Advances and Applications in Mathematical Sciences
Volume 20, Issue 4, February 2021, Pages 541-548

D-ECCENTRIC DOMINATION IN GRAPHS

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Abstract

In a dominating set $D \subseteq V$ of a graph $G(V, E)$ if there exists at least one *D*-eccentric vertex *u* of *v* in *D* for every $v \in V - D$ then it is called a *D*-eccentric dominating set. In this article, the *D*-eccentric dominating set, minimal *D*-eccentric dominating set and *D*-eccentric domination number (γ_{ed}^{D}) in graphs are determined. The *D*-eccentric domination numbers for some standard graphs are established. Some theorems related to *D*-eccentric domination in graphs are declared and verified.

1. Introduction

In 1962 O. Ore proposed a new idea dominating set and domination number [9]. In 1998 T. W. Haynes et al., deliberated various dominating parameters [5]. In 2010 T. N. Janakiraman et al., illustrated eccentric domination in graphs [6]. In 2011 M. Bhanumathi et al., detailed eccentric domination in trees and various bounds of eccentric domination in graph [1]. In 2013 L. N. Varma et al., determined *D*-Distance in graphs [10]. In 2019 A. Mohamed Ismayil et al., developed Detour eccentric domination in

2010 Mathematics Subject Classification: 05C12, 05C69.
Keywords: *D*-eccentric vertex; *D*-eccentric vertex set; *D*-eccentric dominating set; *D*-eccentric domination number.
Received January 10, 2020; Accepted May 13, 2020

graphs [8]. Article [8, 10] inspired us to consider the *D*-eccentric domination in graphs.

2. Preliminaries

Definition 2.1 [10]**.** The *D*-length of a $r - s$ path t is defined as $l^D(t) = d(r, s) + \deg(r) + \deg(s) + \sum \deg(w)$ where sum runs over all intermediate vertices w of t. The *D*-distance $d^D(r, s) = \min\{l^D(t)\}$, where the minimum is taken over all $r - s$ paths in G.

Definition 2.2 [10]**.** The *D*-radius, defined and denoted by $r^D(G) = \min\{e^D(s) : s \in V\}$. The *D*-diameter, defined and denoted by $d^D(G) = \max\{e^D(s) : s \in V\}$.

Definition 2.3. For a vertex s, each vertex at a *D*-distance $e^D(s)$ from s is a *D*-eccentric vertex of s. *D*-eccentric set of a vertex s is defined as $E^D(s) = \{r \in V / d^D(s) = e^D(s)\}$ or any vertex r for which $d^D(r, s) = e^D(s)$ is called *D*-eccentric vertex of s.

Definition 2.4. The *D*-eccentricity of a vertex s is defined by $e^D(s) = \max\{d^D(r, s) / r \in V\}$.

Definition 2.5. The vertex s in G is a *D*-central vertex if $r^D(G) = e^D(s)$ and the *D*-center $C^D(G)$ is the set of all central vertices.

Definition 2.6. The *D*-peripheral of G, $p^D(G) = e^{Dd}(G)$. V is a *D*-peripheral vertex if $e^D(s) = d^D(G)$. The *D*-periphery $P^D(G)$ is the set of all peripheral vertices.

Definition 2.7. A sub graph that has the same vertex set as G is called linear factor the degree of all vertices is one.

In this paper, as it were nontrivial basic associated undirected graphs are considered and for all the other vague terms one can allude [2, 3].

3. *D*-Eccentric Dominating Set

Definition 3.1. Let $P \subseteq V(G)$ be a set of vertices in a graph $G, (V, E)$. Then P is said to be a D-eccentric vertex set of G if for every vertex $s \in V - P$ has at least one vertex r such that $r \in E^D(s)$. A D-eccentric vertex set P of G is called minimal D-eccentric vertex set. If no proper subset P' of P is a D-eccentric vertex set of G. The minimum cardinality of a minimal D-eccentric vertex set of P is called the D-eccentric number and is denoted by $e^D(G)$ and simply denoted by e^D. The maximum cardinality of a minimal D-eccentric vertex set is called the upper D-eccentric number and is denoted by $E^D(G)$ and simply denoted by E^D.

Example 3.1. The D-eccentric vertex set and its numbers are defined in a graph $G(V, E)$ with suitable example as given below

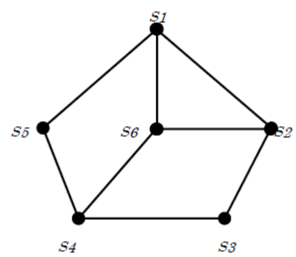

Figure 3.1

In a graph $G(V, E)$ as given figure 3.1, the D-eccentricity of s_1, s_2, s_3, s_4, s_5 and s_6 are respectively $e^D(s_1) = e^D(s_2) = e^D(s_3) = e^D(s_4) = e^D(s_5) = e^D(s_6) = 10$ and the D-eccentric set of s_1, s_2, s_3, s_4, s_5 and s_6 are $E^D(s_1) = \{s_3, s_4\}, E^D(s_2) = \{s_4, s_5\}$ $E^D(s_3) = \{s_1, s_5\}, E^D(s_4) = \{s_1, s_2\}, E^D(s_5) = \{s_2, s_6\}, E^D(s_6) = \{s_3, s_5\}$ respectively. Then the sets $P_1 = \{s_4, s_5\}, P_2 = \{s_1, s_2, s_6\}$ etc., are some D-eccentric vertex sets of $G(V, E)$ and D-eccentric number $e^D = 2$ and upper D-eccentric number $E^D = 3$.

Note: 3.1. r is a D-eccentric vertex of s, then $r \in E^D(s)$.

Observations 3.1.

(1) Every superset of a D-eccentric set is a D-eccentric vertex set.

(2) The subset of a D-eccentric vertex set need not be a D-eccentric vertex set.

(3) In a graph $G(V, E)$, $e^D(G) \leq E^D(G)$.

Definition 3.2. A dominating set $D \subseteq V$ of a graph $G(V, E)$ is said to be a D-eccentric dominating set if for every vertex $s \in V - D$, there exists at least one D-eccentric vertex r of s in D. A D-eccentric dominating set D is a minimal D-eccentric dominating set if there exists a subset $D' \subset D$ which is not a D-eccentric dominating set. The minimum cardinality of a minimal D-eccentric domination set of D is called the D-eccentric domination number and is denoted by γ_{ed}^D. The maximum cardinality of a minimal D-eccentric dominating set of D is called the upper D-eccentric dominating set and is denoted by $\Gamma_{ed}^D(G)$.

Remark 3.1. If P be a minimum D-eccentric vertex set of G then $D \cup S$ is a D-eccentric dominating set of G.

Example 3.2. The D-eccentric dominating set and its numbers are defined in a graph $G(V, E)$ with suitable example as given below

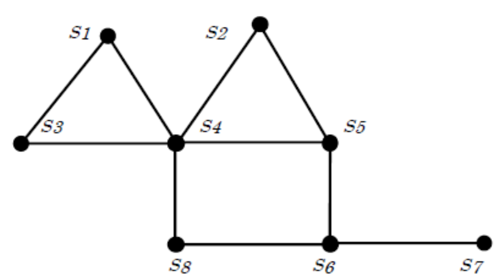

Figure 3.2

In this graph, $E^D(s_1) = \{s_8\}$, $E^D(s_2) = \{s_8\}$, $E^D(s_3) = \{s_8\}$ $E^D(s_4) = \{s_8\}$, $E^D(s_5) = \{s_1, s_3\}$, $E^D(s_6) = \{s_1, s_2, s_3\}$, $E^D(s_7) = \{s_1, s_3\}$, and $E^D(s_8) = \{s_1, s_3\}$. Here $P_1 = \{s_3, s_8\}$, $P_2 = \{s_1, s_8\}$ etc., are some *D*-eccentric vertex sets and $D_1 = \{s_3, s_4, s_8\}$, $D_2 = \{s_1, s_2, s_6, s_8\}$, etc., are some *D*-eccentric dominating sets. The *D*-eccentric domination number is $\gamma_{ed}^D = 3$ and upper *D*-eccentric domination number is $\Gamma_{ed}^D = 4$.

Results 3.1. (i) For any connected graph G, $\gamma(G) \leq \gamma_{ed}^D(G) \leq \Gamma_{ed}^D$.

(ii) Every *D*-eccentric dominating set is a dominating set but the converse is not true.

(iii) If $r^D(G) = d^D(G)$, then $\gamma(G) = \gamma_{ed}^D(G)$.

Observation 3.2. For any connected graph, $\gamma_{ed}^D(G) \leq \gamma(G) + e^D(G)$.

Observation 3.3. If G is disconnected then $\gamma(G) = \gamma_{ed}^D(G)$, since vertices from different components are *D*-eccentric to each other and if G is disconnected graph and r, s are in different components then $d^D(r, s) = \infty$.

Observation 3.4. For any graph $1 \leq \gamma_{ed}^D(G) \leq n$. The bounds are sharp, since $\gamma_{ed}^D(G) = 1$ iff $G = K_n$ and $\gamma_{ed}^D(G) = n$ iff $G = \overline{K_n}$.

4. Bound on D-Eccentric Domination

Observations 4.1

(i) $\gamma_{ed}^D(K_n) = 1$

(ii) $\gamma_{ed}^D(K_{1,n}) = 2$, $n \geq 2$

(iii) $\gamma_{ed}^D(K_{m,n}) = 2$.

(iv)

$$\gamma_{ed}^D(C_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor, & \text{for } n \geq 3 \text{ and } n \neq 5 \text{ where } \lfloor n \rfloor \text{ is a greatest integer less than } n. \\ 3, & n = 5. \end{cases}$$

Theorem 4.1.

$$\gamma_{ed}^{D}(W_n) = \begin{cases} 1, & for\ n = 3. \\ \left\lceil \frac{n}{2} \right\rceil, & for\ n = 4, 5, where\ \lceil n \rceil\ is\ a\ least\ integer\ greatest\ than\ n. \\ \left\lfloor \frac{n}{2} \right\rfloor, & for\ n \geq 6,\ where\ \lfloor n \rfloor\ is\ a\ greatest\ integer\ less\ than\ n. \end{cases}$$

Proof. $G = W_3 = K_4$. Hence $\gamma_{ed}^{D}(W_3) = 1$. When $G = W_4$, consider $D = \{r, s\}$, where r and s are any two adjacent non central vertices. D is a minimum D-eccentric dominating set. Therefore $\gamma_{ed}^{D}(W_4) = 2$. In a graph $G = W_5$, $D = \{r, s, w\}$, where r and s are any two adjacent non central vertices and w is the central vertex. If $G = W_{n,n} \geq 6\ D = \{r, s, w\}$, when r and s are any two adjacent vertices of w is a central vertex, then D is a minimum D-eccentric dominating set of G. Therefore $\gamma_{ed}^{D}(W_n) = \left\lfloor \frac{n}{2} \right\rfloor$ for $n \geq 6$.

Theorem 4.2. *If the graph* K_n *by deleting edges of a linear factor then* $\gamma_{ed}^{D}(G) = \frac{n}{2}$ $(n =$ *even integer*).

Proof. Let G be graph create from a non-trivial K_n has minimum two components. By the result 3.4, $\gamma(D) = \gamma_{ed}^{D}(G)$. Therefore $\gamma(G) = \frac{n}{2}$ where G has an even number of vertices. That is $\gamma(G) = \gamma_{ed}^{D}(G) = \frac{n}{2}$.

Proposition 4.1. *The domination number of path with four vertices is equal to the D-eccentric domination number of path with four vertices.*

Theorem 4.3. *In a path* (P_n) *of order* $n > 2$, $m = 1, 2, \ldots, \frac{n-2}{3}$

$$\gamma_{ed}^{D}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1, & if\ n = 3m, \\ \left\lceil \frac{n}{3} \right\rceil, & if\ n = 3m + 1, \\ \left\lceil \frac{n}{3} \right\rceil + 1, & if\ n = 3m + 2. \end{cases}$$

Proof. Case (i) $n = 3m$.

Let $s_1, s_2, s_3, \ldots, s_{3m}$ represent the path P_n and has all the peripheral vertices. $D = \{s_2, s_5, s_8, \ldots, s_{3m-1}\}$ is the only γ-set of P_n but not $\gamma_{ed}^{D}(P_n)$. That is $\gamma_{ed}^{D}(P_n)$ is $D' = \{s_1, s_4, s_7, \ldots, s_{3m}\}$ where $|D'| = m + 1 = \gamma(P_n) + 1$. Therefore, $\gamma_{ed}^{D}(P_{3m}). + 1 = \left\lceil \frac{n}{3} \right\rceil + 1$.

Case (ii) $n = 3m + 2$.

$D = \{s_1, s_4, s_7, \ldots, s_{3m+2}, s_{3m+1}\}$ is the least dominating set P_n has two peripheral vertices. Hence, $\gamma_D(P_n) = \gamma_{ed}^{D}(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Case (iii) $n = 3m + 2$.

$D = \{s_2, s_5, s_8, \ldots, s_{3m+2}\}$ has end vertices s_{3m+2} and it is not a *D*-eccentric dominating set. Hence, $D \cup \{s_1\}$ is a minimum *D*-eccentric dominating set. Therefore $\gamma_{ed}^{D}(P_n) = \gamma_D(P_n) + 1 = \left\lceil \frac{n}{3} \right\rceil + 1$.

Remark 4.1. In a path (P_n) of order $n = 2$, $\gamma_{ed}^{D}(P_n) = 1$.

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