right)\right\}$$

$$= \min\{\nu^{N}_{\alpha_{!A}}(x * z), \nu^{N}_{\alpha_{!A}}(x * y)\}$$

Therefore $\nu^{N}_{\alpha_{!A}}(y * z) \geq \min\{\nu^{N}_{\alpha_{!A}}(x * z), \nu^{N}_{\alpha_{!A}}(x * y)\}$

Therefore $!A$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem 2. *If A and B are bipolar intuitionistic fuzzy α-ideal of X, then $!(A \cap B) = !A \cap !B$ is also a bipolar intuitionistic fuzzy α-ideal of X.*

Proof. Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider $0, x, y, z \in A \cap B$ then $0, x, y, z \in A$ and $0, x, y, z \in B$.

(i) Now $\mu^{P}_{\alpha_{!(A \cap B)}}(0) = \max\left(\frac{1}{2}, \mu^{P}_{\alpha_{A \cap B}}(0)\right)$

$$= \max\left(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(0), \mu^P_{\alpha_B}(0)\}\right)$$

$$\geq \max\left(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)\}\right)$$

$$= \min\left\{\max\left(\frac{1}{2}, \mu^P_{\alpha_A}(x)\right), \max\left(\frac{1}{2}, \mu^P_{\alpha_B}(x)\right)\right\}$$

$$= \min\{\mu^P_{\alpha_{!A}}(x), \mu^P_{\alpha_{!B}}(x)\}$$

$$= \mu^P_{\alpha_{!A \cap !B}}(x)$$

Therefore $\mu^P_{\alpha_{!(A \cap B)}}(0) \geq \mu^P_{\alpha_{!A \cap !B}}(x)$

Now $\mu^N_{\alpha_{!(A \cap B)}}(0) = \min\left(\frac{-1}{2}, \mu^N_{\alpha_{A \cap B}}(0)\right)$

$$= \min\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_A}(0), \mu^N_{\alpha_B}(0)\}\right)$$

$$\leq \min\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)\}\right)$$

$$= \max\left\{\min\left(\frac{-1}{2}, \mu^N_{\alpha_A}(x)\right), \min\left(\frac{-1}{2}, \mu^N_{\alpha_B}(x)\right)\right\}$$

$$= \max\{\mu^N_{\alpha_{!A}}(x), \mu^N_{\alpha_{!B}}(x)\}$$

$$= \mu^N_{\alpha_{!A \cap !B}}(x)$$

Therefore $\mu^N_{\alpha_{!(A \cap B)}}(0) \leq \mu^N_{\alpha_{!A \cap !B}}(x)$

(ii) Now $\mu^P_{\alpha_{!(A \cap B)}}(y * z) = \max\left(\frac{1}{2}, \mu^P_{\alpha_{A \cap B}}(y * z)\right)$

$$= \max\left(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(y * z), \mu^P_{\alpha_B}(y * z)\}\right)$$

$$\geq \max(\frac{1}{2}, \min\{\min\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\},$$

$$\min\{\mu^P_{\alpha_B}(x*z),\ \mu^P_{\alpha_B}(x*y)\}\})$$

$$= \max(\tfrac{1}{2},\ \min\{\min\{\mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\},$$

$$\min\{\mu^P_{\alpha_B}(x*z),\ \mu^P_{\alpha_B}(x*y)\}\})$$

$$= \max\{\max(\tfrac{1}{2},\ \min\{\mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\},$$

$$\min\{\mu^P_{\alpha_B}(x*z),\ \mu^P_{\alpha_B}(x*y)\}\})\}$$

$$= \min\left\{\min\left\{\max\left(\frac{1}{2},\ \mu^P_{\alpha_A}(x*z)\right),\ \max\left(\frac{1}{2},\ \mu^P_{\alpha_B}(x*z)\right)\right\},\right.$$

$$\left.\min\left\{\max\left(\frac{1}{2},\ \mu^P_{\alpha_A}(x*y)\right),\ \max\left(\frac{1}{2},\ \mu^P_{\alpha_B}(x*y)\right)\right\}\right\}$$

$$= \min\{\min\{\mu^P_{\alpha_{!A}}(x*z),\ \mu^P_{\alpha_{!B}}(x*z)\},\ \min\{\mu^P_{\alpha_{!A}}(x*y),\ \mu^P_{\alpha_{!B}}(x*y)\}\}$$

$$= \min\{\mu^P_{\alpha_{!A\cap\,!B}}(x*z),\ \mu^P_{\alpha_{!A\cap\,!B}}(x*y)\}$$

Therefore $\mu^P_{\alpha_{!(A\cap B)}}(y*z) \geq \min\{\mu^P_{\alpha_{!A\cap\,!B}}(x*z),\ \mu^P_{\alpha_{!A\cap\,!B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{!(A\cap B)}}(y*z) = \min\left(\frac{-1}{2},\ \mu^N_{\alpha_{A\cap B}}(y*z)\right)$

$$= \min\left(\frac{-1}{2},\ \max\{\mu^N_{\alpha_A}(y*z),\ \mu^N_{\alpha_B}(y*z)\}\right)$$

$$\leq \min(\tfrac{-1}{2},\ \max\{\max\{\mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\},$$

$$\max\{\mu^N_{\alpha_B}(x*z),\ \mu^N_{\alpha_B}(x*y)\}\})$$

$$= \min(\tfrac{-1}{2},\ \max\{\max\{\mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\},$$

$$\max\{\mu^N_{\alpha_B}(x*z),\ \mu^N_{\alpha_B}(x*y)\}\})$$

$$= \max\left\{\min\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}\right),\right.$$

$$\left.\min\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_B}(x*z), \mu^N_{\alpha_B}(x*y)\}\right)\right\}$$

$$= \max\left\{\max\left\{\min\left(\frac{-1}{2}, \mu^N_{\alpha_A}(x*z)\right), \min\left(\frac{-1}{2}, \mu^N_{\alpha_B}(x*z)\right)\right\},\right.$$

$$\left.\max\left\{\min\left(\frac{-1}{2}, \mu^N_{\alpha_A}(x*y)\right), \min\left(\frac{-1}{2}, \mu^N_{\alpha_B}(x*y)\right)\right\}\right\}$$

$$= \max\{\max\{\mu^N_{\alpha_{!A}}(x*z), \mu^N_{\alpha_{!B}}(x*z)\}, \max\{\mu^N_{\alpha_{!A}}(x*y), \mu^N_{\alpha_{!B}}(x*y)\}\}$$

$$= \max\{\mu^N_{\alpha_{!A\cap !B}}(x*z), \mu^N_{\alpha_{!A\cap !B}}(x*y)\}$$

Therefore $\mu^N_{\alpha_{!(A\cap B)}}(y*z) \le \max\{\mu^N_{\alpha_{!A\cap !B}}(x*z), \mu^N_{\alpha_{!A\cap !B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{!(A\cap B)}}(0) = \min\left(\frac{1}{2}, \nu^P_{\alpha_{A\cap B}}(0)\right)$

$$= \min\left(\frac{1}{2}, \max(\nu^P_{\alpha_A}(0), \nu^P_{\alpha_B}(0))\right)$$

$$\le \min\left(\frac{1}{2}, \max(\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x))\right)$$

$$= \max\left\{\min\left(\frac{1}{2}, \nu^P_{\alpha_A}(x)\right), \min\left(\frac{1}{2}, \nu^P_{\alpha_B}(x)\right)\right\}$$

$$= \min\{\nu^P_{\alpha_{!A}}(x), \nu^P_{\alpha_{!B}}(x)\}$$

$$= \nu^P_{\alpha_{!A\cap !B}}(x)$$

Therefore $\nu^P_{\alpha_{!(A\cap B)}}(0) \le \nu^P_{\alpha_{!A\cap !B}}(x)$

Now $\nu^N_{\alpha_{!(A\cap B)}}(0) = \max\left(\frac{-1}{2}, \nu^N_{\alpha_{A\cap B}}(0)\right)$

$$= \max\left(\frac{-1}{2}, \min(\nu^N_{\alpha_A}(0), \nu^N_{\alpha_B}(0))\right)$$

$$\geq \max\left(\frac{-1}{2}, \min(\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x))\right)$$

$$= \min\left\{\max\left(\frac{-1}{2}, \nu^N_{\alpha_A}(x)\right), \max\left(\frac{-1}{2}, \nu^N_{\alpha_B}(x)\right)\right\}$$

$$= \min\{\nu^N_{\alpha_{!A}}(x), \nu^N_{\alpha_{!B}}(x)\}$$

$$= \nu^N_{\alpha_{!A \cap !B}}(x)$$

Therefore $\nu^N_{\alpha_{!(A \cap B)}}(0) \geq \nu^N_{\alpha_{!A \cap !B}}(x)$

(v) Now $\nu^P_{\alpha_{!(A \cap B)}}(y * z) = \min\left(\frac{1}{2}, \nu^P_{\alpha_{A \cap B}}(y * z)\right)$

$$= \min\left(\frac{1}{2}, \max(\nu^P_{\alpha_A}(y * z), \nu^P_{\alpha_B}(y * z))\right)$$

$$\leq \min(\frac{1}{2}, \max\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\},$$

$$\max\{\nu^P_{\alpha_B}(x * z), \nu^P_{\alpha_B}(x * y)\}\})$$

$$= \min(\frac{1}{2}, \max\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\},$$

$$\max\{\nu^P_{\alpha_B}(x * z), \nu^P_{\alpha_B}(x * y)\}\})$$

$$= \max\left\{\min\left(\frac{1}{2}, \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}\right),\right.$$

$$\left.\min\left(\frac{1}{2}, \max\{\nu^P_{\alpha_A}(x * y), \nu^P_{\alpha_A}(x * y)\}\right)\right\}$$

$$= \max\left\{\max\left\{\min\left(\frac{1}{2}, \nu^P_{\alpha_A}(x * z)\right), \min\left(\frac{1}{2}, \nu^P_{\alpha_B}(x * z)\right)\right\},\right.$$

$$\left.\max\left\{\min\left(\frac{1}{2}, \nu^P_{\alpha_A}(x * y)\right), \min\left(\frac{1}{2}, \nu^P_{\alpha_B}(x * y)\right)\right\}\right\}$$

$$= \max\{\max\{\nu^P_{\alpha_{!A}}(x*z),\ \nu^P_{\alpha_{!B}}(x*z)\},\ \max\{\nu^P_{\alpha_{!A}}(x*y),\ \nu^P_{\alpha_{!B}}(x*y)\}\}$$

$$= \max\{\nu^P_{\alpha_{!A\cap !B}}(x*z),\ \nu^P_{\alpha_{!A\cap !B}}(x*y)\}$$

Therefore $\nu^P_{\alpha_{!(A\cap B)}}(y*z) \le \max\{\nu^P_{\alpha_{!A\cap !B}}(x*z),\ \nu^P_{\alpha_{!A\cap !B}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{!(A\cap B)}}(y*z) = \max\left(\frac{-1}{2},\ \nu^N_{\alpha_{A\cap B}}(y*z)\right)$

$$= \max\left(\frac{-1}{2},\ \min\{\nu^N_{\alpha_A}(y*z),\ \nu^N_{\alpha_B}(y*z)\}\right)$$

$$\ge \max(\frac{-1}{2},\ \min\{\min\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\},$$

$$\min\{\nu^N_{\alpha_B}(x*z),\ \nu^N_{\alpha_B}(x*y)\}\})$$

$$= \max(\frac{-1}{2},\ \min\{\min\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*z)\},$$

$$\min\{\nu^N_{\alpha_B}(x*y),\ \nu^N_{\alpha_B}(x*y)\}\})$$

$$= \min\left\{\max\left(\frac{-1}{2},\ \min\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}\right),\right.$$

$$\left.\max\left(\frac{-1}{2},\ \min\{\nu^N_{\alpha_B}(x*z),\ \nu^N_{\alpha_B}(x*y)\}\right)\right\}$$

$$= \min\left\{\min\left\{\max\left(\frac{-1}{2},\ \nu^N_{\alpha_A}(x*z)\right),\ \max\left(\frac{-1}{2},\ \nu^N_{\alpha_B}(x*z)\right)\right\},\right.$$

$$\left.\min\left\{\max\left(\frac{-1}{2},\ \nu^N_{\alpha_A}(x*y)\right),\ \max\left(\frac{-1}{2},\ \nu^N_{\alpha_B}(x*y)\right)\right\}\right\}$$

$$= \min\{\min\{\nu^N_{\alpha_{!A}}(x*z),\ \nu^N_{\alpha_{!B}}(x*z)\},\ \min\{\nu^N_{\alpha_{!A}}(x*y),\ \nu^N_{\alpha_{!B}}(x*y)\}\}$$

$$= \min\{\nu^N_{\alpha_{!A\cap !B}}(x*z),\ \nu^N_{\alpha_{!A\cap !B}}(x*y)\}$$

Therefore $\nu^N_{\alpha_{!(A\cap B)}}(y*z) \ge \min\{\nu^N_{\alpha_{!A\cap !B}}(x*z),\ \nu^N_{\alpha_{!A\cap !B}}(x*y)\}$

Therefore $!(A \cap B) = !A \cap !B$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem 3. *If A is a bipolar intuitionistic fuzzy α-ideal of X, then $?A$ is a bipolar intuitionistic fuzzy α-ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider $0, x, y, z \in A$.

(i) Now $\mu^P_{\alpha ?A}(0) = \min\left(\frac{1}{2}, \mu^P_{\alpha_A}(0)\right)$

$$\geq \min\left(\frac{1}{2}, \mu^P_{\alpha_A}(x)\right)$$

$$= \mu^P_{\alpha ?A}(x)$$

Therefore $\mu^P_{\alpha ?A}(0) \geq \mu^P_{\alpha ?A}(x)$

Now $\mu^N_{\alpha ?A}(0) = \max\left(\frac{-1}{2}, \mu^N_{\alpha_A}(0)\right)$

$$\leq \min\left(\frac{-1}{2}, \mu^N_{\alpha_A}(x)\right)$$

$$= \mu^N_{\alpha ?A}(x)$$

Therefore $\mu^N_{\alpha ?A}(0) \leq \mu^N_{\alpha ?A}(x)$

(ii) Now $\mu^P_{\alpha ?A}(y * z) = \min\left(\frac{1}{2}, \mu^P_{\alpha_A}(y * z)\right)$

$$\geq \min\left(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\}\right)$$

$$= \min\left\{\min\left(\frac{1}{2}, \mu^P_{\alpha_A}(x * z)\right), \min\left(\frac{1}{2}, \mu^P_{\alpha_A}(x * y)\right)\right\}$$

$$= \min\{\mu^P_{\alpha ?A}(x * z), \mu^P_{\alpha ?A}(x * y)\}$$

Therefore $\mu^{P}_{\alpha ?\, A}(y * z) \geq \min\{\mu^{P}_{\alpha ?\, A}(x * z), \mu^{P}_{\alpha ?\, A}(x * y)\}$

(iii) Now $\mu^{N}_{\alpha ?\, A}(y * z) = \max\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(y * z)\right)$

$$\leq \max\left(\frac{-1}{2}, \max\{\mu^{N}_{\alpha_A}(x * z), \mu^{N}_{\alpha_A}(x * y)\}\right)$$

$$= \max\left\{\max\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(x * z)\right), \max\left(\frac{-1}{2}, \mu^{N}_{\alpha_A}(x * y)\right)\right\}$$

$$= \max\{\mu^{N}_{\alpha ?\, A}(x * z), \mu^{N}_{\alpha ?\, A}(x * y)\}$$

Therefore $\mu^{N}_{\alpha ?\, A}(y * z) \leq \max\{\mu^{N}_{\alpha ?\, A}(x * z), \mu^{N}_{\alpha ?\, A}(x * y)\}$

(iv) Now $\nu^{P}_{\alpha ?\, A}(0) = \max\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(0)\right)$

$$\leq \max\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(x)\right)$$

$$= \nu^{P}_{\alpha ?\, A}(x)$$

Therefore $\nu^{P}_{\alpha ?\, A}(0) \leq \nu^{P}_{\alpha ?\, A}(x)$

Now $\nu^{N}_{\alpha ?\, A}(0) = \min\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(0)\right)$

$$\geq \max\left(\frac{-1}{2}, \nu^{N}_{\alpha_A}(x)\right)$$

$$= \nu^{N}_{\alpha ?\, A}(x)$$

Therefore $\nu^{N}_{\alpha ?\, A}(0) \geq \nu^{N}_{\alpha ?\, A}(x)$

(v) Now $\nu^{P}_{\alpha ?\, A}(y * z) = \max\left(\frac{1}{2}, \nu^{P}_{\alpha_A}(y * z)\right)$

$$\leq \max\left(\frac{1}{2}, \max\{\nu^{P}_{\alpha_A}(x * z), \nu^{P}_{\alpha_A}(x * y)\}\right)$$

$$= \max\left\{\max\left(\frac{1}{2}, \nu^P_{\alpha_A}(x * z)\right), \max\left(\frac{1}{2}, \nu^P_{\alpha_A}(x * y)\right)\right\}$$

$$= \max\{\nu^P_{\alpha ? A}(x * z), \nu^P_{\alpha ? A}(x * y)\}$$

Therefore $\nu^P_{\alpha ? A}(y * z) \leq \max\{\nu^P_{\alpha ? A}(x * z), \nu^P_{\alpha ? A}(x * y)\}$

(vi) Now $\nu^N_{\alpha ? A}(y * z) = \min\left(\frac{-1}{2}, \nu^N_{\alpha_A}(y * z)\right)$

$$\geq \min\left(\frac{-1}{2}, \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}\right)$$

$$= \min\left\{\min\left(\frac{-1}{2}, \nu^N_{\alpha_A}(x * z)\right), \min\left(\frac{-1}{2}, \nu^N_{\alpha_A}(x * y)\right)\right\}$$

$$= \min\{\nu^N_{\alpha ? A}(x * z), \nu^N_{\alpha ? A}(x * y)\}$$

Therefore $\nu^N_{\alpha ? A}(y * z) \geq \min\{\nu^N_{\alpha ? A}(x * z), \nu^N_{\alpha ? A}(x * y)\}$

Therefore $?A$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem 4. *If A and B are bipolar intuitionistic fuzzy α-ideal of X, then $?(A \cap B) = ?A \cap ?B$ is also a bipolar intuitionistic fuzzy α-ideal of X.*

Proof. Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider $0, x, y, z \in A \cap B$ then $0, x, y, z \in A$ and $0, x, y, z \in B$.

(i) Now $\mu^P_{\alpha ?(A \cap B)}(0) = \min\left(\frac{1}{2}, \mu^P_{\alpha_{A \cap B}}(0)\right)$

$$= \min\left(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(0), \mu^P_{\alpha_B}(0)\}\right)$$

$$\geq \min\left(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)\}\right)$$

$$= \min\left\{\min\left(\frac{1}{2}, \mu^P_{\alpha_A}(x)\right), \min\left(\frac{1}{2}, \mu^P_{\alpha_B}(x)\right)\right\}$$

$$= \min \{\mu^P_{\alpha ? A}(x),\ \mu^P_{\alpha ? B}(x)\}$$

$$= \mu^P_{\alpha ? A \cap ? B}(x)$$

Therefore $\mu^P_{\alpha ?(A \cap B)}(0) \geq \mu^P_{\alpha ? A \cap ? B}(x)$

Now $\mu^N_{\alpha ?(A \cap B)}(0) = \max\left(\frac{-1}{2},\ \mu^N_{\alpha_{A \cap B}}(0)\right)$

$$= \max\left(\frac{-1}{2},\ \max \{\mu^N_{\alpha_A}(0),\ \mu^N_{\alpha_B}(0)\}\right)$$

$$\leq \max\left(\frac{-1}{2},\ \max \{\mu^N_{\alpha_A}(x),\ \mu^N_{\alpha_B}(x)\}\right)$$

$$= \max\left\{\max\left(\frac{-1}{2},\ \mu^N_{\alpha_A}(x)\right),\ \max\left(\frac{-1}{2},\ \mu^N_{\alpha_B}(x)\right)\right\}$$

$$= \max \{\mu^N_{\alpha ? A}(x),\ \mu^N_{\alpha ? B}(x)\}$$

$$= \mu^N_{\alpha ? A \cap ? B}(x)$$

Therefore $\mu^N_{\alpha ?(A \cap B)}(0) \leq \mu^N_{\alpha ? A \cap ? B}(x)$

(ii) Now $\mu^P_{\alpha ?(A \cap B)}(y * z) = \min\left(\frac{1}{2},\ \mu^P_{\alpha_{A \cap B}}(y * z)\right)$

$$= \min\left(\frac{1}{2},\ \min \{\mu^P_{\alpha_A}(y * z),\ \mu^P_{\alpha_B}(y * z)\}\right)$$

$$\geq \min(\frac{1}{2},\ \min \{\min \{\mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_A}(x * y)\},$$

$$\min \{\mu^P_{\alpha_B}(x * z),\ \mu^P_{\alpha_B}(x * y)\}\})$$

$$= \min(\frac{1}{2},\ \min \{\min \{\mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_A}(x * y)\},$$

$$\min \{\mu^P_{\alpha_B}(x * z),\ \mu^P_{\alpha_B}(x * y)\}\})$$

$$= \min\{\min(\frac{1}{2}, \min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\},$$

$$\min\{\mu^P_{\alpha_B}(x*z), \mu^P_{\alpha_B}(x*y)\})\}$$

$$= \min\left\{\min\left\{\min\left(\frac{1}{2}, \mu^P_{\alpha_A}(x*z)\right), \min\left(\frac{1}{2}, \mu^P_{\alpha_B}(x*z)\right)\right\},\right.$$

$$\left.\min\left\{\min\left(\frac{1}{2}, \mu^P_{\alpha_A}(x*y)\right), \min\left(\frac{1}{2}, \mu^P_{\alpha_B}(x*y)\right)\right\}\right\}$$

$$= \min\{\min\{\mu^P_{\alpha_{?A}}(x*z), \mu^P_{\alpha_{?B}}(x*z)\}, \min\{\mu^P_{\alpha_{?A}}(x*y), \mu^P_{\alpha_{?B}}(x*y)\}\}$$

$$= \min\{\mu^P_{\alpha_{?A\cap ?B}}(x*z), \mu^P_{\alpha_{?A\cap ?B}}(x*y)\}$$

Therefore $\mu^P_{\alpha_{?(A\cap B)}}(y*z) \geq \min\{\mu^P_{\alpha_{?A\cap ?B}}(x*z), \mu^P_{\alpha_{?A\cap ?B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{?(A\cap B)}}(y*z) = \max\left(\frac{-1}{2}, \mu^N_{\alpha_{A\cap B}}(y*z)\right)$

$$= \max\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_A}(y*z), \mu^N_{\alpha_B}(y*z)\}\right)$$

$$\leq \max(\frac{-1}{2}, \max\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\},$$

$$\max\{\mu^N_{\alpha_B}(x*z), \mu^N_{\alpha_B}(x*y)\}\})$$

$$= \max(\frac{-1}{2}, \max\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*z)\},$$

$$\max\{\mu^N_{\alpha_B}(x*y), \mu^N_{\alpha_B}(x*y)\}\})$$

$$= \max\left\{\max\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*z)\}\right),\right.$$

$$\left.\max\left(\frac{-1}{2}, \max\{\mu^N_{\alpha_B}(x*y), \mu^N_{\alpha_B}(x*y)\}\right)\right\}$$

$$= \max\left\{\max\left\{\max\left(\frac{-1}{2}, \mu^N_{\alpha_A}(x*z)\right), \max\left(\frac{-1}{2}, \mu^N_{\alpha_B}(x*z)\right)\right\},\right.$$

$$\max\left\{\max\left(\frac{-1}{2},\ \mu^N_{\alpha_A}(x*y)\right),\ \max\left(\frac{-1}{2},\ \mu^N_{\alpha_B}(x*y)\right)\right\}\right\}$$

$$= \max\{\max\{\mu^N_{\alpha_{!A}}(x*z),\ \mu^N_{\alpha_{!B}}(x*z)\},\ \max\{\mu^N_{\alpha_{!A}}(x*y),\ \mu^N_{\alpha_{!B}}(x*y)\}\}$$

$$= \max\{\mu^N_{\alpha_{?A\cap ?B}}(x*z),\ \mu^N_{\alpha_{?A\cap ?B}}(x*y)\}$$

Therefore $\mu^N_{\alpha_{?(A\cap B)}}(y*z) \le \max\{\mu^N_{\alpha_{?A\cap ?B}}(x*z),\ \mu^N_{\alpha_{?A\cap ?B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{?(A\cap B)}}(0) = \max\left(\frac{1}{2},\ \nu^P_{\alpha_{A\cap B}}(0)\right)$

$$= \max\left(\frac{1}{2},\ \max(\nu^P_{\alpha_A}(0),\ \nu^P_{\alpha_B}(0))\right)$$

$$\le \max\left(\frac{1}{2},\ \max(\nu^P_{\alpha_A}(x),\ \nu^P_{\alpha_B}(x))\right)$$

$$= \max\left\{\max\left(\frac{1}{2},\ \nu^P_{\alpha_A}(x)\right),\ \max\left(\frac{1}{2},\ \nu^P_{\alpha_B}(x)\right)\right\}$$

$$= \max\{\nu^P_{\alpha_{?A}}(x),\ \nu^P_{\alpha_{?B}}(x)\}$$

$$= \nu^P_{\alpha_{?A\cap ?B}}(x)$$

Therefore $\nu^P_{\alpha_{?(A\cap B)}}(0) \le \nu^P_{\alpha_{?A\cap ?B}}(x)$

Now $\nu^N_{\alpha_{?(A\cap B)}}(0) = \min\left(\frac{-1}{2},\ \nu^N_{\alpha_{A\cap B}}(0)\right)$

$$= \min\left(\frac{-1}{2},\ \min(\nu^N_{\alpha_A}(0),\ \nu^N_{\alpha_B}(0))\right)$$

$$\ge \min\left(\frac{-1}{2},\ \min(\nu^N_{\alpha_A}(x),\ \nu^N_{\alpha_B}(x))\right)$$

$$= \min\left\{\min\left(\frac{-1}{2},\ \nu^N_{\alpha_A}(x)\right),\ \min\left(\frac{-1}{2},\ \nu^N_{\alpha_B}(x)\right)\right\}$$

$$= \min\{\nu^N_{\alpha_{?A}}(x),\ \nu^N_{\alpha_{?B}}(x)\}$$

$$= \nu^N_{\alpha ? A \cap ? B}(x)$$

Therefore $\nu^N_{\alpha ?(A\cap B)}(0) \geq \nu^N_{\alpha ? A \cap ? B}(x)$

(v) Now $\nu^P_{\alpha ?(A\cap B)}(y * z) = \max\left(\frac{1}{2}, \nu^P_{\alpha_{A\cap B}}(y * z)\right)$

$$= \max\left(\frac{1}{2}, \max\left(\nu^P_{\alpha_A}(y * z), \nu^P_{\alpha_B}(y * z)\right)\right)$$

$$\leq \max(\frac{1}{2}, \max\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\},$$

$$\max\{\nu^P_{\alpha_B}(x * z), \nu^P_{\alpha_B}(x * y)\}\})$$

$$= \max(\frac{1}{2}, \max\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * z)\},$$

$$\max\{\nu^P_{\alpha_B}(x * y), \nu^P_{\alpha_B}(x * y)\}\})$$

$$= \max\left\{\max\left(\frac{1}{2}, \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * z)\}\right),\right.$$

$$\left.\max\left(\frac{1}{2}, \max\{\nu^P_{\alpha_A}(x * y), \nu^P_{\alpha_A}(x * y)\}\right)\right\}$$

$$= \max\left\{\max\left\{\max\left(\frac{1}{2}, \nu^P_{\alpha_A}(x * z)\right), \max\left(\frac{1}{2}, \nu^P_{\alpha_B}(x * z)\right)\right\},\right.$$

$$\left.\max\left\{\max\left(\frac{1}{2}, \nu^P_{\alpha_A}(x * y)\right), \max\left(\frac{1}{2}, \nu^P_{\alpha_B}(x * y)\right)\right\}\right\}$$

$$= \max\{\max\{\nu^P_{\alpha ? A}(x * z), \nu^P_{\alpha ? B}(x * z)\}, \max\{\nu^P_{\alpha ? A}(x * y), \nu^P_{\alpha ? B}(x * y)\}\}$$

$$= \max\{\nu^P_{\alpha ? A \cap ? B}(x * z), \nu^P_{\alpha ? A \cap ? B}(x * y)\}$$

Therefore $\nu^P_{\alpha ?(A\cap B)}(y * z) \leq \max\{\nu^P_{\alpha ? A \cap ? B}(x * z), \nu^P_{\alpha ? A \cap ? B}(x * y)\}$

(vi) Now $\nu^N_{\alpha ?(A\cap B)}(y * z) = \min\left(\frac{-1}{2}, \nu^N_{\alpha_{A\cap B}}(y * z)\right)$

$$= \min\left(\frac{-1}{2}, \min(\nu^N_{\alpha_A}(y * z), \nu^N_{\alpha_B}(y * z))\right)$$

$$\geq \min(\frac{-1}{2}, \min\{\min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\},$$

$$\min\{\nu^N_{\alpha_B}(x * z), \nu^N_{\alpha_B}(x * y)\}\})$$

$$= \min(\frac{-1}{2}, \min\{\min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * z)\},$$

$$\min\{\nu^N_{\alpha_B}(x * y), \nu^N_{\alpha_B}(x * y)\}\})$$

$$= \min\left\{\min\left(\frac{-1}{2}, \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}\right),\right.$$

$$\left.\min\left(\frac{-1}{2}, \min\{\nu^N_{\alpha_B}(x * z), \nu^N_{\alpha_B}(x * y)\}\right)\right\}$$

$$= \min\left\{\min\left\{\min\left(\frac{-1}{2}, \nu^N_{\alpha_A}(x * z)\right), \min\left(\frac{-1}{2}, \nu^N_{\alpha_B}(x * z)\right)\right\},\right.$$

$$\left.\min\left\{\min\left(\frac{-1}{2}, \nu^N_{\alpha_A}(x * y)\right), \min\left(\frac{-1}{2}, \nu^N_{\alpha_B}(x * y)\right)\right\}\right\}$$

$$= \min\{\min\{\nu^N_{\alpha_{?A}}(x * z), \nu^N_{\alpha_{?B}}(x * z)\}, \min\{\nu^N_{\alpha_{?A}}(x * y), \nu^N_{\alpha_{?B}}(x * y)\}\}$$

$$= \min\{\nu^N_{\alpha_{?A \cap ?B}}(x * z), \nu^N_{\alpha_{?A \cap ?B}}(x * y)\}$$

Therefore $\nu^N_{\alpha_{?(A \cap B)}}(y * z) \geq \min\{\nu^N_{\alpha_{?A \cap ?B}}(x * z), \nu^N_{\alpha_{?A \cap ?B}}(x * y)\}$

Therefore $?(A \cap B) = ?A \cap ?B$ is also a bipolar intuitionistic fuzzy α-ideal of X.

Theorem 5. *If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\overline{?\overline{A}} = !A$ is also a bipolar intuitionistic fuzzy α-ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider $0, x, y, z \in A$.

(i) Now $\mu^{P}_{\alpha_{\overline{?\overline{A}}}}(0) = \nu^{P}_{\alpha_{?\overline{A}}}(0)$

$$= \max\left\{\frac{1}{2}, \nu^{P}_{\alpha_{\overline{A}}}(0)\right\}$$

$$= \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_{A}}(0)\right\}$$

$$\geq \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_{A}}(x)\right\}$$

$$= \mu^{P}_{\alpha_{!A}}(x)$$

Therefore $\mu^{P}_{\alpha_{\overline{?\overline{A}}}}(0) \geq \mu^{P}_{\alpha_{!A}}(x)$

Now $\mu^{N}_{\alpha_{\overline{?\overline{A}}}}(0) = \nu^{N}_{\alpha_{?\overline{A}}}(0)$

$$= \min\left\{\frac{-1}{2}, \nu^{N}_{\alpha_{\overline{A}}}(0)\right\}$$

$$= \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_{A}}(0)\right\}$$

$$\leq \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_{A}}(x)\right\}$$

$$= \mu^{N}_{\alpha_{!A}}(x)$$

Therefore $\mu^{N}_{\alpha_{\overline{?\overline{A}}}}(0) \leq \mu^{N}_{\alpha_{!A}}(x)$

(ii) Now $\mu^{P}_{\alpha_{\overline{?\overline{A}}}}(y * z) = \nu^{N}_{\alpha_{?\overline{A}}}(y * z)$

$$= \max\left\{\frac{1}{2}, \nu^{P}_{\alpha_{\overline{A}}}(y * z)\right\}$$

$$= \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_{A}}(y * z)\right\}$$

$$\geq \max\left\{\frac{1}{2}, \min\{\mu^{P}_{\alpha_A}(x * z), \mu^{P}_{\alpha_A}(x * y)\}\right\}$$

$$= \min\left\{\max\left\{\frac{1}{2}, \mu^{P}_{\alpha_A}(x * z)\right\}, \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_A}(x * y)\right\}\right\}$$

$$= \min\{\mu^{P}_{\alpha_{!A}}(x * z), \mu^{P}_{\alpha_{!A}}(x * y)\}$$

Therefore $\mu^{P}_{\alpha_{\overline{?\overline{A}}}}(y * z) \geq \min\{\mu^{P}_{\alpha_{!A}}(x * z), \mu^{P}_{\alpha_{!A}}(x * y)\}$

(iii) Now $\mu^{N}_{\alpha_{\overline{?\overline{A}}}}(y * z) = \nu^{N}_{\alpha_{?\overline{A}}}(y * z)$

$$= \min\left\{\frac{-1}{2}, \nu^{N}_{\alpha_{\overline{A}}}(y * z)\right\}$$

$$= \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_A}(y * z)\right\}$$

$$\leq \min\left\{\frac{-1}{2}, \max\{\mu^{N}_{\alpha_A}(x * z), \mu^{N}_{\alpha_A}(x * y)\}\right\}$$

$$= \max\left\{\min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_A}(x * z)\right\}, \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_A}(x * y)\right\}\right\}$$

$$= \max\{\mu^{N}_{\alpha_{!A}}(x * z), \mu^{N}_{\alpha_{!A}}(x * y)\}$$

Therefore $\mu^{N}_{\alpha_{\overline{?\overline{A}}}}(y * z) \leq \max\{\mu^{N}_{\alpha_{!A}}(x * z), \mu^{N}_{\alpha_{!A}}(x * y)\}$

(iv) Now $\nu^{P}_{\alpha_{\overline{?\overline{A}}}}(0) = \mu^{P}_{\alpha_{?\overline{A}}}(0)$

$$= \min\left\{\frac{1}{2}, \mu^{P}_{\alpha_{\overline{A}}}(0)\right\}$$

$$= \min\left\{\frac{1}{2}, \nu^{P}_{\alpha_A}(0)\right\}$$

$$\leq \min\left\{\frac{1}{2}, \nu^{P}_{\alpha_A}(x)\right\}$$

$$= \nu^{P}_{\alpha_{!A}}(x)$$

Therefore $\nu^{P}_{\alpha_{?\overline{A}}}(0) \leq \nu^{P}_{\alpha_{!A}}(x)$

Now $\nu^{N}_{\alpha_{?\overline{A}}}(0) = \mu^{N}_{\alpha_{?\overline{A}}}(0)$

$$= \max\left\{\frac{-1}{2}, \mu^{N}_{\alpha_{\overline{A}}}(0)\right\}$$

$$= \max\left\{\frac{-1}{2}, \nu^{N}_{\alpha_{A}}(0)\right\}$$

$$\geq \max\left\{\frac{-1}{2}, \nu^{N}_{\alpha_{A}}(x)\right\}$$

$$= \nu^{N}_{\alpha_{!A}}(x)$$

Therefore $\nu^{N}_{\alpha_{?\overline{A}}}(0) \geq \nu^{N}_{\alpha_{!A}}(x)$

(v) Now $\mu^{P}_{\alpha_{?\overline{A}}}(y * z) = \mu^{N}_{\alpha_{?\overline{A}}}(y * z)$

$$= \min\left\{\frac{1}{2}, \mu^{P}_{\alpha_{\overline{A}}}(y * z)\right\}$$

$$= \min\left\{\frac{1}{2}, \nu^{P}_{\alpha_{A}}(y * z)\right\}$$

$$\leq \min\left\{\frac{1}{2}, \max\{\nu^{P}_{\alpha_{A}}(x * z), \nu^{P}_{\alpha_{A}}(x * y)\}\right\}$$

$$= \max\left\{\min\left\{\frac{1}{2}, \nu^{P}_{\alpha_{A}}(x * z)\right\}, \min\left\{\frac{1}{2}, \nu^{P}_{\alpha_{A}}(x * y)\right\}\right\}$$

$$= \max\{\nu^{P}_{\alpha_{!A}}(x * z), \nu^{P}_{\alpha_{!A}}(x * y)\}$$

Therefore $\nu^{P}_{\alpha_{?\overline{A}}}(y * z) \leq \max\{\nu^{P}_{\alpha_{!A}}(x * z), \nu^{P}_{\alpha_{!A}}(x * y)\}$

(vi) Now $\nu^{N}_{\alpha_{?\overline{A}}}(y * z) = \mu^{N}_{\alpha_{?\overline{A}}}(y * z)$

$$= \max\left\{\frac{-1}{2}, \mu^N_{\alpha_{\overline{A}}}(y * z)\right\}$$

$$= \max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(y * z)\right\}$$

$$\geq \max\left\{\frac{-1}{2}, \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}\right\}$$

$$= \min\left\{\max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(x * z)\right\}, \max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(x * y)\right\}\right\}$$

$$= \min\{\nu^N_{\alpha_{!A}}(x * z), \nu^N_{\alpha_{!A}}(x * y)\}$$

Therefore $\nu^N_{\alpha_{\overline{?\overline{A}}}}(y * z) \geq \min\{\nu^N_{\alpha_{!A}}(x * z), \nu^N_{\alpha_{!A}}(x * y)\}$

Therefore $\overline{?\overline{A}} = !A$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem 6. *If A is a bipolar intuitionistic fuzzy α-ideal of X, then $!(?A) = ?(!A)$ is a bipolar intuitionistic fuzzy α-ideal of X.*

Proof. Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider $0, x, y, z \in A$.

(i) Now $\mu^P_{\alpha_{!(?A)}}(0) = \max\left\{\frac{1}{2}, \mu^P_{\alpha_{?A}}(0)\right\}$

$$= \max\left\{\frac{1}{2}, \min\left\{\frac{1}{2}, \mu^P_{\alpha_A}(0)\right\}\right\}$$

$$= \min\left\{\frac{1}{2}, \max\left\{\frac{1}{2}, \mu^P_{\alpha_A}(0)\right\}\right\}$$

$$\geq \min\left\{\frac{1}{2}, \max\left\{\frac{1}{2}, \mu^P_{\alpha_A}(x)\right\}\right\}$$

$$= \min\left\{\frac{1}{2}, \mu^P_{\alpha_{!A}}(x)\right\}$$

$$= \mu^P_{\alpha_{?(!A)}}(x)$$

Therefore $\mu^{P}_{\alpha_{!(?A)}}(0) \geq \mu^{P}_{\alpha_{?(!A)}}(x)$

Now $\mu^{N}_{\alpha_{!(?A)}}(0) = \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_{?A}}(0)\right\}$

$$= \min\left\{\frac{-1}{2}, \max\left\{\frac{-1}{2}, \mu^{N}_{\alpha_A}(0)\right\}\right\}$$

$$= \max\left\{\frac{-1}{2}, \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_A}(0)\right\}\right\}$$

$$\leq \max\left\{\frac{-1}{2}, \min\left\{\frac{-1}{2}, \mu^{N}_{\alpha_A}(x)\right\}\right\}$$

$$= \max\left\{\frac{-1}{2}, \mu^{N}_{\alpha_{!A}}(x)\right\}$$

$$= \mu^{N}_{\alpha_{?(!A)}}(x)$$

Therefore $\mu^{N}_{\alpha_{!(?A)}}(0) \leq \mu^{N}_{\alpha_{?(!A)}}(x)$

(ii) Now $\mu^{P}_{\alpha_{!(?A)}}(y * z) = \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_{?A}}(y * z)\right\}$

$$= \max\left\{\frac{1}{2}, \min\left\{\frac{1}{2}, \mu^{P}_{\alpha_A}(y * z)\right\}\right\}$$

$$= \min\left\{\frac{1}{2}, \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_A}(y * z)\right\}\right\}$$

$$\geq \min\left\{\frac{1}{2}, \max\left\{\frac{1}{2}, \min\{\mu^{P}_{\alpha_A}(x * z), \mu^{P}_{\alpha_A}(x * y)\}\right\}\right\}$$

$$= \min\left\{\frac{1}{2}, \min\left\{\max\left\{\frac{1}{2}, \mu^{P}_{\alpha_A}(x * z)\right\}, \max\left\{\frac{1}{2}, \mu^{P}_{\alpha_A}(x * y)\right\}\right\}\right\}$$

$$= \min\left\{\frac{1}{2}, \min\{\mu^{P}_{\alpha_{!A}}(x * z), \mu^{P}_{\alpha_{!A}}(x * y)\}\right\}$$

$$= \min\left\{\min\left\{\frac{1}{2}, \mu^{P}_{\alpha_{!A}}(x * z)\right\}, \min\left\{\frac{1}{2}, \mu^{P}_{\alpha_{!A}}(x * y)\right\}\right\}$$

$$= \min\{\mu^P_{\alpha_{?(!A)}}(x * z),\ \mu^P_{\alpha_{?(!A)}}(x * y)\}$$

Therefore $\mu^P_{\alpha_{!(?A)}}(y * z) \geq \min\{\mu^P_{\alpha_{?(!A)}}(x * z),\ \mu^P_{\alpha_{?(!A)}}(x * y)\}$

(iii) Now $\mu^N_{\alpha_{!(?A)}}(y * z) = \min\left\{\frac{-1}{2},\ \mu^N_{\alpha_{?A}}(y * z)\right\}$

$$= \min\left\{\frac{-1}{2},\ \max\left\{\frac{-1}{2},\ \mu^N_{\alpha_A}(y * z)\right\}\right\}$$

$$= \max\left\{\frac{-1}{2},\ \min\left\{\frac{-1}{2},\ \mu^N_{\alpha_A}(y * z)\right\}\right\}$$

$$\leq \max\left\{\frac{-1}{2},\ \min\left\{\frac{-1}{2},\ \max\{\mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_A}(x * y)\}\right\}\right\}$$

$$= \max\left\{\frac{-1}{2},\ \max\left\{\min\left\{\frac{-1}{2},\ \mu^N_{\alpha_A}(x * z)\right\},\ \min\left\{\frac{-1}{2},\ \mu^N_{\alpha_A}(x * y)\right\}\right\}\right\}$$

$$= \max\left\{\frac{-1}{2},\ \max\{\mu^N_{\alpha_{!A}}(x * z),\ \mu^N_{\alpha_{!A}}(x * y)\}\right\}$$

$$= \max\left\{\max\left\{\frac{-1}{2},\ \mu^N_{\alpha_{!A}}(x * z)\right\},\ \max\left\{\frac{-1}{2},\ \mu^N_{\alpha_{!A}}(x * y)\right\}\right\}$$

$$= \max\{\mu^N_{\alpha_{?(!A)}}(x * z),\ \mu^N_{\alpha_{?(!A)}}(x * y)\}$$

Therefore $\mu^N_{\alpha_{!(?A)}}(y * z) \leq \max\{\mu^N_{\alpha_{?(!A)}}(x * z),\ \mu^N_{\alpha_{?(!A)}}(x * y)\}$

(iv) Now $\nu^P_{\alpha_{!(?A)}}(0) = \min\left\{\frac{1}{2},\ \nu^P_{\alpha_{?A}}(0)\right\}$

$$= \min\left\{\frac{1}{2},\ \max\left\{\frac{1}{2},\ \nu^P_{\alpha_A}(0)\right\}\right\}$$

$$= \max\left\{\frac{1}{2},\ \min\left\{\frac{1}{2},\ \nu^P_{\alpha_A}(0)\right\}\right\}$$

$$\leq \max\left\{\frac{1}{2},\ \min\left\{\frac{1}{2},\ \nu^P_{\alpha_A}(x)\right\}\right\}$$

$$= \max\left\{\frac{1}{2}, \nu^P_{\alpha_{!A}}(x)\right\}$$

$$= \nu^P_{\alpha_{?(!A)}}(x)$$

Therefore $\nu^P_{\alpha_{!(?A)}}(0) \leq \nu^P_{\alpha_{?(!A)}}(x)$

Now $\nu^N_{\alpha_{!(?A)}}(0) = \max\left\{\frac{-1}{2}, \nu^N_{\alpha_{?A}}(0)\right\}$

$$= \max\left\{\frac{-1}{2}, \min\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(0)\right\}\right\}$$

$$= \min\left\{\frac{-1}{2}, \max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(0)\right\}\right\}$$

$$\geq \min\left\{\frac{-1}{2}, \max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(x)\right\}\right\}$$

$$= \min\left\{\frac{-1}{2}, \nu^N_{\alpha_{!A}}(x)\right\}$$

$$= \nu^N_{\alpha_{?(!A)}}(x)$$

Therefore $\nu^N_{\alpha_{!(?A)}}(0) \geq \nu^N_{\alpha_{?(!A)}}(x)$

(v) Now $\nu^P_{\alpha_{!(?A)}}(y * z) = \min\left\{\frac{1}{2}, \nu^P_{\alpha_{?A}}(y * z)\right\}$

$$= \min\left\{\frac{1}{2}, \max\left\{\frac{1}{2}, \nu^P_{\alpha_A}(y * z)\right\}\right\}$$

$$= \max\left\{\frac{1}{2}, \min\left\{\frac{1}{2}, \nu^P_{\alpha_A}(y * z)\right\}\right\}$$

$$\leq \max\left\{\frac{1}{2}, \min\left\{\frac{1}{2}, \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}\right\}\right\}$$

$$= \max\left\{\frac{1}{2}, \max\left\{\min\left\{\frac{1}{2}, \nu^P_{\alpha_A}(x * z)\right\}, \min\left\{\frac{1}{2}, \nu^P_{\alpha_A}(x * y)\right\}\right\}\right\}$$

$$= \max\left\{\frac{1}{2}, \max\{\nu^P_{\alpha_{!A}}(x*z), \nu^P_{\alpha_{!A}}(x*y)\}\right\}$$

$$= \max\left\{\max\left\{\frac{1}{2}, \nu^P_{\alpha_{!A}}(x*z)\right\}, \max\left\{\frac{1}{2}, \nu^P_{\alpha_{!A}}(x*y)\right\}\right\}$$

$$= \max\{\nu^P_{\alpha_{?(!A)}}(x*z), \nu^P_{\alpha_{?(!A)}}(x*y)\}$$

Therefore $\nu^P_{\alpha_{!(?A)}}(y*z) \le \max\{\nu^P_{\alpha_{?(!A)}}(x*z), \nu^P_{\alpha_{?(!A)}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{!(?A)}}(y*z) = \max\left\{\frac{-1}{2}, \nu^N_{\alpha_{?A}}(y*z)\right\}$

$$= \max\left\{\frac{-1}{2}, \min\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(y*z)\right\}\right\}$$

$$= \min\left\{\frac{-1}{2}, \max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(y*z)\right\}\right\}$$

$$\ge \min\left\{\frac{-1}{2}, \max\left\{\frac{-1}{2}, \min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_A}(x*y)\}\right\}\right\}$$

$$= \min\left\{\frac{-1}{2}, \min\left\{\max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(x*z)\right\}, \max\left\{\frac{-1}{2}, \nu^N_{\alpha_A}(x*y)\right\}\right\}\right\}$$

$$= \min\left\{\frac{-1}{2}, \min\{\nu^N_{\alpha_{!A}}(x*z), \nu^N_{\alpha_{!A}}(x*y)\}\right\}$$

$$= \min\left\{\min\left\{\frac{-1}{2}, \nu^N_{\alpha_{!A}}(x*z)\right\}, \min\left\{\frac{-1}{2}, \nu^N_{\alpha_{!A}}(x*y)\right\}\right\}$$

$$= \min\{\nu^N_{\alpha_{?(!A)}}(x*z), \nu^N_{\alpha_{?(!A)}}(x*y)\}$$

Therefore $\nu^N_{\alpha_{!(?A)}}(y*z) \ge \min\{\nu^N_{\alpha_{?(!A)}}(x*z), \nu^N_{\alpha_{?(!A)}}(x*y)\}$

Therefore $!(?A) = ?(!A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem 7. *If A is a bipolar intuitionistic fuzzy α-ideal of X, then $!(\square A) = \square(!A)$ is a bipolar intuitionistic fuzzy α-ideal of X.*

Theorem 8. *If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\square(?A) = ?(\square A)$ is a bipolar intuitionistic fuzzy α-ideal of X.*

Theorem 9. *If A is a bipolar intuitionistic fuzzy* α*-ideal of X, then* $?(\Diamond A) = \Diamond(?\,A)$ *is a bipolar intuitionistic fuzzy* α*-ideal of X.*

Theorem 10. *If A is a bipolar intuitionistic fuzzy* α*-ideal of X, then* $!\,(\Diamond A) = \Diamond(!\,A)$ *is a bipolar intuitionistic fuzzy* α*-ideal of X.*

Theorem 11. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $!\,A$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 12. *If A and B are bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $!\,(A \cap B) = !\,A \cap !\,B\,!$ *is also a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 13. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $?\,A$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 14. *If A and B are bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $?(A \cap B) = ?\,A \cap ?\,B$ *is also a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 15. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $\overline{?\,\overline{A}} = !\,A$ *is also a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 16. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $!\,(?\,A) = ?(!\,A)$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 17. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $!\,(\Box A) = \Box(!\,A)$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 18. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $\Box(?\,A) = ?(\Box \mathrm{A})$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 19. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $?(\Diamond A) = \Diamond(?\,A)$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

Theorem 20. *If A is a bipolar intuitionistic anti fuzzy* α*-ideal of X, then* $!\,(\Diamond A) = \Diamond(!\,A)$ *is a bipolar intuitionistic anti fuzzy* α*-ideal of X.*

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