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SOLVING FUZZY ASSIGNMENT PROBLEM USING NUMERICAL QUADRATURE FORMULA

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Abstract

Assignment Problem is a special type of transportation problem which deals with the allocation of various resources to an equal number of activities to minimize the total cost or to maximize the profit and used to solve real valued problems. In this paper, the values of the Fuzzy Assignment Problem are considered as Heptagonal fuzzy numbers. First the fuzzy numbers are converted into crisp values using Simpson's three eighth rules. Then the optimum schedule of the Fuzzy Assignment Problem is obtained by usual Hungarian Method. This approach is illustrated by a numerical example.

1. Introduction

The assignment problem (AP) is a special type of linear programming problem in which our objective is to assign various number of jobs to an equal number of persons at a minimum cost. All the algorithms developed to find the optimal solution of Transportation problem are applicable to assignment problem. Kuhn [1] proposed a special algorithm named Hungarian algorithm for finding the optimal solution due to highly degenerate in nature.

In real life situations, time or cost for doing a job by a person or machine

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might vary due to different reasons so that the parameters of Assignment Problem are vague numbers instead of fixed real numbers. In 1965 Fuzzy sets were introduced by Lotfi A. Zadeh [2] which deals with imprecision in real life situations. Chen [3] proposed a fuzzy assignment model that did not consider the differences of individuals and also proved some theorems. Dubois and Fortemphs [4] proposed a flexible algorithm which combines with fuzzy theory, multiple criteria decision making and constraint-directed method. Based on the labelling method for solving the fuzzy assignment problem, the effective algorithm was proposed by Lin and wen [5].

Yuan Feng and Yang [6] investigated a two-objective k-cardinality assignment problem. Liu and Gao [7] proposed an equilibrium optimization problem and extended the assignment problem to the equilibrium multi job assignment problem. K. Rathi and S. Balmohan [8] introduced non-normal heptagonal fuzzy numbers using value and ambiguity index. In this paper the fuzzy assignment problem has been converted into crisp assignment problem using Simpson's three eighth rules and then Hungarian Algorithm has been applied to find an optimal solution.

2. Preliminaries

Definition 2.1 (Fuzzy Set). The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within the specified range i.e. $\mu_{\tilde{A}} : X \to [0, 1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2. A Fuzzy set $\tilde{A}$ defined on universal of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq$ minimum $(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ $\forall x_1, x_2 \in R$ and $0 \leq \lambda \leq 1$.

(ii) $\widetilde{A}$ is normal, there exists $x \in X$ such that $\mu_{\widetilde{A}}(x) = 1$.

(iii) $\mu_{\widetilde{A}}$ is piecewise continuous.

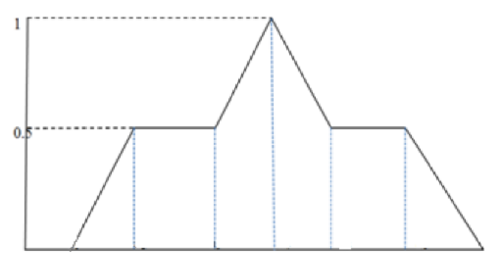
Definition 2.3. A fuzzy number $\widetilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\widetilde{A}}(x) = 0$ for all $x < 0$.

Definition 2.4. A fuzzy number $\widetilde{A} = (a_0, a_1, a_2, a_3, a_4, a_5, a_6)$ is said to be a Heptagonal fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - a_0}{2(a_1 - a_0)} & a_0 \le x \le a_1 \\ \dfrac{1}{2} & a_1 \le x \le a_2 \\ \dfrac{x - a_3}{2(a_3 - a_2)} + 1 & a_2 \le x \le a_3 \\ \dfrac{a_3 - x}{2(a_4 - a_3)} + 1 & a_3 \le x \le a_4 \\ \dfrac{1}{2} & a_4 \le x \le a_5 \\ \dfrac{a_6 - x}{2(a_6 - a_5)} & a_5 \le x \le a_6 \\ 0 & \text{otherwise} \end{cases}$$

where $a_0, a_1, a_2, a_3, a_4, a_5, a_6 \in R$.

Graphical representation of Heptagonal Fuzzy number

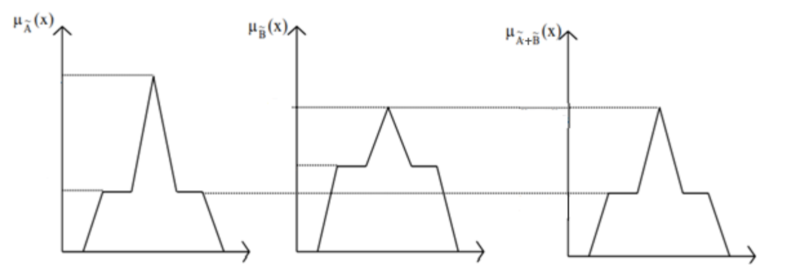


$a_0, a_1, a_2, a_3, a_4, a_5, a_6$

Definition 2.5. The heptagonal fuzzy number $\widetilde{A} = (a_0, a_1, a_2, a_3, a_4, a_5, a_6)$ is said to be symmetric if the product of the difference of two set (a_0, a_1, a_2, a_3) and (a_3, a_4, a_5, a_6) are equal. (i.e.) $| a_0 - a_1 | \cdot | a_1 - a_2 | \cdot | a_2 - a_3 | = | a_3 - a_4 | \cdot | a_4 - a_5 | \cdot | a_5 - a_6 |$.

Definition 2.6. Let $\widetilde{A} = (a_0, a_1, a_2, a_3, a_4, a_5, a_6)$ and $\widetilde{B} = (b_0, b_1, b_2, b_3, b_4, b_5, b_6)$ be two heptagonal fuzzy numbers. Then the addition of two HFNS is given by

$$\widetilde{A} + \widetilde{B} = (c_0, c_1, c_2, c_3, c_4, c_5, c_6) = (a_0 + b_0, a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6).$$



$a_0 a_1 a_2 a_3 a_4 a_5 a_6$ $b_0 b_1 b_2 b_3 b_4 b_5 b_6$ $c_0 c_1 c_2 c_3 c_4 c_5 c_6$.

2.6 Defuzzification.

Defuzzification is the process of finding singleton value (crisp value) from the output of the aggregated fuzzy set. Here Simpson's three eighth rule is used to defuzzify the heptagonal fuzzy number because of its simplicity and accuracy.

3.1. Assignment Problem

The assignment problem can be stated in the form of $n \times n$ cost matrix $[C_{ij}]$ of real numbers as given in the following table:

Jobs→Persons↓	1	2	3	j	n
1	C_{11}	C_{12}	C_{13}	C_{1j}	C_{1n}
2	C_{21}	C_{22}	C_{23}	C_{2j}	C_{2n}

–	–	–	–	–	–
i	C_{i1}	C_{i2}	C_{i3}	C_{ij}	C_{in}
–	–	–	–	–	–
n	C_{n1}	C_{n2}	C_{n3}	C_{nj}	C_n

Mathematically assignment problem can be stated as

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}.$$

$$\text{Subject to } \sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, 3, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad j = 1, 2, 3, \ldots, n.$$

$$\text{Where } x_{ij} = \begin{cases} 1 \text{ if the } i^{th} \text{person assigned the } j^{th} job \\ \quad 0 \text{ othewrwise} \end{cases}$$

and C_{ij} represents the cost of assignment of person i to the job j.

When the costs C_{ij} are fuzzy numbers then the total cost becomes a fuzzy number. Then the fuzzy objective function is

$$\text{Minimize } \bar{z} = \sum_{i=1}^{n} \sum_{i=1}^{n} \bar{C}_{ij} x_{ij}.$$

Where $\bar{C}_{ij} = (a_0, a_1, a_2, a_3, a_4, a_5, a_6)$, the heptagonal fuzzy numbers. It cannot be minimized directly. We defuzzify the fuzzy cost coefficients into crisp ones by our proposed method.

4. Algorithms

4.1. Hungarian Assignment Algorithm

Check whether the assignment problem is balanced or not. If not add dummy row or dummy column with cost value 0 and make it as a balanced one.

In each row subtract the smallest cost element from each element so that there will be at least one zero in each row. In the same way proceed in the column also.

Examine the rows successively until a row with exactly one zero is found. Circle the zero as an assigned cell and cross out all other zero in its column. Proceed in this manner until all the rows have been examined. If there are more than one zero in any row do not consider that row and pass on to the next row.

Repeat the procedure for the columns of the cost matrix. If there is no single zero in any row or column then arbitrarily choose a row or column having the minimum number of zeros. Repeat the steps 3 and 4 until all the zeros are either assigned or crossed out.

If the number of assigned cells equals the number of rows then there is an optimal assignment schedule. If not go to next step.

Draw the minimum number of horizontal or vertical lines through all the zeros as follows:

(i) Mark (√) to those rows where no assignment has been made.

(ii) Mark (√) to those columns which have zeros in the marked rows.

(iii) Mark (√) rows (not already marked) which have assignment in marked columns.

(iv) Draw straight lines through unmarked rows and marked columns.

If the number of lines is equal to the number of rows or columns. The optimum solution is attained by arbitrary allocation in the position of the zeros not crossed in step 3. If not go to the next Step.

Choose the smallest element from the uncrossed elements and subtract this element from them and add the same at the point of intersection of two lines. Other elements crossed by the lines remain unchanged.

Repeat the procedure till an optimum solution is attained.

4.2. Algorithm to solve fuzzy assignment problem

Step 1. First convert the cost values of the fuzzy Assignment Problem which are all in of fuzzy numbers into crisp value by using the following formula.

(i.e.) If $C_{ij} = (a_0, a_1, a_2, \ldots, a_n)$ then the crisp value of C_{ij} which is denoted by c_{ij} is defined by $C_{ij} = 3h/8[(a_0 + a_n) + 3(a_1 + a_2 + a_4 + a_5 + \ldots + a_{n-1}) + 2(a_3 + a_6 + \ldots + a_{n-3})]$ where $h =$ difference between first and last terms (i.e.) $h = a_n - a_0$.

Step 2. Check whether the assignment problem is balanced or not. If not add dummy row or dummy column with cost value 0 and make it as a balanced one.

Step 3. Using Hungarian method, find the optimum assignment schedule.

5. Numerical Example

To illustrate a fuzzy assignment problem whose elements are Heptogonal fuzzy numbers by using the proposed method taken from the paper 'Fuzzy assignment Problem Using Normalised Heptagonal Fuzzy numbers' by P. Selvam, A. Rajkumar and J. Sudha Easwari.

Let us consider a fuzzy assignment problem with rows representing three jobs J_1, J_2, J_3 and columns representing the three Machines M_1, M_2, M_3 and the cost matrix $(\overline{C}_{ij})$ is given whose elements are heptagonal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

	M_1	M_2	M_3
J_1	(1,4,6,8,11,14,16)	(8,9,10,13,15,17,18)	(6,9,10,12,13,14,15)
J_2	(11,14,16,18,22,24,26)	(9,13,15,18,20,22,23)	(15,18,20,22,24,27,28)
J_3	(7,10,12,14,16,18,20)	(11,12,15,17,19,20,21)	(10,12,14,15,18,20,22)

Solution:

The heptagonal fuzzy number $(a_0, a_1, a_2, a_3, a_4, a_5, a_6)$ is changed to a crisp one by applying Simpson's three eighth rules

$$C_{ij} = 3h/8[(a_0 + a_n) + 3(a_1 + a_2 + a_4 + a_5 + \ldots + a_{n-1}) + 2(a_3 + a_6 + \ldots + a_{n-3})]$$

where $h =$ difference between first and last terms (i.e.) $h = a_n - a_0$.

Here the formula becomes, $C_{ij} = 3h/8[(a_0 + a_6) + 3(a_1 + a_2 + a_4 + a_5) + 2(a_3)]$

$$C'_{11} = (1, 4, 6, 8, 11, 14, 16)' = 776.25, C'_{12} = (8, 9, 10, 13, 15, 17, 18)' = 768.75,$$

$$C'_{13} = (6, 9, 10, 12, 13, 14, 15)' = 617.23, C'_{21} = (11, 14, 16, 18, 22, 24, 26)' = 1693.13$$

$$C'_{22} = (9, 13, 15, 18, 20, 22, 23)' = 1459.5, C'_{23} = (15, 18, 20, 22, 24, 27, 28)' = 1725.75$$

$$C'_{31} = (7, 10, 12, 14, 16, 18, 20)' = 1087.13, C'_{32} = (11, 12, 15, 17, 19, 20, 21)' = 990$$

$$C'_{33} = (10, 12, 14, 15, 18, 20, 22)' = 1143.$$

The new cost table is

	M_1	M_2	M_3
J_1	776.25	768.75	617.63
J_2	1693.13	1459.5	1725.75
J_3	1087.13	990	1143

Using Hungarian Algorithm, the optimal allocations are

	M_1	M_2	M_3
J_1	158.62	248.25	**0**
J_2	136.5	**0**	169.12
J_3	**0**	0	153

The optimum schedule is $J_1 \to M_3$, $J_2 \to M_2$, $J_3 \to M_1$.

The optimum assignment cost is

$$(6,\ 9,\ 10,\ 12,\ 13,\ 14,\ 15) + (9,\ 13,\ 15,\ 18,\ 20,\ 22,\ 23) + (7,\ 10,\ 12,\ 14,\ 16,\ 18,\ 20)$$

$$= (22,\ 32,\ 37,\ 44,\ 49,\ 54,\ 58).$$

Comparing the assignment cost which has been found in the above example with assignment cost calculated by existing method is same but this is the easiest method to calculate.

6. Conclusion

In this paper the fuzzy costs of heptagonal fuzzy assignment problem have been defuzzified into crisp value by using our new formula and then solved by Hungarian method. We hope that this approach will be effective in fuzzy assignment problem involving imprecise data. Not only the heptagonal fuzzy numbers, we can use this method for the type of fuzzy numbers $(a_0, a_1, a_2, \ldots, a_n)$ where n is the multiple of 6.

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